Using Special Relativity (SR) as a starting point, then noting a few empirical 4-Vector facts, one can instead *derive* the Principles that are normally considered to be the Axioms of Quantum Mechanics (QM). Hence, [SR→QM]

Since many of the QM Axioms are rather obscure, this seems a far more logical and understandable paradigm than QM as a separate theory from SR, and sheds light on the origin and meaning of the QM Principles. For instance, the properties of SR can be “quantized by the Metric”, while SpaceTime & the Metric are not themselves “quantized”, in agreement with all known experiments and observations to-date.

The SRQM or [SR→QM] Interpretation of Quantum Mechanics

A Tensor Study of Physical 4-Vectors

or: Why General Relativity (GR) is *NOT* wrong
or: Don’t bet against Einstein ;)
or: QM, the easy way...

SRQM: A treatise of SR→QM by John B. Wilson (SciRealm@aol.com) version 2021-Jan-02 .7
4-Vectors = 4D (1,0)-Tensors are a fantastic language/tool for describing the physics of both relativistic and quantum phenomena. They easily show many interesting properties and relations of our Universe, and do so in a simple and concise mathematical way. Due to their tensorial nature, these 4-Vectors are automatically 4D SpaceTime coordinate-frame invariant, and can be used to generate “ALL” of the physical Lorentz Scalar (0,0)-Tensors and higher-rank Tensors of Special Relativity (SR).

Let me repeat: You can mathematically build “ALL” of the SR Lorentz Scalars and “larger” SR Tensors from empirically discovered SR 4-Vectors.

SR 4-Vectors are likewise easily shown to be related to the standard physics 3D vectors {3-vectors = 3D (1,0)-tensors} that are used in Newtonian Classical Mechanics (CM), Maxwellian Classical ElectroMagnetism (EM), and standard Quantum Mechanics (QM).

In addition, each and every SR 4-Vector also fundamentally connects a special-relativistically-related physical temporal scalar to a spatial 3-vector:

- Temporal time (t) & Spatial 3-position (x, y, z) as SR 4-Position R = Rμ = (ct, r)
- Temporal energy (E) & Spatial 3-momentum (p→(px, py, pz)) as SR 4-Momentum P = Pμ = (E/c, p)
- Temporal charge-density (ρ) & Spatial 3-current-density (j→(jx, jy, jz)) as SR 4-CurrentDensity J = Jμ = (ρc, j)

Why 4-Vectors and Tensors as opposed to some of the more abstract mathematical approaches to Quantum Mechanics?

Experiment is the ultimate arbiter of which theories actually correspond to reality. If your quantum logics and string theories give no testable/measurable predictions, and if your invented/hypothetical/wished-for particles can’t be detected, then they are basically useless for real, actual, empirical physics. The components of 4-Vectors are the physical properties of real particles that *CAN* actually be empirically detected/measured/tested, and Tensors are the well-known and well-established mathematical/physical objects which describe these concepts in an invariant, coordinate-independent way.

In this treatise, I will first extensively demonstrate how 4-Vectors are used in the context of Special Relativity (SR), and then show that their use in Relativistic Quantum Mechanics (RQM) is really not fundamentally different.

Quantum Principles, without need of QM Axioms, then emerge in a natural and elegant way.

SR is a theory of Measurement, even in QM.

I also introduce the SRQM (Physical) Diagramming Method: a highly instructive, graphical charting-method, which visually shows how the SRQM 4-Vectors, Lorentz 4-Scalars, and higher rank 4-Tensors are all related to each other. This symbolic representation clarifies a lot of physics and is a great tool for teaching and understanding.

SRQM: A treatise of SR → QM by John B. Wilson (SciRealm@aol.com)

Many concepts herein inspired by works of renowned Physicist & SR/GR Expert Wolfgang Rindler
GR = General Relativity  

SR = Special Relativity  

CM = Classical Mechanics  

QM = Quantum Mechanics  

RQM = Relativistic Quantum Mechanics  

EM = Electromagnetism/Electromagnetics

NRQM = Non-Relativistic Quantum Mechanics = (standard QM)

QFT = Quantum Field Theory = (multiple particle QM)

QED = Quantum Electrodynamics = QFT for (e^)’s & photons

RWE/QWE = Relativistic/Quantum Wave Equation

KG = Klein-Gordon (Relativistic Quantum) Equation: Relation

PDE = Partial Differential Equation

MCRF = Momentarily Co-Moving Reference: Rest Frame

EoS = Equation of State (Scalar Invariant) = \(w = p_\gamma/\rho_\delta\)

\(P_T\) = 4-TotalMomentum = \((H/c) = E/c, p_T\) = \(\Sigma_i\) All 4-Momenta \(P_i\)

\(H = \) the Hamiltonian = \(\gamma (P_T \cdot U)\)

\(L = \) the Lagrangian = \(\gamma (P_T \cdot U)/\gamma\)

\(V = \) vertical” Projection Tensor, also just to match this ordering convention of SpaceTime

SRQM = The [SR → QM] Interpretation of Quantum Mechanics, by John B. Wilson

In full, with each a subset of the former: [GR→SR→RQM→QM→(EM & CM)]
Some Physics: Mathematics
Conventions & Notation

SRQM

A Tensor Study of Physical 4-Vectors

Quantum Mechanics is derivable from Special Relativity

SRQM

Both are the SpaceTime-reversed situations of the other...
and equivalent under CPT symmetry.
Special Relativity → Quantum Mechanics
The SRQM Interpretation: Links

See also:
http://scirealm.org/SRQM.html (alt discussion)
http://scirealm.org/SRQM-RoadMap.html (main SRQM website)
http://scirealm.org/4Vectors.html (4-Vector study)
http://scirealm.org/SRQM-Tensors.html (Tensor & 4-Vector Calculator)
http://scirealm.org/SciCalculator.html (Complex-capable RPN Calculator)

or Google “SRQM”

http://scirealm.org/SRQM.pdf (this document: most current ver. at SciRealm.org)
SRQM Study: Physical / Mathematical Tensors

4D Tensor Types: 4-Scalar, 4-Vector, 4-Tensor

Component Types: Temporal, Spatial, Mixed

Matrix Format

SR 4-Scalar S

A Tensor Study of Physical 4-Vectors

SR 4-Vector V^μ
an "arrow": magnitude and 1 direction

SR 4-Tensor T^μν
a 'matrix' or dyadic: magnitude and 2 directions

SRQM Diagram Format

Each 4D index = \{0,1..3\} = Tensor Dimension 4

SRQM Diagram Ellipse:
4-Scalars, 0 index = rank 0
4'0 = 0 corners in diagram
4'0 = (1) = 1 component

SR 4-Vector
4D (1,0)-Tensor V = \vec{V}
uses a single upper index or upper bar
V^μ = (v^0, v^1, v^2, v^3)

SR 4-Scalar
4D (0,0)-Tensor S or S_0
Lorentz Scalar

SR 4-CoVector: One-Form

SR 4-CoVector = 4D One-Form
4D (0,1)-Tensor V
uses a single lower index or lower bar
C_\mu = \eta_{\mu\sigma}^C \sigma = (C_0, C_1, C_2)
= (c^0, -c) \rightarrow (c^-, c^+, c^0, c^-)

SR Mixed 4-Tensor 4D (1,1)-Tensor T^μν = η^μν T^ρτ

SR Mixed 4-Tensor 4D (1,1)-Tensor
T^ρτ = η^ρτ V^μ = V^μ \rightarrow

SR Lowered 4-Tensor 4D (0,2)-Tensor T^μν = η^μν V^σ = V^σ \rightarrow

SR: Minkowski Metric
\[ \delta[R] = \delta[R^a] = \eta^{\mu\nu} = V^\mu + H^\mu \rightarrow \]
Diag[+1,-1,-1,-1] = Diag[1,-1,-1,-1]

{in Cartesian form} "Particle Physics" Convention
\( \{\eta_{\mu\nu}\} = 1/\{\eta_{\mu\nu}\} : \eta_{\mu\nu} = \delta_\mu^\nu \)
Tr[\eta^{\mu\nu}] = 4

4-Gradient \( \partial_\mu \)

4-Position \( R^\mu = (ct,r) \rightarrow (Event) \)

SpaceTime
\( \partial_R = \partial_\mu R^\mu = 4 \) Dimension

Tensor Property:
Rank = # of indices
{0 = a Scalar}
{1 = a Vector}
{2 = a Dyadic/Matrix}
etc...

Dimension = # of values a tensor index can take
(SR Tensors = 4D) (Classical = 3D)

Trace[T^\mu\nu] = \eta^\mu_\mu T^\mu_\nu = T^\mu_\nu = T
V\cdot V = V^\mu V^\nu = (V^0)^2 - V^1 V^1 - V^2 V^2 - V^3 V^3
= (V^0)^2 = Lorentz Scalar Invariant

Technically, all these objects are "SR 4 Tensors", but we usually reserve
the name "4-Tensor" for 4D objects with 2 (or more) indices, and use
the (m,n)-Tensor notation to specify all the objects more precisely.
**A Tensor Study**

**Physics**

*most all cases except strong gravity*

**Regime**

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Classical</th>
<th>Relativistic</th>
<th>Euclidean</th>
<th>Minkowskian</th>
<th>Galilean</th>
<th>Lorentzian/Poincaré</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D</td>
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<td>Lorentzian/Poincaré</td>
</tr>
</tbody>
</table>

**Metric**

<table>
<thead>
<tr>
<th>Dimension</th>
<th>SR 4-Tensor</th>
<th>SR 4-Vector</th>
<th>SR 4-Vector</th>
<th>SR 4-Scalar</th>
<th>3-D Tensor</th>
<th>3-Vector</th>
<th>3-Scalar</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D</td>
<td>(2,0)-Tensor</td>
<td>(1,1)-Tensor</td>
<td>T_{ij} \rightarrow V_{ij} = (v_0, v_1)</td>
<td>(0,0)-Tensor</td>
<td>T_{ij} \rightarrow V_{ij} = (v_0, v_1)</td>
<td>(0,0)-Tensor</td>
<td>S_{ij} \rightarrow V_{ij} = (v_0, v_1)</td>
</tr>
<tr>
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<td>S_{ij} \rightarrow V_{ij} = (v_0, v_1)</td>
</tr>
</tbody>
</table>

**Transform: Invariance**

3D Classical: Euclidean Metric

\[ \nabla [r] = \nabla [r'] = E^k = - H^k \rightarrow \text{Kronecker delta } \delta_{ik} \]

Diag\[+1,+1,+1,+1] = \text{Diag}[1] = \text{Diag}[\delta_{ik}] \]

(in Cartesian form)

\{ \delta_{kk} \} = 1/\{ \delta_{kk} \} \rightarrow \text{Tr}[\delta_{kk}] = \delta_{11}^1 + \delta_{22}^2 + \delta_{33}^3 = 1 + 1 + 1 = 3 \]

3D Space

\[ \nabla \vec{r} = \nabla \vec{r}' = \vec{V} = (x) + \vec{v} = (r) \left[ \begin{array}{c} x \\ y \\ z \end{array} \right] (1 + 1 + 1) \]

Dimension

\[ 3D \text{ SR: Minkowski Metric} \]

\[ \delta[R] = \delta^2[R] = \eta_{\mu \nu} = \eta_{\mu \nu} = V_{\mu \nu} + H_{\mu \nu} \rightarrow \]

Diag\[+1,-1,-1,-1] = \text{Diag}[1, -1, -1, -1] = \text{Diag}[\delta_{ik}] \]

(in Cartesian form) "Particle Physics" Convention

\{ \eta_{\mu \nu} \} = 1/\{ \eta_{\mu \nu} \} \rightarrow \eta_{\nu \gamma} = \delta_{\nu \gamma} = \text{Diag}[1, 1, 1, 1] \]

\[ \text{Tr}[\eta_{\mu \nu}] = \eta_{00}^0 + \eta_{11}^1 + \eta_{22}^2 + \eta_{33}^3 = 1 + 1 + 1 = 4 \]

**Dimension**

- # of components required to specify a vector object, 3 for 3D, 4 for 4D

**Regime:**

- Classical (old = 1D time + 3D space)
- Relativistic (modern = 4D Time-Space)

**Metric:**

- How specific the “intervals” between events are measured, e.g. Diag\[1, -1, -1, -1\]

**Transform:**

- How coordinate reference-frames are related to each other

Invariant:

- Tensor value which doesn’t change with a change in the reference-frame: e.g. c, h, m

**Galilean Transformations**

- 1-time t
  - = (r/c) = (t)
  - = <time>

- 3-vector
  - = r i = (r i) → (x,y,z)
  - = (r i, r j, r k)

**Lorentz Transformations**

- 3-vector
  - = r i = (r i) → (x,y,z)
  - = (r i, r j, r k)

**Time-Space-Boots, Spatial-Rotations, (CPT)**

- 4-Position R
  - = R i = (r i) = (ct, r i) → (ct, x, y, z)
  - = (r i, r j, r k, r l)

- Event = <time> <location>

**Lorentz Invariant**

\[ R \cdot R = R^i R^i = \text{ct}^2 - r \cdot r = (ct)^2 \]

Interval ct = |R|

**3D & 4D Vector internal components labeled with superscript index, not an exponent**

Only scalars outside of a vector will have exponents, such as in the Lorentz Scalar Product
In Classical Mechanics (CM), an ex. of the magnitude of a 3-vector is the length $|\mathbf{r}|$ of a 3-displacement $\mathbf{r}=(x,y,z)$. The magnitude of a 3-vector is the length $r=|\mathbf{r}|$.

The magnitude of an SR 4-Vector is very similar to the magnitude of a 3-vector, but there are some interesting differences. One uses the Lorentz Scalar Product, a 4D Dot Product, which includes time & space components, and is based on the SR-Minkowski Metric Tensor. "Particle Physics" sign-convention ($\{\text{temporal},0,0\}$) gives the Minkowski Metric $\eta_{\mu\nu}$ with entries zero. Positive signs are used for 4D explicit expressions of SR 4-Vector Magnitudes.

Using **Einstein Summation Convention** which has upper-lower paired-indices summed over:

$$\mathbf{r} \cdot \mathbf{r} = r^2 = (ct)^2 + (x)^2 + (y)^2 + (z)^2 = r^2$$

The 3D magnitude $r = |\mathbf{r}|$ is the Pythagorean Theorem.

**3D Classical:**

$$\sqrt{r} = \sqrt{r^2} = E^\mu = -H^\mu \rightarrow \text{Kronecker delta} \delta_{jk}$$

The SR-Minkowski Metric $\eta_{\mu\nu}$ gives $\eta_{00} = 1$ (Cartesian form), with the entries zero. Note the 3D $\{\text{spatial},1\}$ part is negative.

Using Einstein Summation Convention which has upper-lower paired-indices summed over:

$$\mathbf{r} \cdot \mathbf{r} = (ct)^2 + (x)^2 + (y)^2 + (z)^2 = r^2$$

**4D SR-Minkowski Metric**

$$\delta_{\mu\nu} = \eta^\mu = \eta_{\mu\nu} = V^\mu + H_{\mu} \rightarrow Diag[+1,-1,-1,-1] = Diag[1,1,1,1]$$

for 4-Position $R=(ct,x,y,z)$

4D SpaceTime $\delta\mathbf{R} = \delta r^\mu \mathbf{R}^\nu = \delta_\mu^\nu \mathbf{R}^\nu = 4$

using **3D Classical:**

$$\sqrt{r^2} = \sqrt{(ct)^2 + (x)^2 + (y)^2 + (z)^2} = r$$

The 4D-Vector version has the Pythagorean elements in the **spatial** components, the **temporal** component is of opposite sign. This gives a “causality condition”, with SpaceTime intervals (in the $\{+1,1,1,1\}$ SR-Minkowski Metric) that can be:

$$\Delta x \cdot \Delta x = (\Delta t)^2 = (\Delta r)^2$$

**SR 4-Tensor**

$$\{\text{Time-like:Temporal} =)$$

$$\{) \text{causal = 1D temporally-ordered, spatially-relaxed}$$

$$\{\text{Space-like: Spatial} =)$$

$$\{) \text{causal & topological, maximum signal speed} (|\Delta \mathbf{r}|/|\Delta t|\text{=c})$$

Uses **Matrix Representation**, **4D or 3D**.

- **SR 4-Vector**: $V^\mu$ or $V_\nu$
- **SR 3-Vector**: $T^\mu$, $T_\nu$, or $T_{\nu\mu}$
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- **SR 3-Vector**: $T^\mu$, $T_\nu$, or $T_{\nu\mu}$

**3D**

$$\Delta x \cdot \Delta x = (\Delta r)^2$$

**SR 4-Vector**

$$\{\text{Light-like:Null} =)$$

$$\{) \text{causal & topological, maximum signal speed} (|\Delta \mathbf{r}|/|\Delta t|\text{=c})$$

$$\{) \text{temporally relative, topological = 3D spatially-ordered}$$

- **SR 4-Vector**: $V^\mu$ or $V_\nu$
- **SR 3-Vector**: $T^\mu$, $T_\nu$, or $T_{\nu\mu}$
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**3D**

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SR QM Study: SR Minkowski SpaceTime

Invariant Lorentz Scalar Product
Tensor Index Raising, Lowering, & Gymnastics

Both GR and SR use a metric tensor (gμν) to describe measurements in SpaceTime (Time-Space).
SR uses the “flat” Minkowski Metric gμν → ημν, → Diag[1,−1,−1,−1] = Diag[1,−1,−1,−1] [Causal family], which is the (curvature ~ 0 limit = low-mass limit) of the GR metric gμν. SR is valid everywhere except extreme gravity, ex. near BH’s.

4-Vectors are tensorial entities of Minkowski SpaceTime which maintain covariance for inertial observers, meaning that they may have different relativistic components for different observers, but describe the same physical object. (like viewing a sculpture from different angles – snapshot pictures look different, but it's actually the same object)
There are also (4-CoVectors = One-Forms = 4D (0,1)-Tensors) which are dual to (4-Vectors = 4D (1,0)-Tensors).

4-Vectors = 4D (1,0)-Tensors

A → A' = A'' = ημ'ν' Aμ = (a0,a1,a2,a3) = (a0,a1,a0,a0) → (a0,a0,a0,a0)
B → B' = B'' = ημ'ν' Bμ = (b0,b1,b2,b3) = (b0,b0,b0,b0)

4-CoVectors = 4D (0,1)-Tensors = DualVectors = One-Forms
Aα = Aα = ηαβ Aβ = (a0,a1,a2,a3) = (a0,a1,a2,a3) = (a0,a1,a2,a3)
Bβ = Bβ = ηαβ Bα = (b0,b1,b2,b3) = (b0,b1,b2,b3) = (b0,b1,b2,b3)

Index Raising & Lowering with SR: Lorentz Transform
ημν Aμ = ημν Aν = ηνμ Aμ = ηνμ Aν
4-Vectors = 4D (1,0)-Tensors
4-CoVectors = 4D (0,1)-Tensors

Invariant Lorentz Scalar Product (0,0)-Tensors

AμBν = AμBν = AμBν = ημρ ηνσ AρBσ = AρBσ = AρBσ
= Aνμ Bν Λμν Λνσ Λρσ Λρσ
= (a0,b0,a0,b0) 
= (a0,a0,a0,a0)

Index Gymnastics = Only Single Upper-Lower Pairs Allowed

Proof of invariance (using Tensor gymnastics and the properties of the Minkowski Metric η & Lorentz Transforms Λ):

Aμ'Bν' = Λμν Aμ
(Aμ'Bν' Λμν = Λμν Aμ)
AμBν = Λμν Aμ
(AμBν = Λμν Aμ)

Lorentz Scalar Product of 4-Vectors (A·B) → Lorentz Invariant Scalars = 4D (0,0)-Tensors.

They have the same measured value for all inertial observers, i.e. the same value in all 4D inertial reference-frames.

Einstein & Lorentz “saw” the physics of SR, Minkowski & Poincaré “saw” the mathematics of SR. We are indebted to all of them for the simplicity, beauty, and power of how SR and 4-vectors work...
The **SRQM (Physics) Diagramming Method** shows the properties & relationships of various physical objects/tensors in a graphical way. This “flowchart” method aids understanding.

**Representation:** 4-Scalars by ellipses, 4-Vectors by rectangles, 4-Tensors by octagons. Physical/mathematical equations and descriptions inside each shape/object. Sometimes there will be additional clarifying descriptions around a shape/object.

**Relationships:** Lorentz Scalar Products or tensor compositions of different 4-Vectors are on simple lines(→) between related 4-Vectors. Lorentz Scalar Products of a single 4-Vector, or Invariants of Tensors, are next to that object and often highlighted in a different color.

**Flow:** Objects that are some function of a Lorentz 4-Scalar with another 4-Vector or 4-Tensor are on lines with arrows(→) indicating the direction of flow. (ex. multiplication by SR scalar, or scalar function of SR 4-Vector indicated by [..])

**Properties:** Some objects will also have a symbol representing its properties nearby, and sometimes there will be color highlighting within the object to emphasize temporal-spatial properties. I use blue=Temporal \[\rightarrow\] red=Spatial \[\rightarrow\] purple=mixed Time-Space.

**Alternate ways of writing 4-Vector expressions in physics:**

\[(\mathbf{A} \cdot \mathbf{B})\] is a 4-Vector style, which uses vector-notation (ex. inner product "\(\cdot\)" or exterior product "\(\wedge\)"). and is typically more compact, always using **bold** UPPERCASE to represent the 4-Vector, ex. \((\mathbf{A} \cdot \mathbf{B}) = (A^\mu \eta_{\mu\nu} B^\nu)\), and **bold lowercase** to represent 3-vectors, ex. \((\mathbf{a} \cdot \mathbf{b}) = (a^i \delta^i_j b^j)\). Most 3-vector rules have analogues in 4-Vector mathematics.

\((A^\mu_0 B^\nu)\) is a Ricci Calculus style, which uses tensor-index-notation and is useful for more complicated expressions, especially to clarify those expressions involving tensors with more than one index, such as the Faraday EM Tensor \(F^{\mu\nu} = (\partial A^{\nu} - \partial A^{\mu}) = (\partial \mathbf{A})\)

\[(\mathbf{A} \cdot \mathbf{B})\] is a Ricci Calculus style, which uses tensor-index-notation and is useful for more complicated expressions, especially to clarify those expressions involving tensors with more than one index, such as the Faraday EM Tensor \(F^{\mu\nu} = (\partial A^{\nu} - \partial A^{\mu}) = (\partial \mathbf{A})\)
**Special Relativity → Quantum Mechanics**

**SRQM Tensor Invariants**

**Inherent 4D SpaceTime Properties**

One of the extremely important properties of Tensor Mathematics is the fact that there are numerous ways to generate Tensor Invariants. These Invariants lead to Physical Properties that are fundamental in our Universe, and are totally independent of any coordinate-systems used to measure them. Thus, they represent symmetry properties that are inherent in the fabric of SpaceTime (Time·Space). See the Cayley-Hamilton Theorem, esp. for the Anti-Symmetric Tensor Products.

**Trace Tensor Invariant:** $\text{Tr}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T_{\nu} = T_{\mu} = \Sigma\text{[EigenValues } \lambda_{\mu}\text{]}$ for $T^{\nu}$

**Determinant Tensor Invariant:** $\text{Det}\{T^{\mu\nu}\} = II\text{[EigenValues } \lambda_{\mu}\text{]}$ for $T^{\nu}$,→ (Pfaffian$[T^{\mu\nu}]$)$^2$ for anti-symmetric $T^{\mu\nu}$

**Inner Product Tensor Invariant:** $I[P^{\mu\nu}] = T^{\mu\nu}T_{\rho\sigma} = I_{\rho\sigma} : I[P^{\mu\nu}] = LSP[T^{\mu\nu},T_{\nu}] = T_{\nu}\eta_{\mu\nu}T^{\mu\nu} = T^{\nu}T_{\mu} = TT$

**4-Divergence Tensor Invariant:** $4\text{-Div}[T^{\mu\nu}] = \partial\mu T^{\mu\nu} = \partial T^{\nu}/dx^{\mu} = \partial T : 4\text{-Div}[T^{\mu\nu}] = \partial\mu T^{\mu\nu} = \partial T^{\nu}/dx^{\mu} = S'$

**Lorentz Scalar Product Tensor Invariant:** $LSP[T^{\mu\nu},S'\nu] = T^{\mu\nu}S_{\nu} = T_{\mu}S_{\nu} = T_{\nu}S_{\mu} = T\cdotS = t^{o}\delta_{\mu\nu} = t^{o}\cdotS = t^{o}S^{'o}$

**Phase Space Tensor Invariant:** $PS[T^{\mu\nu}] = (dt^{\mu}/t^{0}) = (dt^{\mu}dt^{\nu}dt^{3}dt^{0}/t^{0})$ for $(T\cdot T) =$ constant

**The Ratio of 4-Vector Magnitudes (Ratio of Rest Value 4-Scalars):** $T\cdot V / S\cdot V = (t^{o}/S^{o})$

**Tensor EigenValues $\lambda_{\mu} = \{\lambda_{1}, \lambda_{2}, \lambda_{3}, \lambda_{4}\}:$ could also be indexed $0..3$

**The various Anti-Symmetric Tensor Products:** index bracket $[ ]$ notation indicates anti-symmetric indices:

$\text{Trace } T_{\gamma}^{\mu\nu} = \text{Trace } \Sigma\text{[EigenValues } \lambda_{\mu}\text{]}$ for $(1,1)-$Tensors

$\text{AntiSymm Bi-Product } T_{[\mu\nu]}^{\sigma\rho} = \text{Inner Product}$

$\text{AntiSymm Tri-Product } T_{[\mu\nu\rho]}^{\sigma\rho\tau} = ?\text{Name?}$

$\text{AntiSymm Quad-Product } T_{[\mu\nu\rho\sigma]}^{\tau\sigma\tau\tau} = 4\text{D Determinant } = II\text{[EigenValues } \lambda_{\mu}\text{]}$ for $4D\text{(1,1)-Tensors}$

These invariants are not all always independent, some invariants are functions of other invariants.

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**SRQM (Physics) Diagrammoting Method**

**4-Gradient $\partial^\mu$**

$$\partial^\mu = \partial / (c_0\cdot V) = \partial / \partial R_\mu$$

**Minkowski Metric**

$$\delta [R^\mu] = \delta [R^\mu] / \delta [R^\nu] = \lambda_\mu^\nu$$

**Transform**

$$\Lambda_\mu^\nu \lambda_\nu^\sigma = \Lambda_\mu^\nu \lambda_\nu^\nu = \Lambda_\mu^\nu$$

**Determinant Tensor Invariant**

$$\text{Det} [\Lambda_\mu^\nu] = \pm 1$$

**Inner Product Tensor Invariant**

$$\Lambda_\mu^\nu \lambda_\mu^\rho = \Lambda_\mu^\nu \lambda_\nu^\rho = \Lambda_\mu^\nu$$

**SpaceTime $\partial \cdot R = \partial R^\mu / \partial R^\nu = 4$ Dimension**

**4-Momentum $P^\mu = (mc, p) = (E/c, p) = mU$$

**4-Velocity $U^\mu = \gamma (c, u) = dR / d\tau$$

**Einstein’s**

$$E = \gamma mc = \gamma m c^2 = \gamma E_o$$

Rest Mass $m_o = $ Rest Energy $E_o$

**4-Position $R^\mu = (ct, r) = \{<\text{Event}>\}$

**4-Vector SRQM Interpretation of QM**

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John B. Wilson

http://scirealm.org/SRQM.pdf

---

**SR 4-Tensor**

$$T_{(2,0)}^{\mu\nu}$$

**SR 4-Vector**

$$V = (v^\mu, v_\mu)$$

**SR 4-Scalar**

$$\text{Det}S = \Sigma\text{[EigenValues } \lambda_{\mu}\text{]}$$

**Lorentz Scalar Invariant**

Speed of Light (c) from LSP$[\cdot]$ of 4-Velocity

$$U^\mu U_\mu = c^2$$

Rest Mass $m_o = $ Rest Energy $E_o$

**Relativistic Gamma**

$$\gamma = \sqrt{1 - \beta \cdot \beta}$$

$$\beta = u/c$$

---

**A Tensor Study of Physical 4-Vectors**

4-Vector SRQM Interpretation of QM
SRQM Study: Physical/Mathematical Tensors

Tensor Types: 4-Scalar, 4-Vector, 4-Tensor

Physical Examples – Venn Diagram

0 index-count Tensors:
- EM Charge \( Q = \rho d^4x \)
- Lorentz Scalar \( S \)
- Planck’s Const \( h \)
- ProperTime \( d^4p/E \)
- Speed-of-Light \( c = \sqrt{|U-U|} \)

1 index-count Tensors:
- SR 4-Vector
  - 4-Position \( \mathbf{R} = (c, t) \)
  - 4-Velocity \( \mathbf{U} = (c, u) \)
  - 4-Momentum \( \mathbf{P} = (m, p) \)

SR 4-CoVector = “Dual” 4-Vector
- 4-Position \( \mathbf{R} = (c, t) \)
- 4-Velocity \( \mathbf{U} = (c, u) \)
- 4-Momentum \( \mathbf{P} = (m, p) \)

2 index-count Tensors:
- Minkowski Metric \( \eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \)
- Faraday EM 4-Tensor \( F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \)
- Perfect Fluid 4-Tensor \( T_{\mu\nu} = (\rho u_\mu) u_\nu + (p) u_\mu u_\nu \)

SR Mixed 4-Tensor
- 4D (1,1)-Tensors \( T_{\mu\nu} = \eta_{\mu\nu} T_{\rho\sigma} \)

SR Lowered 4-Tensor
- 4D (0,2)-Tensors \( T_{\mu\nu} = \eta_{\mu\nu} T_{\rho\sigma} \)

1+(2) index-count Tensors:
- SR & GR 4-Tensors \( T_{\mu\nu} \)

Riemann Curvature Tensor
- Ricci Decomposition of Riemann Tensor
- Weyl (Conformal) Curvature Tensor

Trace of Ricci Tensor
- Lorentz Scalar Invariant

Physical 4-Tensors: Objects of Reality which have Invariant 4D SpaceTime properties

- Time:Space

5D Physical/4V SciRealm.org

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SciRealm.org
http://scirealm.org/SRQM.pdf
SRQM Study:

**SRQM 4-Vectors = 4D (1,0)-Tensors**

**SRQM 4-Tensors = 4D (2,0)-Tensors**

{or higher index #}

*Made from 4-Vector relations*

---

**SRQM 4-Vectors**

- Faraday EM Tensor $F^{\mu\nu}$
  - $[0, -e^{\phi}/c, e$]
  - Antisymmetric Tensor
  - $[+e^{\phi}/c, -e^0]$

- 3-electric-field ($e = e^0$)
  - 3-magnetic-field ($b = b^0$)

- 3 charge-flux: $\partial A^0 = \partial A^0 = -\partial A^0$

- Antisymmetric Tensor

- Temporal-Temporal: Temporal-Spatial: Spatial-Spatial

---

**SRQM 4-Tensors**

- $[0, 3-mass-moment (n = n^0); 3-angular-momentum (l = l^0)]$

- $M^{\mu\nu} = X^{\mu}P^{\nu} - X^{\nu}P^{\mu}$

- $[1:0; -1^{l\mu} = -\delta^{l\mu}]$

- $[0:0; 0^{l\mu} = 0^{l\mu}]$

- $[0:0; 0^{l\mu} = 0^{l\mu}]$

---

4-Tensors can be constructed from the Tensor Products of 4-Vectors. Technically, 4-Tensors refer to all SR objects (4-Scalars, 4-Vectors, etc.), but typically reserve the name 4-Tensor for SR Tensors of 2 or more indices. Use (m,n)-Tensor notation to specify types more precisely.
SRQM Study:

4-Scalars = 4D (0,0)-Tensors = Lorentz Scalars = 4D SR Invariants ↔ Physical Constants

*Made from 4-Vector relations*

4-Scalar = 4D (0,0)-Tensor = SR Invariant

SI Dimensional Units

<table>
<thead>
<tr>
<th>Variable</th>
<th>SI Unit</th>
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<tbody>
<tr>
<td>[s]</td>
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4-Scalar = 4D (0,0)-Tensor (generally composed of 4-Vector combinations with LSP)

(\tau) = [R \cdot U]/[U \cdot U] = [R \cdot R]/[U \cdot R] **Time as measured in the at-rest frame**

(\alpha) = [dR \cdot U]/[U \cdot U] **Differential Time as measured in the at-rest frame**

(\nu) = [U \cdot \dot{\alpha}]/(\alpha \cdot \alpha) **Note that the 4-Gradient operator is to the right of 4-Velocity**

(\rho) = V/(U \cdot U) = (q)(N \cdot U)/(U \cdot U) = (q)(n)

\(\Omega = [A \cdot U] \) (\varphi \leftrightarrow \varphi_{EP}) as the EM version RestScalarPotential

(\nu) = [T \cdot U]/[U \cdot U] \(\Phi_{phase,free} = -[K \cdot R] = (Kr - \omega t) \) \(\Phi_{phase} = -[K \cdot R] = (Kr - \omega t) \) **Units [Angle] = [WaveVec] [Length] = [Freq] [Time]**

(\nu_{action}) = -[P \cdot R] = (p \cdot r - Et) \(\nu_{action} = -[P \cdot R] = (p \cdot r - Et) \) **Units [Action] = [Momentum] [Length] = [Energy] [Time]**

(\delta) = [P \cdot U]/[K \cdot U] = [P \cdot R]/[K \cdot R] \(\nu = K/(2\pi) \)

(\delta) = [P \cdot U]/[K \cdot U] = [P \cdot R]/[K \cdot R] \(\nu = K/(2\pi) \)

(\delta) = [P \cdot U]/[K \cdot U] = [P \cdot R]/[K \cdot R] \(\nu = K/(2\pi) \)

\(\delta = (1/\gamma)F \) Lorentz Force Eqn. \(\gamma = -e \) as Electron Charge

(\delta) = (1/\gamma)F \(\nu = V_{el} \) \(T_{el} \) \(\delta = (1/\gamma)F \) Lorentz Force Eqn. \(\gamma = -e \) as Electron Charge

<table>
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<th>Variable</th>
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Faraday EM Inner Product Invariant \(2(b \cdot b \cdot e(c))^2\)

Faraday EM Determinant Invariant \(e(b/c)^2\)

\(T^2 = (kg^2/C^2 \cdot s^2)\)

\(T^4 = (kg^4/C^4 \cdot s^4)\)

Lorentz Scalars = (0,0)-Tensors can be constructed from the Lorentz Scalar Products (LSP) of 4-Vectors: \((A \cdot B)\) = Lorentz Scalar
SRQM Study: Physical 4-Vectors

Some SR 4-Vectors and Symbols

- 4-Gradient: $\partial = \partial = \partial = (\partial/c, \nabla)$
- 4-Position: $\vec{r} = (x, y, z, t)$
- 4-Displacement: $\Delta \vec{r} = (\Delta x, \Delta y, \Delta z)$
- 4-Unit Temporal: $T = \gamma(1, \beta)$
- 4-Velocity: $\vec{u} = \gamma(c, \vec{u})$
- 4-Acceleration: $\vec{a} = \gamma(c^2, \vec{a})$
- 4-Spin: $\vec{s} = \gamma_{sn}(\beta, \gamma, \vec{n})$
- 4-Momentum: $P = (mc, \vec{p})$
- 4-Force: $F = (F/c, \vec{f})$
- 4-Mass Flux: $G = \gamma(\rho(c, \vec{u}))$
- 4-Heat Energy Flux: $Q = \gamma(\rho E(c, \vec{u}))$
- 4-Pure Entropy Flux: $S_{ent pure} = \gamma(S_{ent pure})$
- 4-Entropy Heat Flux: $S_{ent heat} = \gamma(S_{ent heat})$

4-Vector SRQM Interpretation of QM

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http://scirealm.org/SRQM.pdf
SRQM Study:
Primary/Primitive/Elemental 4-Vectors:
4-UnitTemporal \( T \) & 4-UnitSpatial \( S \)

**SR 4-Tensor**

1. \( (1,0) \)-Tensor \( T^{\mu} \) or \( T^0 \)
2. \( (0,1) \)-Tensor \( V_\nu = \text{V} \) or \( V^\nu \)
3. Lorentz Scalar \( \text{S} \)

**SR 4-Vector**

1. \( (1,0) \)-Vector \( U^\mu = \text{U} \) or \( \text{U}^\mu \)
2. \( (0,1) \)-Vector \( V_\mu = \text{V} \) or \( \text{V}^\mu \)

**SR 4-Scalar**

1. \( (0,0) \)-Scalar \( \text{S} \)
2. Lorentz Scalar \( \text{S} \)

**Relativistic Gamma**

\[ \gamma = 1/\sqrt{1 - \beta \cdot \beta} \]

\( \beta = u/c \)

**4-Velocity**

\[ U = U^\mu = c \gamma (1, \beta) = (\text{U} \cdot \beta) \text{R} = d\text{R}/d\tau = c \text{T} \]

**4-Spin**

\[ S_{\text{spin}} = S^\mu_{\text{spin}} = s^{\mu}_{\text{S}} = (s^0, s) = (c, s) \]

**SR LightCone**

- 4-UnitTemporal, \([\text{dimensionless}]\)
  - Magnitude* = +1
  - "Magnitude" = \((\pm 1)\)
  - |Magnitude| = (1)

- 4-UnitSpatial, \([\text{dimensionless}]\)
  - Magnitude* = 0
  - "Magnitude" = \((\pm 1)\)
  - |Magnitude| = (0)

**4-UnitTemporal**

\[ \mathbf{T} = T^\mu = \gamma (1, \beta) \]

\[ \mathbf{T} \cdot \mathbf{T} = \gamma (1, \beta) \gamma (1, \beta) = \gamma^2 (1 + \beta \cdot \beta) = 1 \]

**4-UnitSpatial**

\[ \mathbf{S} = S^\mu = \gamma (1, \beta) \gamma (1, \beta) \]

\[ \mathbf{S} \cdot \mathbf{S} = \gamma (1, \beta) \gamma (1, \beta) = \gamma^2 (1 - \beta \cdot \beta) = -1 \]

\[ \mathbf{T} \cdot \mathbf{S} = \gamma (1, \beta) \gamma (1, \beta) \]

\[ \mathbf{S} \cdot \mathbf{T} = \gamma (1, \beta) \gamma (1, \beta) \]

**4-UnitTemporal**

- Orthogonal to (\( T \))
- 4-UnitSpatial
- 3 independent components

**4-UnitSpatial**

- Invariant (\( \beta \cdot \beta \))
- 3 independent components

**SR 4-Vector Interpretation of QM**

- Lorentz Scalar
  - \( \gamma = 1/\sqrt{1 - \beta \cdot \beta} \)
  - \( \beta = u/c \)

- 4-Velocity
  - \( U = U^\mu = c \gamma (1, \beta) \)
  - \( (U \cdot \beta) \text{R} = d\text{R}/d\tau = c \text{T} \)

- 4-Spin
  - \( S_{\text{spin}} = S^\mu_{\text{spin}} = s^{\mu}_{\text{S}} = (s^0, s) = (c, s) \)

- Lorentz Scalar Invariant
  - \( \text{S} = s^\mu \)
  - \( \text{S} \cdot \text{S} = s^\mu s^\nu \eta_{\mu \nu} = s^0 s^0 - s^1 s^1 = -c^2 \)
  - \( \text{T} \cdot \text{S} = \gamma (1, \beta) (1, \beta) \)
  - \( \text{S} \cdot \text{T} = \gamma (1, \beta) (1, \beta) \)

**Trace**

\[ \text{SS} = s^\mu s^\nu \eta_{\mu \nu} = s^0 s^0 - s^1 s^1 = -c^2 \]

**Interpretation**

- Lorentz Invariant
  - \( \text{SS} = s^0 s^0 - s^1 s^1 = -c^2 \)

**SRQM Study**

- Physical 4-Vectors
- Lorentz Transformations
- Invariance Properties
SRQM: Some Basic 4-Vectors

4-Position, 4-Velocity, 4-Differential, 4-Gradient
SR SpaceTime Calculus & Invariants

Invariant ProperTime
\[ R \cdot \frac{U}{U} = (ct, r) \gamma (c, u) = \gamma (c^2 t - r \cdot u) = \gamma \left( c^2 t_0 - c^2 \right) = \gamma \frac{c^2 t_0}{c^2} = t_0 = \tau \]

Invariant LightSpeed
\[ R \cdot U = \gamma (c, u) = \gamma (c^2 - u \cdot u) = c^2 - 1 \gamma^2 = c^2 \]

Invariant ProperTime Derivative
\[ U \cdot \frac{d}{dt} = \frac{d}{dt}[\gamma (c, u)] = \gamma \frac{d}{dt} = \frac{d}{dt} \]

Invariant d'Alembertian
\[ \partial \cdot \partial U = (\gamma c, u) \cdot (\gamma c, U) = \gamma (c^2 - u \cdot U) = \gamma (c^2 - c^2) = 0 \]

Wave Equation
\[ U \cdot \partial U = (\gamma c, u) \cdot (\gamma c, U) = \gamma (c^2 - c^2) = 0 \]

The 4-Velocity is interesting in that it sort of bootstrap itself into existence:
\[ U = (\gamma c, u) = \frac{d}{dt} = (U \cdot \partial U = U \cdot \partial U) = U \cdot \frac{d}{dt} = \frac{d}{dt} = \]

The bootstrap is because of:
\[ U = (d/d\tau)(R) \]

Math/Phys Wave Solution

4-WaveVector
\[ K = K \equiv \omega = \frac{2\pi n(v, c)}{c} = -\partial [\Phi] = (\omega/c, \omega n/v_{phase}) = (1/c \mathcal{I}, \mathcal{A}) \]

Relativistic Gamma \( \gamma = 1/\sqrt{1 - \beta \cdot \beta} \), \( \beta = u/c \)
A Tensor Study of Physical 4-Vectors

SRQM Study: Physical 4-Vectors
Some 4-Velocity Relations

4-Position
\[ R = R^\mu = (c, r) \]
alt. notation \( X = X^\mu \)

4-Velocity
\[ U = U^\mu = \gamma (c, u) \]

4-Acceleration
\[ A = A^\mu = \gamma c (\gamma u + y a) \]

Relativistic Gamma \( \gamma \) combines with Invariant Rest-Scalars to create Relativistic components. This allows the 4-Velocity to create many other standard SR 4-Vectors. ex. with 4-Momentum
\[ P = (mc, p) = (mc, mu) = m(c, u) = m_0 \gamma (c, u) = m_0 U \]

SR 4-Tensor
\[
\begin{align*}
T^{\mu \nu} & = \left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right) \\
V & = (v_0, v) \\
\end{align*}
\]

SR 4-Vector
\[
\begin{align*}
V & = V^\mu = (v^0, v) = (v, v) \\
\end{align*}
\]

SR 4-Vector
\[
\begin{align*}
V & = V^\mu = (\text{scalar} \times c^2, 3\text{-vector}) \\
\end{align*}
\]

Rest Mass/Energy/c^2
\[ m_0 E/c^2 \]

Rest Ang. Frequency/c^2
\[ \omega/c^2 \]

Rest EM Potential/c^2
\[ \phi/c^2 \]

Rest Charge Density
\[ \rho_c = q_0 \]

Rest Number Density
\[ n_0 = n(c, u) = n_0 \gamma (c, u) = n_0 U \]

Rest Inv. Thermal Energy
\[ 1/\kappa_b T_0 \]

4-Momentum
\[ P = P^\mu = (mc, mu) = m_0 \gamma (c, u) = m_0 U = -S[S] = (E/c, p) = (E/c^2) \gamma (c, u) = (E/c^2) U = (E/c^2) T \]

4-WaveVector
\[ K = K^\mu = (\omega/c, c k) = (\omega/c) U = -S[\Phi] = (\omega/c, \omega/\gamma_{\text{phase}}) = (1/c T, \hbar/\lambda) = (\omega/c) T \]

4-(EM) Vector Potential
\[ A = A^\mu = (\phi/c, a) = (\phi_0/c^2) U \]

4-(EM) Vector Potential Momentum
\[ Q = Q^\mu = (q_0 c, qa) = (V/c, q) \]

4-Charge Flux: 4-Current Density
\[ J = J^\mu = (pc, j) = p(c, u) = p_0 \gamma (c, u) = p_0 U = q n_0 U = q N \]

4-(Dust) Number Flux
\[ N = N^\mu = (nc, n) = n(c, u) = n_0 \gamma (c, u) = n_0 U \]

4-Thermal Vector
4-Inverse Temperature Momentum
\[ \Theta = \Theta^\mu = (\Theta_0^\mu, \Theta) = (c/\kappa_b T_0, u/k_b T_0) = (\Theta_0/c_0 U) = (1/\kappa_b T)(c, u) = (1/\kappa_b T)(c, u) U = (1/\kappa_b T)(c, u) \]

Trace[T^\mu\nu] = \eta^\mu\nu T^\mu\nu = \Theta = \text{Lorentz Invariant} \]

[4x413] = (\begin{array}{cc}
\frac{\partial}{\partial t} & \gamma \frac{\partial}{\partial x} \\
\gamma \frac{\partial}{\partial x} & \gamma (\gamma + u) \end{array}) \]

Lorentz Scalar
\[ \gamma = \frac{1}{\sqrt{1 - (u^2/c^2)}} \]

Proper Time Derivative
\[ \frac{\partial}{\partial \tau} = \frac{\partial}{\partial t} \]

Inertial Light Speed (c)
\[ c = \frac{1}{\sqrt{1 - (u^2/c^2)}} \]

http://scirealm.org/SRQM.pdf

SRQM.org
John B. Wilson
SciRealm.org
SciRealm@ao.com
SRQM Study:
Physical 4-Vectors
Some 4-Acceleration Relations

4-Velocity
U = U = γ(c, u) = γ(c²-u·u) = c²

4-Acceleration
A = A = γ(c', γ'U + γa)
γ²(γ'(u·a)/c, γ'(u·a)u/c²+a)
= γ²((u·a)c, (u·u·a)/c²+a)
dU/dτ = d²R/dτ² = {γ'γ'dγ'dτ}
SRQM Study: Physical 4-Vectors

Some 4-Gradient Relations

The relations below are for the 4-(Position)Gradient $\partial_R$, 4-Gradients wrt. other 4-Vector variables exist also... ex. 4-WaveGradient $\partial_k$

Minkowski Metric $\delta = [\delta_{ij}] = \eta^{ij}$

Lorentz Transform $\partial_R = \partial \gamma \mathbf{N} = [\delta_{ij}] \partial^i \mathbf{V} = \eta^{ij}$

SpaceTime 4-R, R=4 Dimension $\partial_R \gamma = \partial \gamma \mathbf{N} = [\delta_{ij}] \partial^i \mathbf{V}$

SRQM Non-Zero Commutation $[\partial_R, \partial_R] = \partial_R \gamma = \partial \gamma \mathbf{N} = [\delta_{ij}] \partial^i \mathbf{V}$

Calculus $dR \cdot \partial dR = \partial \mathcal{D}$ Total Derivative Chain Rule

4-Position:4-Differential $R = R^\nu = (ct, r) = \mathbf{r} = \langle \mathbf{r} \rangle$

Equation of State $\mathcal{D} = dR \gamma = \partial \gamma \mathbf{N} = [\delta_{ij}] \partial^i \mathbf{V}$

4-Momentum $\mathcal{D} = \gamma \mathbf{N} = [\delta_{ij}] \partial^i \mathbf{V}$

4-WaveVector $\mathcal{D} = \gamma \mathbf{N} = [\delta_{ij}] \partial^i \mathbf{V}$

4-(EM)VectorPotential $\mathcal{D} = \gamma \mathbf{N} = [\delta_{ij}] \partial^i \mathbf{V}$

3-CurrentDensity $\mathcal{D} = \gamma \mathbf{N} = [\delta_{ij}] \partial^i \mathbf{V}$

3-NumberFlux $\mathcal{D} = \gamma \mathbf{N} = [\delta_{ij}] \partial^i \mathbf{V}$

SR 4-Tensor $\mathcal{D} = \gamma \mathbf{N} = [\delta_{ij}] \partial^i \mathbf{V}$

SR 4-Vector $\mathcal{D} = \gamma \mathbf{N} = [\delta_{ij}] \partial^i \mathbf{V}$

SR 4-Scalar $\mathcal{D} = \gamma \mathbf{N} = [\delta_{ij}] \partial^i \mathbf{V}$

4-Vector $\mathcal{D} = \gamma \mathbf{N} = [\delta_{ij}] \partial^i \mathbf{V}$
### SRQM Study: Physical 4-Tensors

#### Some SR 4-Tensors and Symbols

<table>
<thead>
<tr>
<th>4-Tensor</th>
<th>Unit Dimensionless</th>
<th>Dimension</th>
<th>SR: Lorentz Transforms</th>
</tr>
</thead>
</table>
| Lorentz Identity Transform | $\Lambda^\mu'_{\nu} = \delta^\mu_\nu$ | 0 | SR: Lorentz Identity (
| | | | $I_{(4)}$) |
| Lorentz Space-Reversal (Parity Inverse) Transform | $\Lambda^\mu'_{\nu} = -I_{(4)}$ | 0 | SR: Lorentz Space-Reversal (
| | | | $\mu'=-\nu$) |
| Lorentz Time-Reversal Transform | $\Lambda^\mu'_{\nu} = 0$ | 0 | SR: Lorentz Time-Reversal (
| | | | $\mu'=0$, $\nu'=0$) |
| Lorentz ComboPT Transform | $\Lambda^\mu'_{\nu} = 0$ | 0 | SR: Lorentz ComboPT (
| | | | $\mu'=-\nu$, $\nu'=0$) |

#### Perfect Fluid

$$T^\mu_{\nu} = (\rho c^2 + p) u^\mu u^\nu$$

#### Faraday EM

$$F = \partial A = \partial \times E$$

#### 4-Angular Momentum

$$M^\mu = x^\mu p^\nu - x^\nu p^\mu$$

#### Trace[4-Tensor]

$$\text{Trace}[T^\mu_{\nu}] = \eta^\mu_{\nu} T^\mu_{\nu} = T^\nu_{\nu} = T$$

### Lorentz Transform

$$\Lambda^\mu'_{\nu} = (\gamma, -\beta y, 0, 0)$$

### Minkowski Metric

$$\eta_{\mu\nu} = \begin{pmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{pmatrix}$$

### General Time-Space Boost

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

### Non-symmetric Mixed 4-Tensor

$$\Lambda^\mu'_{\nu} = (\gamma, \beta y, 0, 0)$$

### Symmetric, Spatial Isotropic

$$\Lambda^\mu_{\nu} = (\gamma, 0, 0, 0)$$

### Symmetric Mixed 4-Tensor

$$\Lambda^\mu_{\nu} = (\gamma, -\beta y, 0, 0)$$

### Anti-symmetric (skew)

$$\Lambda^\mu_{\nu} = (\gamma, \beta y, 0, 0)$$

### Unit Dimensionless

$$\Lambda^\mu_{\nu} = (\gamma, 0, 0, 0)$$

### Component Dimensions

- $\text{Energy density} = \rho = \gamma M^0$ [kg/m$^3$]
- $\text{Pressure} = \rho c^2 = \gamma M^0 c^2$ [kg/m$^3$]
SRQM Study: Physical 4-Tensors

Some SR 4-Tensors and Symbols

4-Unit Temporal Tensor $T = T_{\mu \nu} = (1, \beta)$

Spatial "(H)orizontal" Projection (2,0)-Tensor $P_{\mu \nu} \rightarrow \nu = \beta_{\mu \nu} = \propto H_{\mu \nu} \rightarrow \text{Diag}[1,1]$ [MCRF]

Temporal "(V)ertical" Projection (2,0)-Tensor $P_{\mu \nu} \rightarrow \nu = \beta_{\mu \nu} = \propto V_{\mu \nu} \rightarrow \text{Diag}[0,1]$ [MCRF]

4-Force Density $F_{\mu \nu} = F_{\mu \nu} \rightarrow \beta_{\mu \nu} = \propto T_{\mu \nu} \rightarrow \{= 0 \nu \text{ if conserved} \}$

Perfect Fluid Stress-Energy $T_{\mu \nu} := (p_\rho T_{\mu \nu} + (p_\rho - \rho) H_{\mu \nu}) \rightarrow \text{(MCRF)}$

Spatial Isotropic 4-Tensor Symmetric, Spatial Isotropic

4-Tensor $T_{\mu \nu}$ Symmetric, Spatial Isotropic

Faraday EM Tensor $F_{\mu \nu} = \partial A_\rho / \partial x^\nu - \partial A_\rho / \partial x^\nu = \partial \wedge A$

SR: Minkowski Metric $\delta[R] = \beta_{\mu \nu} = \propto V_{\mu \nu} \rightarrow \text{Diag}[0,1]$ [MCRF]

Perfect Fluid 4-Tensor $T_{\mu \nu} = 0$

Stress-Energy 4-Tensor Symmetric, Spatial Isotropic

Maxwell 4D EM Stress-Energy Tensor $T_{\mu \nu} := -1/(\mu_0) F_{\mu \nu} F^{\mu \nu} [\text{(1/14)}] \rightarrow \text{(MCRF)}$

SR-QM Study: Physical 4-Tensors

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm.org/JohnWilsie
http://www.scirealm.org/SRQM.pdf

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SRQM Study: Physical 4-Tensors

Metric Sign-Convention: Signature

4-Gradient

\[ \partial = \frac{\partial}{\partial \theta} = \frac{\partial}{\partial \varphi} = \frac{\partial}{\partial t} \]

for \((-+,+,+),\) Spacelike+, Relativity, EastCoast convention

\[ \frac{\partial}{\partial \theta} = \frac{\partial}{\partial \varphi} = -t \]

for \((+,-,-,+)\), Timelike+, ParticlePhys, WestCoast convention

4-Position

\[ R = \text{R} = (c,t,x,y,z) \]

4-Tensor Unit Dimensionless

Symmetric, Spatial Isotropic

4-Vector SRQM Interpretation of QM

4-Vector SRQM Study: Physical 4-Vectors

A Tensor Study of Physical 4-Vectors

Ultimately, the Sign-Convention or Metric Signature is based on the definition of the 4-Gradient \(\partial\), since it is used to create the Minkowski Metric \(\eta^{\mu\nu}\).

I prefer the \((+,-,-,+)\) Metric Signature: SignConvention, and use it in this study = \{temporal,0\^\(0\),+\}, Timelike+, ParticlePhysics, WestCoast convention

Perfect Fluid 4D Stress-Energy Tensor (technically an EnergyDensity)

\[ T^{\mu\nu} = (\rho_o)V^{\mu\nu} + (p_o)\delta^{\mu\nu} \]

\[ T^{\mu\nu} = \begin{pmatrix} \rho_o & p_o \\ p_o & -\rho_o \end{pmatrix} \]

for \((+,+,+)\), Spacelike+, Relativity, EastCoast convention

\[ T^{\mu\nu} = \begin{pmatrix} \rho_o & 0 \\ 0 & -\rho_o \end{pmatrix} \]

for \((+,+,+)\), Timelike+, ParticlePhys, WestCoast convention

Maxwell 4D EM Stress-Energy Tensor (technically an EnergyDensity)

\[ T^{\mu\nu} = \left\{ \begin{array}{c} \text{(1/}\mu_o)\left[ F_{\mu\nu} F^{\alpha\beta} \right] \right. \\ \left. \text{(1/}\mu_o)\left[ F_{\mu\nu} F^{\alpha\beta} \right] \right. \end{array} \]

for \((+,+,+)\), Spacelike+, Relativity, EastCoast conv.

\[ T^{\mu\nu} = \left\{ \begin{array}{c} \text{(1/}\mu_o)\left[ F_{\mu\nu} F^{\alpha\beta} \right] \right. \\ \left. \text{(1/}\mu_o)\left[ F_{\mu\nu} F^{\alpha\beta} \right] \right. \end{array} \]

for \((+,+,+)\), Timelike+, ParticlePhys, WestCoast convention

Stress-Energy Tensors are technically EnergyDensities, not Energies: EnergyDensity (temporal) & Pressure: Stress (spatial) have the same dimensional measurement units. \([Pa = J/m^2 = kg/m^2s^2]\)

\( \rho_o = p_m c^2 \)

EnergyDensity (temporal) & Pressure: Stress (spatial) have the same dimensional measurement units. \([Pa = J/m^2 = kg/m^2s^2]\)
SRQM Study: Physical 4-Tensors

Projection 4-Tensors \( \{ P_{\mu\nu} : P_{\mu}^\nu : P_{\mu\nu} \} \)

4-Vector SRQM Interpretation of QM

SR Perfect Fluid Stress-Energy 4-Tensor
\[
T_{\mu\nu}^{\text{perfectfluid}} = (\rho_0 c^2 V_{\mu\nu} + (-p_0) T_{\mu\nu}) \quad \text{(MCRF)}
\]

The projection tensors can work on 4-Vectors to give a new 4-Vector, or on 4-Tensors to give either a 4-Scalar component or a new 4-Tensor.

4-Unit Temporal \( T^0 = (1, \bar{\nu}) \)
4-Generic \( A^\epsilon = (\alpha^\epsilon, \alpha^\epsilon) = (a^0, a^\nu) (a^0, a^\nu) \) from \( T^0 = (0, 0) \) Temporal Projection

\[ V^\mu, A^\mu = \begin{cases} (1 : a^0 + a^\nu + a^\nu + a^\mu) = 0, & \text{Spatial Projection} \\ (0 : a^0 + a^\nu + a^\nu + a^\nu) = 0 & \end{cases} \]

Minkowski Metric
\[ \delta[R]^\mu_{\nu} = \delta[R]^\mu_{\nu} = V^\mu V^\nu + H^\mu H^\nu \]

4-Spatial \( S = S^\mu = (\bar{\nu}, \bar{\nu}) \) from \( T^0 = (0, 0) \) Spatial Projection

\[ V_{\mu} H^\mu = 0 \]

\[ \tilde{\delta}[R]^\mu_{\nu} = \delta[R]^\mu_{\nu} = V^\mu V^\nu + H^\mu H^\nu \]

Note that the Projection Tensors have unit dimensionless = 1:
the object projected retains its own dimensional measurement units
Note that the (2,0)- & (0,2)-Spatial Projectors have opposite signs from the mixed (1,1)-Spatial due to the (-1,1) Metric Signature convention

Trace \( [T] = \delta_{\mu\nu} T^\mu_{\nu} = T^0_{\nu} = T \)

\[ V \cdot V = V^\mu V^\nu = (v^\mu v^\nu) - V^2 = (v^\mu v^\nu)^2 \]

Lorentz Scalar Invariant
SRQM Chart:
Special Relativity → Quantum Mechanics
SR → QM Interpretation Simplified

SRQM: The [SR → QM] Interpretation of Quantum Mechanics

Special Relativity (SR) Axioms: Invariant Interval + LightSpeed (c) as Universal Physical Constant lead to SR, although technically SR is itself the Minkowski-SpaceTime low-curvature: "flat" limiting-case of GR.

\{c, \tau, m_0, \hbar, i\} = \{\text{SpeedOfLight, ProperTime, RestMass, Dirac/Planck Reduced Constant(h=h/2\pi), ImaginaryNumber}\}

are all Empirically-Measured SR Lorentz-Invariant Physical and/or Mathematical Constants = (0,1)

Standard SR 4-Vectors:
- **4-Position** \( R = (ct, r) \) = <Event>
- **4-Velocity** \( U = \gamma(c, u) \) = (U·R)=(t/d\tau)R=dR/d\tau
- **4-Momentum** \( P = (E/c, p) \) = m_0U
- **4-WaveVector** \( K = (\omega/c, k) \) = P/\hbar
- **4-Gradient** \( \partial = (\partial_t/c, \nabla) \) = -iK

SR + Empirically Measured Physical Constants lead to RQM via the (KG) Klein-Gordon Quantum Eqn, and thence to QM via the low-velocity limit \( \{ |v| << c \} \), giving the Schrödinger Eqn. This fundamental KG Relation also leads to the other Quantum Wave Equations:

- **spin=0** boson field = 4-Scalar:
  - Free Scalar Wave (Higgs)
- **spin=1/2** fermion field = 4-Spinor:
  - Weyl
- **spin=1** boson field = 4-Vector:
  - Maxwell (EM photonic)

SRQM: A treatise of SR → QM by John B. Wilson (SciRealm@aol.com)
There are some paradigm assumptions that need to be cleared up:

The real, physical world *IS NOT* Euclidean 3-dimensional (3D) with absolute background time. Classical and quantum 3D physics is a great approximation; but only for Galilean, slow-moving objects |v|<<c. 3D physics uses \{3-vectors = 3D (1,0)-tensors\}, has 3D Euclidean invariants like lengths (Pythagorean theorem), has 1D Euclidean scalar invariants like absolute time, but it does not contain or predict many of the physical properties and relationships that we now know to be true from SR & RQM.

Also, these 1D & 3D Euclidean invariants have been empirically-proven to *NOT* be invariant in the real world. This is based on a century+ of physics experiments and observations confirming the fact of 4D Relativity.

The real, physical world *IS* a locally Minkowskian 4-Dimensional SpaceTime (4D), with relativistically-interconnected (1 time + 3 space) dimensions. Time and space are interconnected in a very specific Lorentzian way, via SpaceTime 4D Relativistic Metrics, which give a great many special relationships and invariances that 3D physics misses entirely. These properties are easily explained using SR:Minkowskian Physical \{4-Vectors = 4D (1,0)-Tensors\}.

3D physics can be obtained as a limiting-case approximation from 4D Physics by using relative speed |v|<<c. Classical Mechanics (CM) is just a low-speed limiting-case of Special Relativity (SR) Quantum Mechanics (QM) is just the low-speed limiting-case of Relativistic Quantum Mechanics (RQM)

SRQM: A treatise of SR→QM by John B. Wilson (SciRealm@aol.com)
There are some paradigm assumptions that need to be cleared up:

Minkowskian:SR 4D Physical 4-Vectors *ARE NOT* generalizations of Classical/Quantum 3D physical 3-vectors.
While a "mathematical" Euclidean (n+1)D-vector is the generalization of a Euclidean (n)D-vector, the "physical/physics" analogy ends there.

Minkowskian:SR 4D Physical 4-Vectors *ARE* the primitive elements of 4D Minkowski:SR SpaceTime. Classical/Quantum physical 3-vectors are just the spatial components of SR Physical {4-Vectors = 4D (1,0)-Tensors}. There is also a fundamentally-related classical/quantum physical scalar related to each 3-vector, which is just the temporal component scalar of a given SR Physical SpaceTime 4-Vector.

4-Position \( \mathbf{R} = R^\mu = (r^\mu, r) = (ct, \mathbf{r}) \rightarrow (ct=cr^0, r^x, r^y, r^z) = (ct, x, y, z) \)
4-Momentum \( \mathbf{P} = P^\mu = (p^\mu, p) = (p^0, \mathbf{p}) = (E/c, \mathbf{p}) \rightarrow (E/c=p^0/c, p^x, p^y, p^z) \)

These Classical/Quantum {scalar}+{3-vector} are the dual {temporal}+{spatial} components of a single SR Time·Space 4-Vector = (temporal scalar \( c^4 \), spatial 3-vector) with SR LightSpeed factor \( c^4 \) to give correct overall dimensional measurement units.

While different observers may see different relative "values" of the Classical/Quantum components from their point-of-view:frame-of-reference in SpaceTime, each will see the same actual SR 4-Vector \( \mathbf{V} = V^\mu \) and its magnitude\(^2 = \mathbf{V} \cdot \mathbf{V} = V^\mu V^\nu = [(v^0)^2 - \mathbf{v} \cdot \mathbf{v}] = (v^0)^2 \) at a given <Event> in SpaceTime.
Magnitudes\(^2 \) can be \(+ =\text{temporal} \; 0 =\text{null} \; - =\text{spatial} \) in (+,−,−,−) Special Relativity, due to the Lorentzian=pseudo-Riemannian metric (non-positive-definite)

SRQM: A treatise of SR → QM by John B. Wilson (SciRealm@aol.com)
There are some paradigm assumptions that need to be cleared up:

Relativistic Physics **IS NOT** the generalization of Classical or Quantum Physics.

Classical & Quantum Physics **ARE** the low-relative-speed \( |v| << c \) limiting-case approximation of Relativistic Physics.

This includes (Newtonian) Classical Mechanics and Classical QM (NRQM: meaning the Non-Relativistic Schrödinger QM Equation – it is not fundamental). The rules of standard QM are just the low-relative-speed approx. of RQM rules. Classical EM is for the most part already compatible with Special Relativity. However, Classical EM doesn't include or take into account intrinsic spin, even though spin is a result of SR Poincaré Invariance, not QM.

So far, in all of my research, if there was a way to get a result classically, then there was usually a much simpler way to get the result using tensorial 4-Vectors and SRQM relativistic thinking. Likewise, a lot of QM results make much more sense when approached from SRQM (ex: **Spatial** relations).

**Einstein Energy:**
\[
P = m \cdot U \rightarrow \{ E = mc^2 = \gamma m_o c^2 = \gamma E_o : p = m \cdot u = \gamma m_u \}
\]

**Hamiltonian:**
\[
H = \gamma (P \cdot U) \quad \text{(Relativistic)} \rightarrow \{ T + V \} = \{ \text{Ekinetic} + E_{\text{potential}} \} \quad \text{(Classical-limit only, } |u| << c \}
\]

**Lagrangian:**
\[
L = -(P \cdot U)/\gamma \quad \text{(Relativistic)} \rightarrow \{ T - V \} = \{ \text{Ekinetic} - E_{\text{potential}} \} \quad \text{(Classical-limit only, } |u| << c \}
\]

\[
\text{(differential 4-Vector formats)}
\]

**SR/QM Wave Eqn** (inv of Phase Eqn):
\[
K_T = -\partial[\Phi_{\text{phase}}]/\hbar \rightarrow \{ \omega_T = -\partial[\Phi] : k_T = \nabla[\Phi] \}
\]

**Hamilton-Jacobi Eqn** (inv of Action Eqn):
\[
P_T = -\partial[S_{\text{action}}]/\hbar \rightarrow \{ E_T = -\partial[S] : p_T = \nabla[S] \}
\]

\[
\text{(integral 4-Scalar formats)}
\]

**SR Action Eqn** (inv of H-J Eqn):
\[
\Delta S_{\text{action}} = -\int \text{path} P_T \cdot dX = -\int \text{path} (P_T \cdot U) dt = \int \text{path} L dt
\]

**SR/QM Phase Eqn** (inv of Wave Eqn):
\[
\Delta \Phi_{\text{phase}} = -\int \text{path} K_T \cdot dX = -\int \text{path} (K_T \cdot U) dt = \Delta S_{\text{action}}/\hbar
\]

\[
\text{(advanced mechanics)}
\]

**Euler-Lagrange Equation:**
\[
(U = (d/dt)[X]) \rightarrow \{ \partial_R = (d/dt)\partial_U \} \quad \text{(the easy derivation)}
\]

**Hamilton’s Equations:**
\[
(d/dt)[X] = (\partial/d\partial P) [H]_0 \& (d/dt)[P_i] = (\partial/d\partial X) [H]_0
\]

\[
\text{(SR wave mechanics - requires a 4-WaveVector K as solution)}
\]

**d’Alembertian Wave Equation:**
\[
\partial^2 \Phi = (\partial/c)^2 - \nabla \cdot \nabla, \text{ with solutions } \sim \Sigma_n (A_n) e^{\pm (kn \cdot X)}
\]

**Einstein-de Broglie Relation:**
\[
P = \hbar K \rightarrow \{ E = \hbar \omega : p = \hbar k \}
\]

**Complex Plane-Wave Relation:**
\[
K = i\partial \rightarrow \{ \omega = i\partial : k = -i\nabla \}
\]

**Schrödinger Relations:**
\[
P = i\hbar \partial \rightarrow \{ E = i\hbar \partial : p = -i\nabla \}
\]

**Canonical QM Commutation Relations inc. QM Time-Energy:**
\[
[P^{\mu},X^{\nu}] = i\hbar \eta^{\mu\nu} \rightarrow \{ [x^0, p^i] = [ct, E/c] = [t, E] = -i\hbar : [x^i, p^j] = i\hbar \delta^{ij} \}
\]

\[
[\partial^\mu, X^\nu] = \eta^{\mu\nu} \rightarrow \{ [x^0, \partial^i] = [ct, \partial/c] = [t, \partial] = -1 : [x^i, \partial^j] = +\delta^{ij} \}
\]

**Total Momentum:**
\[
P_T = P + qA \rightarrow \{ E_T = E + q\phi : p_T = p + qa \}
\]

**Minimal Coupling:**
\[
P = P_T - qA \rightarrow \{ E_T = E - q\phi : p = p - qa \}
\]

\[
\text{(Physical Inverse Effects)}
\]

**Josephson-Junction (differential 4-Vector format):**
\[
A = -(h/q)\partial[\Delta \Phi_{\text{pot}}]/\partial X
\]

**Aharonov-Bohm (integral 4-Scalar format):**
\[
\Delta \Phi_{\text{pot}} = -(q/h)\int \text{path} A \cdot dX
\]

**Compton Scattering:**
\[
\Delta \lambda = (\lambda' - \lambda) = (h/m_c) (1 - \cos[\sigma])
\]

**Klein-Gordon Relativistic Quantum Wave Eqn:**
\[
\partial^2 \Phi = -(m_c/h)^2 \Phi
\]
There are some paradigm assumptions that need to be cleared up:

We will **NOT** be employing the commonly-(mis)used Newtonian classical limits \( \{c \to \infty\} \) and \( \{\hbar \to 0\} \).

Neither of these is a valid physical assumption, for the following reasons:

Both \( c \) and \( \hbar = \hbar/2\pi \) are unchanging Universal Physical Constants and Lorentz Scalar Invariants. Taking a limit where these change is non-physical. They are CONSTANT. Tensor math shows them invariant.

Many, many experiments verify that these physical constants have not changed over the lifetime of the universe. This is one reason for the 2019 Redefinition of SI Base Units on Fundamental Constants \( \{c, h, e, k_B, N_A, K_B, \Delta \nu_{Cs}\} \).

Photons/waves have energy \( E \) via momentum \( E=pc \) & frequency \( E=\hbar \omega \) : \( (\omega = 2\pi \nu) \) \{ angular [rad/s], circular[cycle/s], \( 2\pi \text{ rad} = 1 \text{ cycle} \}.

Let \( E = pc \). If \( c \to \infty \), then \( E \to \infty \). Then Classical EM light rays/waves have infinite energy.

Let \( E = \hbar \omega = hv \). If \( \hbar \to 0 \), then \( E \to 0 \). Then Classical EM light rays/waves have zero energy.

Obviously neither of these is true in the Newtonian/Classical limit.

In Classical EM and Classical Mechanics, LightSpeed \( (c) \) remains a large but finite constant.
Likewise, Dirac's (Planck-reduced) Constant \( (\hbar = \hbar/2\pi) \) remains very small but never becomes zero.

The correct way to take the limits is via:

The low-speed non-relativistic limit \( \{ |v| << c \} \), which is a physically-occurring situation.

The Hamilton-Jacobi non-quantum limit \( \{ \hbar|\nabla \cdot p| << (p \cdot p) \} \) or \( \{ |\nabla \cdot k| << (k \cdot k) \} \), which is a physically-occurring situation.
There are some paradigm assumptions that need to be cleared up:

We will *NOT* be implementing the common \{→lazy and extremely misguided\} convention of setting physical constants to the value of (dimensionless) unity, often called “Natural Units”, to hide them from equations; nor using mass \((m)\) instead of \((m^0)\) as the RestMass. Likewise for other components vs Lorentz Scalars with rest-value-naughts \((\_0)\), like energy \((E)\) vs \((E^0)\) as the RestEnergy.

One sees this very often in the literature. The usual excuse cited is “For the sake of brevity”. Well, the “sake of brevity” forsakes “clarity”. There is nothing physically “natural” about “natural units”.

The *ONLY* situations in which setting constants to unity \((1)\) is practical or advisable is in numerical simulation or mathematical analysis.

When teaching physics, or trying to understand physics: it helps when equations are dimensionally correct.

In other words, the physics technique of dimensional analysis is a powerful tool that should not be disdained.

\[ i.e. \text{Brevity only aids speed of computation, Clarity aids understanding} \]

The situation of using “naught = \(\_0\)”, for rest-values, such as \((m^0)\) for RestMass and \((E^0)\) for RestEnergy:

is intrinsic to SR, is a very good idea, absolutely adds clarity, identifies Lorentz Scalar Invariants, and will be explained in more detail later.

Essentially, relativistic gamma \((\gamma)\) pairs with invariant (Lorentz scalar:rest value \(\_0\)) to make a relativistic component: \{ \(m = \gamma m^0\); \(E = \gamma E^0\); \(p = \gamma p^0\); \(\omega = \gamma \omega^0\) \}

Note the multiple equivalent ways that one can write 4-Vectors of SpaceTime \((\text{Time} \cdot \text{Space})\) using these rules:

\[ 4\text{-Momentum } \mathbf{P} = P^\mu = (p^0, \mathbf{p}) = (mc=E/c, \mathbf{p}=mu) = (\gamma m^0, \gamma E^0/c, p=\gamma m^0) = -\partial [ S_{\text{action,free}} ] \]
\[ = m, \mathbf{U} = m, \gamma (c, \mathbf{u}) = \gamma m, (c, \mathbf{u}) = m, (c, \mathbf{u}) = (mc, \mathbf{mu}) = (mc, p) = mc(1, \beta) = m, c(1, \beta) = (m, c) \mathbf{T} \]
\[ = (E/c^2) \mathbf{U} = (E/c^2) \gamma (c, \mathbf{u}) = \gamma (E/c^2)(c, \mathbf{u}) = (E/c) \gamma (c, \mathbf{u}) = (E/c, \mathbf{Eu}/c^2) = (E/c, p) = (E/c)(1, \beta) = (E/c, \gamma (1, \beta) = (E/c)(1, \beta) = (E/c) \mathbf{T} \]

This notation makes clear what is \{ relativistically-varying=(frame-dependent) \} vs. Invariant=(frame-independent) \} and \{ Temporal vs. Spatial \}

BTW, I prefer the “Particle Physics” Metric-Signature-Convention \((+,-,-,-)\)=\{temporal:0\:+\}. \{Makes rest values positive, fewer minus signs to deal with\}

Show the physical constants and rest naughts \((\_0)\) in the work. They deserve the respect and you will benefit.

You can always set constants to unity later, when you are doing your numerical simulations.
There are some paradigm assumptions that need to be cleared up:

Some physics books on ElectroMagnetism (EM) say that the Electric field \( E \) and the Magnetic field \( B \) are the "real" physical objects, and that the EM scalar-potential \( \varphi \) and the EM 3-vector-potential \( \mathbf{A} \) are just "calculational/mathematical" artifacts.

Neither of these statements is relativistically correct.

All of these physical EM properties: \( \{E, B, \varphi, \mathbf{A} \} \) are actually just the components of SR tensors, and as such, their values will relativistically vary in different observers' reference-frames.

Given this SR knowledge, and to match our 4-Vector notation, we demote the physical property symbols, (the tensor "components") to their lower-case equivalents \( \{e, b, \varphi, a\} \).

see Wolfgang Rindler's works as example

The truly SR invariant physical objects are:

The 4-Gradient \( \partial \), the 4-VectorPotential \( \mathbf{A} \), their combination via the exterior (wedge=\( ^{\wedge} \)) product into the Faraday EM 4-Tensor \( F^{\alpha \beta} = \partial ^{\alpha} A^\beta - \partial ^{\beta} A^\alpha = (\partial ^{\wedge} \mathbf{A}) \), and their combination via the inner (dot=\( \cdot \)) product into the Lorenz Gauge 4-Scalar \( \partial \cdot \mathbf{A} = 0 \). Yes, Lorenz, not Lorentz.

\( \text{Lorentz Gauge: Conservation of EM (Vector)Potential} \)

\[ \partial \cdot \mathbf{A} = \left( \partial /c \right) \left( \varphi /c \right) - \nabla \cdot \mathbf{a} = 0 \]

\[ (\partial /c \cdot \mathbf{a}) + \nabla \cdot \mathbf{a} = 0 \]

Temporal-spatial components of 4-Tensor \( F^{\alpha \beta} \): electric 3-vector field \( \mathbf{e} = e^i = e^0 \)

Spatial-spatial components of 4-Tensor \( F^{\alpha \beta} \): magnetic 3-vector field \( \mathbf{b} = b^i = (\frac{1}{2}) \, \epsilon_{ij} \, F^{ij} \)

\( \text{Temporal component of 4-Vector} \mathbf{A} = A^0 = \text{EM scalar-potential} \varphi \)

\( \text{Spatial components of 4-Vector} \mathbf{A} = A^i = \text{EM 3-vector-potential} \mathbf{a} \)

Note that the Speed-of-Light \( c \) plays a prominent role in the component definitions.

Also, QM requires the 4-VectorPotential \( \mathbf{A} \) as explanation of the Aharonov-Bohm Effect. The physical measurability of the AB Effect proves the reality of the 4-Vector Potential \( \mathbf{A} \). Again, all the lower and higher-rank SR tensors can be built from fundamental 4-Vectors.
There are some paradigm assumptions that need to be cleared up:

A number of QM philosophies make the assertion that particle “properties” do not “exist” until measured. The assertion is based on the QM Heisenberg Uncertainty Principle, and more specifically on quantum non-zero commutation, in which a measurement on one property of a particle alters a different non-commuting property of the same particle.

That is an incorrect analysis. Properties define particles: what they do & how they interact with other particles. Particles and their properties “exist” as <events> independently of human intervention or observation. The correct way to analyze this is to understand what a measurement is: an arrangement of some number of particles in a particular manner as to allow an observer to get information about one or more of the “subject particle’s” properties. Typically this involves “counting” spacetime <events> and using SR invariant intervals as a basis-of-measurement.

Some properties are indeed non-commuting. This simply means that it is not possible to arrange a set of particles (the measuring device) in such a way as to measure (ie. obtain “complete” information about) both of the “subject particle’s” non-commuting properties at the same spacetime <event>. The measurement arrangement <events> can be done at best sequentially, and the temporal order of these <events> makes a difference in observed results. EPR-Bell, however, allows one to “infer” (due to conservation:continuity laws) properties on a “distant” subject particle by making a measurement on a different “local” {space-like-separated but entangled} particle. This does *not* imply FTL signaling nor non-locality.

The (psi-epistemic) measurement just updates local partial-information one already has about particles that interacted/entangled then separated.

So, a better way to think about it is this: The “measurement—updated information” of a property does not “exist” until a physical setup <event> is arranged. Non-commuting properties require different physical arrangements in order for the properties to be measured, and the temporally-first measurement alters that particle’s properties in a minimum sort of way, which affects the latter measurement. All observers agree on Causality, the time-order of temporally-separated spacetime <events>. However, individual observers may have different sets of partial information about the same particle(s).

This objective, realist view makes way more sense than the subjective belief that a particle’s actual properties don’t exist until it is “observed”, which is about as unscientific and laughable a statement as I can imagine.

**Relativity is the System-of-Measurement that QM has been looking for: Psi-Epistemic**
There are some paradigm assumptions that need to be cleared up:

**A well-formulated and correctly-used notation is critical for understanding physics**

Unfortunately, there are a number of “sloppy” notations seen in relativistic and quantum physics.

Incorrect: Using $T^{ii}$ as a Trace of 3D tensor $T^{ij}$, or $T^{\mu\mu}$ as a Trace of 4D Tensor $T^{\mu\nu}$

The Trace operation requires a paired upper-lower index combination (Einstein Sum), which then gets summed over.

Incorrect: Hiding factors of LightSpeed ($c$) in relativistic equations, ex. $E = m$

Wrong: $E = m$: Energy $[J = \text{kg} \cdot \text{m}^2/\text{s}^2]$ is *not* identical to mass $[\text{kg}]$, not in dimensional units nor in reality.

Correct: $E = mc^2$: Energy is related to mass via the Speed-of-Light ($c$) $[\text{m/s}]$, ie. mass is a type of concentrated energy.

Incorrect: Using $m$ instead of $m_o$ for rest mass; Using $E$ instead of $E_o$ for rest energy

Correct: $E = mc^2 = \gamma m_o c^2 = \gamma E_o$

$E$ & $m$, and $p$ are relativistic internal components of 4-Momentum $P = (E/c, p) = (mc, p)$ which vary in different reference-frames.

$E_o$ & $m_o$ are Lorentz Scalar Invariants, the SR Rest Values, which are the same, even in different reference-frames: $P = m_o U = (E_o/c^2) U$
There are some paradigm assumptions that need to be cleared up:

Incorrect: Using the same symbol for a tensor-index and a component
The biggest offender in many books for this one is quantum commutation. Unclear because (i) means two different things in the same equation.
Correct way: \(i = \sqrt{-1}\) is the imaginary unit; \(\{j,k\}\) are tensor-indices
In general, any equation which uses complex-number math should reserve \(i\) for the imaginary, not as a tensor-index.

Incorrect: Using the 4-Gradient: Gradient One-Form notation incorrectly
The 4-Gradient is a 4-Vector, a (1,0)-Tensor, uses an upper index, and has a negative spatial component \((-\nabla)\) in \((+,+,-,-)\) SR.
The Gradient (4D) One-Form, its more natural tensor form, a (0,1)-Tensor, uses a lower index in SR.

Incorrect: Mixing styles in 4-Vector naming conventions
There is pretty much universal agreement on the 4-Momentum \(P=P^\mu=(p^0,p^\nu)=(E/c,p)=(mc,p)=(E/c,p)=(mc,p)\)
Do not in the same document use 4-Potential \(A=(\phi,A)\): This is wrong on many levels, inc. dimensional units.
The correct form is 4-Vector Potential \(A=A^\mu=(a^0,a^\nu)=(\phi/c,a)\), with \(\phi\) the scalar-potential and \(a\) the 3-vector-potential

For all SR 4-Vectors, one should use a consistent notation:

The UPPER-CASE SpaceTime (Time·Space) 4-Vector Names match the lower-case spatial 3-vector names
There is a LightSpeed (c) factor in the temporal component to give overall matching dimensional units for the entire 4-Vector
4-Vector components are typically lower-case with a few exceptions, mainly energy (E) vs. energy-density \(\{e,\rho_e,\rho_c\}\)
Old Paradigm: QM (as I was taught...)

SR and QM as separate theories

Simple GR Axioms:
- Principle of Equivalence
- Invariant Interval Measure
- Tensors describe Physics
- SpaceTime Metric $g^{\mu\nu}$
- $\{c, G\} = \text{physical constants}$

$\text{GR}$

$\text{SR}$

$\text{SR}$ limiting-case: $g^{\mu\nu} \rightarrow \eta^{\mu\nu}$
Minkowski "Flat" SpaceTime Metric = (Curvature $\sim 0$)

SR limiting-case: $|v| \ll c$

Obscure QM Axioms:
- Wave-Particle Duality
- Unitary Evolution
- Operator Formalism
- Hilbert Space Representation
- Principle of Superposition
- Canonical Commutation Relation
- Heisenberg Uncertainty Principle
- Pauli Exclusion Principle (FD-statistics)
- Bose Aggregation Principle (BE-statistics)
- Hermitian Generators
- Correspondence Principle to CM
- Born Probability Interpretation
- $\{\hbar = h/2\pi\} = \text{physical constant}$

$\text{QM}$

$\text{QM}$ limiting-case: # particles $N \gg 1$
but not always...
see Bose-Einstein condensate, superfluidity

$\text{RQM}$

$\text{QFT}$

$\text{CM}$

Quantum Gravity ???

Multiple Particles

This was the QM paradigm that I was taught while in Grad School: everyone trying for Quantum Gravity
Old Paradigm: QM (years later...)

SR and QM still as separate theories

QM limiting-case better defined, still no QG

Simple GR Axioms:
- Principle of Equivalence
- Invariant Interval Measure
- Tensors describe Physics
- SpaceTime Metric $g^{\mu\nu}$
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- $\{\hbar|\nabla\cdot p| \ll (p\cdot p)\}$ or $\{|\nabla\cdot k| \ll (k\cdot k)\}$ or $\psi \rightarrow \text{Re}[\psi]$ 

GR limiting-case: $g^{\mu\nu} \rightarrow \eta^{\mu\nu}$

Yet another “would be” fortuitous merging??

SR limiting-case: $|v| << c$

A fortuitous merging?

QM limiting-case:

It is known that QM + SR “join nicely” together to form RQM, but problems with RQM + GR...
SRQM Study:
Physical Theories as Venn Diagram
Which regions are empirically real?

Many QM physicists believe that the regions outside of QM don’t exist…
SRQM Interpretation would say that the regions outside of GR probably don’t exist...

**GR**: General Relativity

**QM**: Quantum Mechanics

**SR**: Special Relativity

**CM**: Classical Mechanics

**CM** limiting-case: $|v| << c$

**QM** limiting-case: $\hbar |\nabla \cdot p| << (p \cdot p)$

**RQM**: Relativistic QM

- Many-Worlds Interpretations
- Non-local interactions
- Instantaneous QM entangled connections
- Instantaneous Physical Wavefunction Collapse
- Spacetime Dimensions $>4$
- Hidden: Alternate Dimensions
- Super-Symmetry
- String Theory
- Alternate Gravity Theories
- Slews of hypothetical new particles etc.

**SR → QM (SRQM) fits the facts...**

**Quantum Mysticism**

Basically lots of stuff for which there is no empirical evidence... & loads of hype...

Many QM physicists believe that the regions outside of QM don’t exist...
SRQM Interpretation would say that the regions outside of GR probably don’t exist...
SRQM Study: Physical Limit-Cases as Venn Diagram
Which limit-regions use which physics?

Instead of taking the Physical Theories as set, examine Physical Reality and then apply various limiting-conditions.

What do we then call the various regions?

As we move inwards from any region on the diagram, we are adding more stringent conditions which give physical limiting-cases of “larger, more encompassing” theories.

If one is in Classical GR, one can get Classical SR by moving toward the Minkowski SpaceTime limit.

If one is in RQM, one can get Classical SR by moving toward the Hamilton-Jacobi non-QM limit, or to standard QM by moving toward the SR low-velocity limit.

Looking at it this way, I can define SRQM to be equivalent to Minkowski SpaceTime, which contains RQM, and leads to Classical SR, or QM, or CM by taking additional limits.

My assertion:
There is no “Quantized Gravity”
Actual GR contains SRQM and Classical GR.
Perhaps “Gravitizing QM”...

SRQM: A treatise of SR → QM by John B. Wilson (SciRealm@aol.com)
Special Relativity → Quantum Mechanics

Background: Proven Physics

Both General Relativity (GR) and Special Relativity (SR) have passed very stringent tests of multiple varieties. Likewise, Relativistic Quantum Mechanics (RQM) and standard Quantum Mechanics (QM) have passed all tests within their realms of validity:

- Generally micro-scale systems: e.g., single particles, ions, atoms, molecules, electric circuits, atomic-force microscopes, etc., but a few special macro-scale systems: e.g., Bose-Einstein condensates, super-currents, super-fluids, long-distance entanglement, etc.

To-date, however, there is no observational/experimental indication that quantum effects "alter" the fundamentals of either SR or GR. Likewise, there are no known violations, QM or otherwise, of Local Lorentz Invariance (LLI) nor of Local Position/Poincaré Invariance (LPI).

In fact, in all known experiments where both SR/GR and QM are present, QM respects the principles of SR/GR, whereas SR/GR modify the results of QM. All tested quantum-level particles, atoms, isotopes, super-positions, spin-states, etc. obey GR's Universality of Free-Fall & Equivalence Principle and SR's $E = mc^2$ and speed-of-light (c) communication/signaling limit. Meanwhile, quantum-level atomic clocks are used to measure gravitational red/blue-shift effects.

Think about that for a moment...

Some might argue that QM modifies the results of SR, such as via non-commuting measurements. However, that is an alteration of CM expectations, not SR expectations. In fact, there is a basic non-zero commutation relation fully within SR: \( \left[ \partial_\mu, X^\nu \right] = \eta_{\mu\nu} \). The actual commutation part (Commutator \([a,b]\)) is not about \(\hbar\) or \(i\), which are just invariant Lorentz Scalar multipliers.

On the other hand, GR Gravity *does* induce changes in quantum interference patterns and hence modifies QM:

- See the COW gravity-induced neutron QM interference experiments, the LIGO & VIRGO & KAGRA gravitational-wave detections via QM interferometry, and now also QM atomic matter-wave gravimeters via QM interferometry.
- Likewise, SR induces fine-structure splitting of spectral lines of atoms, "quantum" spin, spin magnetic moments, spin-statistics (fermions & bosons), antimatter, QED, Lamb shift, relativistic heavy-atom effects (liquid mercury, yellowish color of gold, lead batteries having higher voltage than classically predicted, heavy noble-gas interactions, relativistic chemistry...), etc. - essentially requiring QM to be RQM to be valid. QM is instead seen to be the limiting-case of RQM for \(|v| << c\).

Some QM scientists say that quantum entanglement is "non-local", but you still can't send any real messages/signals/information/particles faster than SR's speed-of-light (c). The only "non-local" aspect is the alteration of probability-distributions based on knowledge-changes obtained via measurement. A local measurement can only alter the "partial information" already-known about the probability-distribution of a distant (entangled) system.

There is no FTL-communication-with nor alteration-of the distant particle. Getting a Stern-Gerlach "up" here doesn't cause the distant entangled particle to suddenly start moving "down" there. One only knows "now" that it "would" go down "if" the distant experimenter actually performs the measurement.

QM respects the principles of SR/GR, whereas SR/GR modify the results of QM.
Special Relativity → Quantum Mechanics

Background: GR Principles

Known Physics ↔ Empirically Tested

**Principles/Axioms and Mathematical Consequences of General Relativity (GR):**

**Equivalence Principle:** Inertial Motion = Geodesic Motion, Universality of Free-Fall, Mass Equivalency (Mass\_inertial = Mass\_gravitational)

**Relativity Principle:** SpaceTime (M) has a Lorentzian=pseudo-Riemannian Metric (g\_μν), SR:Minkowski Space rules apply locally (g\_μν → η\_μν) (Minkowski)

**General Covariance Principle:** Tensors describe Physics, General Laws of Physics are independent of arbitrarily chosen Coordinate-Systems

**Invariance Principle:** Invariant Interval Measure comes from Tensor Invariance Properties, 4D SpaceTime from Invariant Trace[g\_μν] = 4

**Causality Principle:** Minkowski Diagram/Light-Cone give {Time-Like (+), Light-Like (Null=0), Space-Like (-)} Measures and Causality Conditions

Einstein:Riemann’s Ideas about Matter & Curvature:
Riemann(g) has 20 independent components → too many
Ricci(g) has 10 independent components = enough to describe/specify a gravitational field

{c,G} are Fundamental Physical Constants; so is {h}, but less well-known that this comes from SR

**To-date, there are no known violations of any of these GR Principles.**

GR has passed EVERY observational test to-date, in both weak and strong field regimes.

It is vitally important to keep the mathematics grounded in known physics.
There are too many instances of trying to apply top-down, theoretical-only mathematics to physics.
(ex. String Theory, SuperSymmetry: no physical evidence to-date; SuperGravity: physically disproven)
Progress in science doesn’t work that way. Nature itself is the arbiter of what math works with physics. Tensor mathematics applies well to known physics {SR and GR}, which have been empirically extremely well-tested in a huge variety of physical situations. Tensors describe physics.

All known experiments to date comply with all of these Principles, including QM and RQM
Old Paradigm: QM Axioms (for comparison)

SR and QM still as separate theories
QM limiting-case better defined, still no QG

Simple GR Axioms:
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- Tensors describe Physics
- SpaceTime Metric $g^{\mu\nu}$
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GR limiting-case: $g^{\mu\nu} \rightarrow \eta^{\mu\nu}$
Minkowski “Flat” SpaceTime Metric = (Curvature ~ 0)

Obscure QM Axioms:
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- Hermitian Generators
- Correspondence Principle to CM
- Born Probability Interpretation

$\{h = h/2\pi\} = \text{physical constant}$

It is known that QM + SR “join nicely” together to form RQM, but problems with RQM + GR...

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf
*New Paradigm: SRQM or [SR→QM]*

QM derived from SR + a few empirical facts
Simple and fits the data

This new paradigm explains why RQM “miraculously fits” SR, but not necessarily GR
This new paradigm explains why RQM “miraculously fits” SR, but not necessarily GR.
Classical SR w/ EM Paradigm (for comparison)

CM & EM derived from SR + a few empirical facts

The entire classical SR→{EM,CM} structure is based on the limiting-case of quantum effects being negligible.

Notice that only the SR 4-Vector relation: \( \textbf{K} = (1/\hbar)\textbf{P} \)

is missing from the Classical Interpretation…

All of the SR 4-Vectors, including \( \textbf{K} \) & \( \partial \), are still present in the Classical setting.

\( \textbf{K} \) is used in the Relativistic Doppler Effect and EM waves. \( \partial \) is used in the SR Conservation/Continuity Equations, Maxwell Equations, Hamilton-Jacobi, Lorenz Gauge, etc.

\( \partial = (-i)\textbf{K} \) may be somewhat controversial, but it is the equation for complex plane-waves, which are still used in classical EM.

Background Inherent Assumption

QM limiting-case:

\( \hbar |\mathbf{V} \cdot \mathbf{p}| < \epsilon (\mathbf{p} \cdot \mathbf{p}) \) or \( \psi \rightarrow \text{Re}[\psi] \) or

\( |\mathbf{V} \cdot \mathbf{k}| < (\mathbf{k} \cdot \mathbf{k}) \)

(\( \text{doesnt depend on } \hbar \))

Hamilton-Jacobi non-quantum limit

Change by a few quanta has negligible effect on overall state

This (Classical=non-QM) SR→{EM,CM} approx. paradigm has been working successfully for decades...
The SRQM view:
Each level (range of validity) is a subset of the larger level.

General Relativity (GR)

Special Relativity (SR) → Relativistic Quantum Mechanics (QM)

GR limiting-case: \( g^{\mu \nu} \rightarrow \eta^{\mu \nu} \) Minkowski “Flat” SpaceTime = (Curvature ~ 0)

Non-relativistic Quantum Mechanics (QM)

QM limiting-case: \( |v| \ll c \)

Non-relativistic Quantum Mechanics (QM)

CM

Classical Mechanics

QM limiting-case: \( \hbar |\nabla \cdot p| \ll (p \cdot p) \) or \( \psi \rightarrow \text{Re}[\psi] \) or \( |\nabla \cdot k| \ll (k \cdot k) \)

Change by a few quanta has negligible effect on overall state

SRQM: A treatise of SR→QM by John B. Wilson (SciRealm@aol.com)
SRQM = New Paradigm: SRQM w/ EM View as Venn Diagram Ranges of Validity

The SRQM view: Each level (range of validity) is a subset of the larger level.

GR
General Relativity

SRQM
Special Relativity → Relativistic QM
GR limiting-case: $g^{\mu\nu} \to \eta^{\mu\nu}$ Minkowski “Flat” SpaceTime = (Curvature ~ 0)

QM
Non-relativistic Quantum Mechanics
SRQM limiting-case: $|v| << c$

CM
Classical Mechanics
QM limiting-case: $\hbar|\nabla \cdot p| << (p \cdot p)$ or $\psi \to \text{Re}[\psi]$ or $|\nabla \cdot k| << (k \cdot k)$
Change by a few quanta has negligible effect on overall state

EM w/ spin
ex. Stern-Gerlach
ex. photoelectric effect
ex. electron diffraction
ex. Aharonov-Bohm

QED
ex. Hawking-Unruh
radiation

$q = \text{EM charge}$
$A = 4-\text{EM Vector Potential}$

SRQM: A treatise of SR→QM by John B. Wilson (SciRealm@aol.com)
SRQM: SR language beautifully expressed with Physical 4-Vectors

Newton's laws of classical physics are greatly simplified by the use of physical 3-vector notation, which converts 3 separate space components, which may be different (relative) in various coordinate systems, into a single invariant object: a 3D vector, with an invariant 3D magnitude (but not 4D invariant). The basis-values of these components can differ in certain invariant ways, via Galilean transforms, yet still refer to the same overall 3-vector object.

SR is able to expand the concept of mathematical vectors into the Physical 4-Vector, which combines both (time) and (space) components into a single (Time\times Space) object: These 4-Vectors are elements of Minkowski 4D SR Space-Time. They have Lorentzian invariant magnitude that combines both classical (3D) and relativistic (4D) magnitudes. There is a Speed-of-Light factor (invariant) in the temporal component to make the dimensional units match. However, unlike the 3D magnitude^2 (only +Riemann=positive-definite, the 4D magnitude^2 can be (+/0/-)pseudo-Riemannian=CausalConditions.

In this presentation: I style classical 3D objects this way (by a triangle/wedge Δ) to emphasize that they are actually just the separated components of SR 4-Vectors. The triangle/wedge Δ (3 sides) represents splitting the components into a scalar and 3-vector.

In this presentation:
I use the \{(Time,0^0,+)\}=(\{+,-,-\}) metric signature, giving \[ A\cdot A = A\cdot\eta_{iv}A^i = [(a^i)^2 - a\cdot a] = (a^i)^2 \]
For 4-Vectors will use Upper-Case Letters, ex. A; 3-vectors will use lower-case letters, ex. a; I always put the (c) dimensional factor in the temporal component. Vectors of both types will be in bold font; components and scalars in normal font and usually lower-case. 4-Vector name will match with 3-vector name. Tensor form will usually be normal font with tensor indices: { Greek \( Time\times Space \) index \((0,1..3)\): ex. \( A = A^i \) \} or { Latin \( SpaceOnly \) index \((1..3)\): ex. \( a = a^k \) }

<table>
<thead>
<tr>
<th>SR 4-Tensor (2,0)-Tensor T^iv</th>
<th>SR 4-Vector (1,0)-Tensor V^i = (v^0, v)</th>
<th>SR 4-Vector (1,0)-Tensor V^i = (v^0, v)</th>
<th>SR 4-Vector (0,1)-Tensor V_i^v = (v_0, -v)</th>
<th>SR 4-Scalar (0,0)-Tensor S or ( S^i_i ) = Lorentz Scalar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classical (scalar) 3-vector</td>
<td>not Lorentz Invariant</td>
<td>Lorentz Invariant</td>
<td>Lorentz Invariant</td>
<td>Lorentz Scalar Invariant</td>
</tr>
<tr>
<td>Classical 4-Vector 4-Position ( R )</td>
<td>Classical 3-vector (3D)</td>
<td>Classical scalar (1D)</td>
<td>Lorentz 4-Scalar [m/s]</td>
<td>Trace[T^iv] = ( \eta_{iv}T^iv = T^i_i = T )</td>
</tr>
<tr>
<td>( R^u = (r^u) = (ct,r) )</td>
<td>( r^i = (r^i) \rightarrow (x,y,z) )</td>
<td>( 1-time \ t \ &lt;time&gt; )</td>
<td>( c )</td>
<td>( v\cdot v = V^i\eta_{iv}V^v = (v^0)^2 - v\cdot v = (v^0)^2 )</td>
</tr>
</tbody>
</table>
4-Vectors are 4D (1,0)-Tensors, Lorentz 4-Scalars are 4D (0,0)-Tensors, 4-CoVectors are 4D (0,1)-Tensors, (m,n)-Tensors have (m) upper-indices and (n) lower-indices. Any equation which employs only Tensors, such as those with only 4-Vectors and Lorentz 4-Scalars, is automatically Frame-Invariant, or coordinate-frame-independent. One’s frame-of-reference plays no role in the form of the overall equations. This is also known as being “Manifestly-Invariant”, when no inner components are used. This is exactly what Einstein meant by his postulate: “The laws of physics should have the same form for all inertial observers”. Use of the RestFrame-naught (ο) helps show this.

It is seen when the spatial part (τ) of a magnitude can be set to zero (= at-rest). The temporal part (τ0) would then equal the rest value (τ0ο).

The components (τ0, τ1, τ2, τ3) of the 4-Vector V can relativistically vary depending on the observer and their choice of coordinate system, but the 4-Vector V = V0 itself is invariant. Equations using only 4-Tensors, 4-Vectors, and Lorentz 4-Scalars are true for all inertial observers. The SRQM Diagramming Method makes this easy to see in a visual format, and will be used throughout this treatise.

The following examples are SR Time·Space frame-invariant equations:
SR 4-Vectors are the primitive elements of Minkowski SpaceTime $(1+3)$D. We want to be clear, however, that SR 4-Vectors are NOT generalizations of Classical or Quantum 3-vectors.

SR 4-Vectors are the primitive elements of Minkowski SpaceTime (TimeSpace) $4D \rightarrow (1+3)D$, which incorporate both: a (temporal scalar element) and a (spatial 3-vector element) as components. Temporals and Spatials are metrically distinct, but can mix in SR. 4-Vector $A = A^\mu = (a^0,a^1,a^2,a^3) = (a'_0,a'_1,a'_2,a'_3)$ with component scalar $(a^0) \rightarrow (a'_0)$ & component 3-vector $(a^1 = a) \rightarrow (a'_1,a'_2,a'_3)$

It is the {Classical (Newtonian) or Quantum} 3-vector $(a)$ which is a limiting-case approximation of the spatial part of SR 4-Vector $(A)$ for $|v| \ll c$.

i.e. The energy $(E)$ and 3-momentum $(p)$ as “separate” entities occurs only in the low-velocity limit $|v| \ll c$ of the Lorentz Boost Transform. They are actually part of a single 4D entity: the 4-Momentum $P = (E/c,p)$ with the components: temporal energy $(E)$, spatial 3-momentum $(p)$, dependent on a frame-of-reference, while the overall 4-Vector $P$ is invariant. Likewise with time $(t)$, space 3-position $(r)$ in the 4-Position $R=(ct,r)$.

SR is 4D Minkowskian; obeys Lorentz/Poincaré Invariance.

CM is 3D Euclidean; obeys Galilean Invariance.

**4-Vector SRQM Interpretation of QM**

http://scirealm.org/SRQM.pdf

SciRealm.org
John B. Wilson
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Classical (scalar) not Lorentz Invariant

3D Galilean Invariant

$Lorentz \text{ Scalar Invariant}

Trace[T^{νμ}] = \eta_{νμ}T^{νμ} = T_{νμ} = T^μ_ν

V\cdot V = V^ν\eta_νV^μ = [(v^ν)^2 - v^ν_0 c]^2 = (v^ν)^2

= Lorentz Scalar Invariant

SR 4-Vector

$(1,0)$-Tensor $V^\nu = V = (v^0,v)$

SR 4-Vector

$(1,1)$-Tensor $T^\nu_\nu$, or $T^μ_ν$

SR 4-Scalar

$(0,0)$-Tensor $S$ or $S_0$

Lorentz Scalar

A Tensor Study of Physical 4-Vectors
**SRQMQ:
SR 4-Vectors & Lorentz Scalars**

**Rest Values (“naughts”=o) are Lorentz Scalars**

\[ \mathbf{A} = (a^0, a, a^1, a^2) \]

The rest-values of several physical properties are all Lorentz scalars. We can choose a frame that may simplify the expressions.

Choose a frame in which the spatial component is zero. This is known as the “rest-frame” of the 4-Vector. It is not moving spatially.

\[ \mathbf{P} = (m_\text{o}) = (E/c)_\text{o} \rightarrow (\mathbf{E}/c)_\text{o} \]

\[ \mathbf{P} \cdot \mathbf{U} = m_\text{o}c^2 = (E/c)_\text{o} \]

This leads to simple relations between 4-Vectors.

\[ \mathbf{P} = (m_\text{o}) U = (E/c)_\text{o} \]

And gives nice Scalar Product relations between 4-Vectors as well.

\[ \mathbf{P} \cdot \mathbf{K} = (m_\text{o}c^2) \mathbf{K} = (E/c)_\text{o} \]

This property of SR equations is a very good reason to use the “naught” convention for specifying the difference between relativistic component values which can vary, like (m), versus RestValue Invariant Scalars, like (m_0), which do not vary. They are usually related via a Lorentz Factor: \( \gamma = \gamma_{m_0} ; E = \gamma E_0 ; \omega = \gamma \omega_0 \), as seen in the relations of \( \mathbf{P} , \mathbf{K} , \mathbf{U} , \) and \( \mathbf{T} \).

\[ \mathbf{P} = (mc, p) = (m_\text{o}) U = (E/c)_\text{o} \]

\[ \mathbf{P} = (E/c, p) = (E/c)_\text{o} \gamma (c, u) = (yc_{\text{o}}, yc_{\text{o}} mu) = (mc, p) = (m_\text{o}) T = (mc) \gamma (1, \beta) = (mc)(1, \beta) \]

\[ \mathbf{P} \cdot \mathbf{K} = (m_\text{o}c^2) \mathbf{K} = (E/c)_\text{o} \mathbf{K} = \mathbf{P} = (\text{scalar invariant}) \mathbf{K} \]

\[ \mathbf{K} = (\omega/c, k) = (\omega/c, \omega \hat{n} / V_{\text{phase}}) \]

Trace\([\mathbf{T}^{\nu \nu}] = \eta_{\mu \nu} T^{\mu \nu} = T_{\nu \nu} = T \]

\[ \mathbf{V} \cdot \mathbf{V} = V_{\nu} V^{\nu} = (V^0)^2 - V_{\text{phase}}^2 = (V^0)^2 \]

\[ \mathbf{V} \cdot \mathbf{V} = \text{Lorentz Scalar Invariant} \]

---

**Notation:**

- **"0" for rest values { naughts, "(o)bserver value" }**
- **"0" for temporal components { 0\textsuperscript{th} index }**

---

**SR 4-Tensor (2,0)-Tensor T\text{iv}**

**SR 4-Vector (1,0)-Tensor V\textsuperscript{v} = V = (v^0, v)\]**

**SR 4-CoVector:OneForm (0,1)-Tensor V\textsubscript{0} = (v_0, v)\]**

**SR 4-Scalar (0,0)-Tensor S or S\textsubscript{0} Lorentz Scalar**
SRQM Study: Manifest Invariance

Invariant SR 4-Vector Relations

Consider a particle at a SpaceTime (Time·Space) <Event> that has properties described by 4-Vectors A and B:

One possible relationship is that the two 4-Vectors are related by a Lorentz 4-Scalar (S): ex. B = (S) A.

How can one determine this? Answer: Make an experiment that empirically measures the tensor invariant [ B·C / A·C ].

If B = (S) A then B·C = (S) A·C, giving (S) = [ B·C / A·C ]

if C=A, then (S) = [ B·A / A·A ] This basically a standard vector projection.

if C=other, Invariant result mediated by another 4-Vector C, always possible.

Run the experiment many times. If you always get the same result for (S), then it is likely that the relationship is true, and thus invariant.

Example: Measure (S_p) = [ P·U / U·U ] for a given particle type.
Repeated measurement always give (S_p) = m_v
This makes sense because we know [ P·U ] = γ(E - p·u) = E_v and [ U·U ] = c^2
Thus, 4-Momentum P = (E_v/c^2)U = (m_v)U = (m_v)c^2 4-Velocity U

Example: Measure (S_K) = [ K·U / U·U ] for a given particle type.
Repeated measurement always give (S_K) = (ω/c^2)
This makes sense because we know [ K·U ] = γ(ω - k·u) = ω_v and [ U·U ] = c^2
Thus, 4-WaveVector K = (ω/c^2)U = (ω_v/c^2)c^2 4-Velocity U

Since P and K are both related to U, this would also mean that the 4-Momentum P is related to the 4-WaveVector K in a particular Lorentz Invariant manner for each given particle type… a major hint for later...

SR 4-Tensor
(2,0)-Tensor T^uv
(1,1)-Tensor T^v_u, or T_{uv}
(0,2)-Tensor T^ux
SR 4-Vector
(1,0)-Tensor V^u = V = (v^0, v)
SR 4-CoVector:OneForm
(0,1)-Tensor V_μ = (v_μ, v)
SR 4-Scalar
(0,0)-Tensor S or S_0
Lorentz Scalar

4-Vector SRQM Interpretation of QM

http://scirealm.org/SRQM.pdf
A Tensor Study

Some SR Mathematical Tools
Definitions, Approximations, Misc.

\( \beta = \frac{v}{c} ; \beta = |\beta|; \)

\( \gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\beta \cdot \beta}}; \)

dimensionless Velocity Beta Factor

dimensionless Lorentz Relativistic Gamma Factor

\( \{ \beta=(0..1); \text{rest at} (\beta=0); \text{speed-of-light} (c) \text{ at} (\beta=1) \} \)

\( \{ \gamma=(1..\infty); \text{rest at} (\gamma=1); \text{speed-of-light} (c) \text{ at} (\gamma=\infty) \} \)

\((1+x)^n \sim (1 + nx + O[x^2]) \) for \( \{ |x| \ll 1 \} \) Approximation used for \( \text{SR} \rightarrow \text{Classical limiting-cases} \)

Lorentz Transformation \( \Lambda^\mu{}_{\nu} = \frac{\partial X'{}^\mu}{\partial X^\nu} = \delta[|X'|] \): a relativistic frame-shift, such as a Rotation or Velocity-Boost.

It transforms a 4-Vector in the following way: \( X'^\mu = \Lambda^\mu{}_{\nu} X^\nu \) : with Einstein summation over the paired indices, and the (') indicating an alternate frame.

A typical Lorentz Boost Transformation \( \Lambda^\mu{}_{\nu} \rightarrow B^\mu{}_{\nu} \) for a linear-velocity frame-shift \((x,t)-\text{Boost in the} \ x \text{-direction:} \)

\[ \begin{align*}
\text{Original } A^\nu &= (a^i, a^i, a^i, a^i) \\
\text{Boosted } A'^\nu &= (a^i, a^i, a^i, a^i) = \Lambda^\nu{}_{\nu} \rightarrow B^\nu{}_{\nu} = (\gamma a^i - \gamma \beta a^i + \gamma a^i, a^i, a^i, a^i) \\
A^i \cdot B^k &= (\Lambda^i{}_{i}) \cdot (\Lambda^k{}_{k}) = A^i \cdot B^k = (a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3 - a^0 b^0) = (a^0 b^0 - a^i b^i) \\
&= A^i B^k = \Sigma_{i=0}^{3} (a^0 b^0) = (a^0 b^0 + a^0 b^1 - a^0 b^2 - a^0 b^3) \\
&= a_0 b_0 + \Sigma_{i=0}^{3} a_i b_i \\
\text{using the Einstein Summation Convention where upper:lower paired-indices are summed over.}
\end{align*} \]

\( \delta[X] = \nabla \partial[X] = (\partial_x/c, \nabla) (ct, x) = \text{Diag}[\partial_x/c, \nabla] = \text{Diag}[1, -1, -1, 1] = \eta^{\mu\nu} \) Minkowski “Flat” SpaceTime Metric

\( \text{SR: Minkowski Metric} \)

\[ \begin{align*}
\partial[R] &= \partial[R'] = \eta^{\mu\nu} = V^{\mu\nu} + H^{\mu\nu} \\
\text{Diag}[+1, -1, -1, -1] &= \text{Diag}[1, -1, -1, -1] = \text{Diag}[1, -\delta^{i\nu}] \\
\{ \text{in Cartesian form, “Particle Physics” Convention} \} \\
\{ \eta_{\mu\nu} \} &= 1/\{ \eta^{\mu\nu} \} = \delta^{\mu\nu} = \delta_{\nu}^{\mu} \\
\text{Tr}[\eta^{\mu\nu}] &= 4
\end{align*} \]

\( \text{SR: Lorentz Transform} \)

\[ \frac{\partial[R']}{\partial[R]} = \delta^{\mu\nu} \]

\[ \Lambda^\mu{}_{\nu} = (\Lambda^{-1})^\nu{}_{\mu} = \Lambda^{\mu}_{\nu} \Lambda^{\nu}_{\mu} = \eta_{\mu\nu} = \delta_{\nu}^{\mu} \]

\[ \Lambda_{\mu}^{\nu} \Lambda_{\nu}^{\mu} = 4 = \Lambda_{\mu}^{\nu} \Lambda_{\nu}^{\mu} \\
\text{Det}[\Lambda_{\mu}^{\nu}] = \pm 1
\]

\[ \text{Tr}[\Lambda_{\nu}^{\mu}] = \{\infty, \infty\} \]

\[ \text{Lorentz Transform Type} \]

\[ \text{SpaceTime} \]

\( \partial R = \partial R^{\mu\nu} = 4 \text{ Dimension} \)

\( \text{Trace}[T_{\nu}^{\mu}] = \eta_{\mu\nu} T_{\nu}^{\mu} = T_{\nu}^{\nu} = T \)

\( V \cdot V = V^{\mu} V_{\nu} = [(V^{\mu})^2 - V^{\mu} V_{\nu} = (V^0)^2 \]

\( \text{Lorentz Scalar Invariant} \)
SRQM Study: Ordering of Time·Space <Events>

Temporal Causality vs. Spatial Topology
Simultaneity vs. Stationarity

Venn Diagram

Properties of Minkowski:SR SpaceTime <Events>

**Time-Like Ordering of...**

- **Time-Like Separated <Events>**
  - **Causal:** Invariant = Absolute Temporal Order (A→B→C)
    - ProperTime (\(t = \tau\)) for | clock at-rest |}
    - Time Dilation (\(t = \gamma t\) for ...→ moving clock →)
  - All observers agree on temporal order of time-separated events, although temporal event separation may be ←[Time-Dilated]→

- **Light-Like (Null) Separated <Events>**
  - **Causal:** Invariant = Absolute Temporal Order (A→B→C)
    - All observers agree on temporal order of light-separated events, and on the invariant TimeSpace <Event> interval measurement.
    - All observers measure invariant LightSpeed (c) in their own frames.

- **Space-Like Separated <Events>**
  - **Non-Causal:** Relative → Relativity of Simultaneity (A→?→B)
    - Simultaneity: (only if in reference-frame with Same-Time occurrence)
      - ("no wait" for stationary particle/worldline, "motion" in all other frames)
      - Any 2 time-separated events may occur in any spatial order = frame-dependent

**Space-Like Ordering of...**

- **Time-Like Separated <Events>**
  - Non-Topological: Relative → Relativity of Stationarity (A→?→B)
    - Stationarity: (only if in reference-frame with Same-Place occurrence)
      - ("no motion" for stationary particle/worldline, "motion" in all other frames)
      - Any 2 time-separated events may occur in any spatial order = frame-dependent

- **Light-Like (Null) Separated <Events>**
  - **Topological:** Invariant = Absolute Spatial Order (A→B→C)
    - All observers agree on spatial order/topology of light-separated events, and on the invariant TimeSpace <Event> interval measurement.
    - All observers measure invariant LightSpeed (c) in their own frames.

- **Space-Like Separated <Events>**
  - **Topological:** Invariant = Absolute Spatial Order (A→B→C) or (C→B→A)
    - ProperLength (\(L_o\)) for | ruler at-rest |}
    - Length Contraction (\(L = L_o/\gamma\)) for ...→ moving ruler ←)
    - All observers agree on spatial order/topology of space-separated events, although spatial event separation may be ←[Length-Contracted]→

**Spatial Topology**

- **4-Displacement (between <events>)**
  - \(\Delta R \cdot \Delta R^\prime = (c\Delta t \Delta r)^2 - \Delta r \cdot \Delta r \rightarrow 0\)

- **Space-Like Invariant Interval**
  - \(\Delta R \cdot \Delta R^\prime = (c\Delta t \Delta r)^2 - \Delta r \cdot \Delta r \rightarrow 0\)

- **Trace[\(T^{\mu\nu}\)] = \eta_{\mu\nu}T^{\mu\nu} = T^\nu = T \eta^\nu = V \cdot V = [ (v^\nu)^2 - v \cdot v ] = (v^\nu)^2\)
  - Lorentz Scalar Invariant
SRQM Diagram: 
The Basis of Classical SR Physics Special Relativity via 4-Vectors

Focus on a few of the main SR Physical 4-Vectors:

4-Position
\[ R = R^\mu(r^\mu) = (c, r) = (ct, r) \]

4-Velocity
\[ U = U^\mu = dR^\mu/dr^\mu = \gamma(c, u) \]

4-Gradient
\[ \partial = \partial_\mu = \partial/\partial r^\mu = (\partial_x, \partial_y, \partial_z) \]

These 4-Vectors give some of the main classical results of Special Relativity, including 4D SR: Minkowski Space concepts like:

Relativity: Time Dilation (\( \rightarrow \) clock moving \( \rightarrow \)), Length Contraction (\( \rightarrow \) ruler moving \( \rightarrow \))

Invariants: Proper Time (\( \rightarrow \) clock at rest \( \rightarrow \)), Proper Length (\( \rightarrow \) ruler at rest \( \rightarrow \))

Temporal 1D Ordering: Time-like event separations \( \rightarrow \) Causality is Absolute, Space-like event separations \( \rightarrow \) Simultaneity is Relative

Spatial 3D Ordering: Time-like event separations \( \rightarrow \) Stationarity is Relative, Space-like event separations \( \rightarrow \) Topology is Absolute

Use of the Lorentz Scalar Product to make Lorentz Invariants, Continuity Equations, etc. The Invariant Speed-of-Light \( c \), Invariant Proper Measurements (Time & Space)

Invariant SR Wave Equations, via the d'Alembertian (Lorentz Scalar Product of 4-Gradient with itself), leads to 4-WaveVector \( K \) solution.

\[ \text{SR 4-Tensor} \]
\[ (2,0)-Tensor \ T_{x^\mu}^{\nu} \]
\[ (1,1)-Tensor \ T^\nu_{\nu} \text{ or } T^\nu_{\nu} \]
\[ (0,2)-Tensor \ T_{x^\mu}^{\nu} \]

\[ \text{SR 4-Vector} \]
\[ (1,0)-Tensor \ V^\nu = V = (v^0, v) \]
\[ \text{SR 4-CoVector: OneForm} \]
\[ (0,1)-Tensor \ V_{\nu} = (v_0, v) \]

\[ \text{SR 4-Scalar} \]
\[ (0,0)-Tensor \ S \text{ or } S = \xi^\mu \xi_\mu = \gamma(c, v) \]

\[ \text{Lorentz Scalar} \]

\[ V \cdot V = V^\mu \eta_{\mu \nu} V^\nu = \left((v^0)^2 - v^0 v^0\right) = (v^0)^2 \]

\[ \text{Lorentz Scalar Invariant} \]
SRQM Diagram: The Basis of Classical SR Physics
Special Relativity via 4-Vectors

The Basis of most all Classical SR Physics is in the SR Minkowski Metric of “Flat” SpaceTime $\eta^{\mu\nu} = \delta^{\mu}[R] = \delta[R]$, which is generated from the 4-Gradient $\partial = \partial^\mu$ and 4-Position $\bf{R} = R^\mu$ and determines the invariant measurement interval $\bf{R} \cdot \bf{R} = R^\mu \eta_{\mu\nu} R^\nu$ between <Events>.

This Minkowski Metric $\eta^{\mu\nu}$ provides the relations between the 4-Vectors of SR: 4-Position $\bf{R} = R^\mu$, 4-Gradient $\partial = \partial^\mu$, 4-Velocity $\bf{U} = U^\mu$.

The Tensor Invariants of these 4-Vectors give the:

- Invariant Interval Measures $\bf{R} \cdot \bf{R}$
- Invariant [Magnitude] LightSpeed ($c$), from $\bf{U} \cdot \bf{U}$
- Invariant d’Alembertian Wave Equation & 4-WaveVector $\bf{K}$, from $\partial \partial$

The relation between 4-Gradient $\partial$ and 4-Position $\bf{R}$ gives the Dimension of SpaceTime = (4), the Minkowski Metric ($\eta^{\mu\nu}$), and the Lorentz Transformations ($\Lambda^\mu_\nu$).

The relation between 4-Gradient $\partial$ and 4-Velocity $\bf{U}$ gives the invariant ProperTime Derivative (d$\tau$/d$t$).

Rearranging gives the invariant ProperTime Differential (d$\tau$/d$t$), which gives relativistic $\partial \partial$ [Time Dilation] $\rightarrow$ (temporal) & $\partial \partial$ [Length Contraction] $\rightarrow$ (spatial).

The ProperTime Derivative d$\tau$/d$t$: acting on 4-Position $\bf{R}$ gives 4-Velocity $\bf{U}$ acting on the SpaceTime Dimension Lorentz Scalar gives the Continuity of 4-Velocity Flow.

The relation between 4-Displacement $\Delta \bf{R}$ and 4-Velocity $\bf{U}$ gives Relativity of Simultaneity:Stationary.

One of the most important properties is the Tensor Invariant Lorentz Scalar Product ($\partial \partial = \cdot$), provided by the lowered-index form of the Minkowski Metric $\eta_{\mu\nu}$.

From here, each object will be examined in turn...
The 4-Differential(Displacement) is just the infinitesimal version of the finite 4-Displacement, $L^2\Delta R = (c\Delta t, \Delta r)$.

As such, any “defined” 4-Position, like the 4-Zero, is Lorentz Invariant (point rotations and boosts), since the translations can move it. The 4-Position $R = (ct, r)$ relates time to space via the fundamental invariant physical constant ($c$): the Speed-of-Light = “(c)elerity ; (c)eleritas”, which is used to give consistent dimensional units across all SR 4-Vectors.

The 4-Position $R = (ct, r)$ relates time to space via the fundamental invariant physical constant ($c$): the Speed-of-Light = “(c)elerity ; (c)eleritas”, which is used to give consistent dimensional units across all SR 4-Vectors.

As such, any “defined” 4-Position, like the 4-Zero, is Lorentz Invariant (point rotations and boosts), but not Poincaré Invariant (Lorentz + time & space translations), since the translations can move it.

The 4-Differential(Displacement) is just the infinitesimal version of the finite 4-Displacement, and is used in the calculus of SR. $U = dR/dt$.

4-Displacement $\Delta R = (c\Delta t, \Delta r)$, where $\Delta R = \Delta R_\mu = \xi^\mu$, $\xi^\mu = (\Delta r^\mu)$, $\Delta r^\mu = (\Delta r^\mu)$, $\Delta r^\mu = (\Delta r^\mu)$.

4-Position $R = (ct, r)$, $dR = (c dt, dr)$.

4-Vector $V = (\nu^\mu)$, $V^\mu = (\nu^\mu)$.

4-SpaceTime Dimension is the fundamental invariant of SR.

4-Velocity $U = (\gamma(c, u))$, $\gamma = \sqrt{1 - u^2/c^2}$.

4-Gradient $\partial = (\partial / \partial c, \partial / \partial v, \partial / \partial u, \partial / \partial t)$.

4-Position $R = (ct, r)$, $dR = (c dt, dr)$.

4-Displacement $\Delta R = (c\Delta t, \Delta r)$, $dR = (c dt, dr)$, $\Delta R = (c\Delta t, \Delta r)$.

4-Displacement $\Delta R = (c\Delta t, \Delta r)$, $dR = (c dt, dr)$, $\Delta R = (c\Delta t, \Delta r)$.

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4-Vector $V = (\nu^\mu)$, $V^\mu = (\nu^\mu)$.

4-SpaceTime Dimension is the fundamental invariant of SR.

4-Velocity $U = (\gamma(c, u))$, $\gamma = \sqrt{1 - u^2/c^2}$.

4-Gradient $\partial = (\partial / \partial c, \partial / \partial v, \partial / \partial u, \partial / \partial t)$.
The Invariant Interval is the Lorentz Scalar Product of the (4-Position, 4-Displacement, 4-Differential) with itself, giving a magnitude-squared, which may be (+/0/-).

\( R \cdot R = (c_0^2) - r \cdot r = (c_0^2) - (r_0^2) \)
\( \Delta R \cdot \Delta R = (c_0^2) - \Delta r \cdot \Delta r = (c_0^2) - (\Delta r_0^2) \)
\( \text{d}R \cdot \text{d}R = (c_0^2) - (c_0^2) = -(\text{d}r_0^2) \)

Space-like interval (-)

The 4D SpaceTime Intervals are Invariant, meaning that all observers must agree on their magnitudes, regardless of differing reference frames. This leads to the idea of ProperTime (\( \Delta t = \Delta t_0 \)), which is the time-displacement measured by a clock at-rest, and ProperLength (\( L_0 = |\Delta x_0| \)), which is the space-displacement measured by a ruler at-rest. This also leads to the various Causality Conditions of SR, and the concept of the (Minkowski Diagram) Light Cone. The differential form "\( d\mathbf{R} \cdot d\mathbf{R} \)" is apparently also still true in the curved spacetime of GR.
SRQM Diagram: The Basis of Classical SR Physics

SpaceTime Dimension = 4D = (1+3)D

4-Vector SRQM Interpretation of QM

A Tensor Study of Physical 4-Vectors

4-Gradient \( \partial^\mu \) 
\[ \partial \equiv \partial R = (\partial, \partial c, -\nabla) \]
Dimension

\( \partial \cdot R = 4 : \) The 4-Divergence SpaceTime Dimension Relation
\[ = (\partial, \partial c, -\nabla) \cdot (ct, r) \]
\[ = (\partial t) + \nabla \cdot r \]
\[ = (\partial t) + \partial [x] + \partial [y] + \partial [z] \]
\[ = (1+1+1+1) \]
\[ = 4 \]

Alternate Tensorial Derivation:
\[ (\partial \cdot R) = (\partial^\mu R^\mu) = (\partial^\mu \eta_{\mu\nu} R^\nu) = \eta_{\mu\nu}(\partial^\nu R^\mu) = \eta_{\nu\mu} = \delta_\nu^\mu = \partial^\nu \]
\[ = (\partial_0 + \partial_1 + \partial_2 + \partial_3) = (1+1+1+1) = 4 \]

This Tensor Invariant Lorentz Scalar relation gives the dimension of SpaceTime. The only way there can more dimensions is if there is another SpaceTime direction available. 4-Divergence (\( \partial [ ] \)) is also used in SR Conservation Laws, ex. (\( \partial J \)) = 0

All empirical evidence to-date indicates that there are only the 4 known dimensions: 1 temporal (t); measured in SI units = [s], with (ct): measured in SI units [m]
3 spatial (x, y, z); measured in SI units = [m]

These are the 4 components that appear in:
4-Position
\[ R = (ct, r) \rightarrow (ct, x, y, z) \]
measured in SI units [m]

SR 4-Tensor
(2,0)-Tensor \( T^{\mu \nu} \)
(1,1)-Tensor \( T^\tau \) or \( T^\tau_\tau \)
(0,2)-Tensor \( V_V \)

SR 4-Vector
\( V = (\nabla, v) \)

SR 4-CoVector: OneForm
(0,1)-Tensor \( V_\mu \)

SR 4-Scalar
(0,0)-Tensor \( S \) or \( S_\mu^\mu \)

Lorentz Scalar

\( \delta^{\mu \nu} = \delta^\mu \nu = \delta_{\mu \nu} = \delta_{\mu \nu} = \{1 \) if \( \mu = \nu \), else 0 \} = \text{Diag}[1,1,1,1] \)

4D Kronecker Delta = 4D Identity

4-Displacement
\[ \Delta R = (c\Delta t, \Delta r) \]
\[ dr = (cdt, dr) \]

4-Position
\[ R = (ct, r) \]

Invariant Interval
\[ R \cdot R = (ct)^2 + r^2 \]
\[ \Delta R \cdot \Delta R = (c\Delta t)^2 + \Delta r^2 \]
\[ dR \cdot dR = (cdt)^2 + dr^2 \]

SpaceTime Dimension
\[ \partial \cdot R = 4 \]

ProperTime Derivative
\[ U \cdot \partial U = \gamma (\partial + (dx/dt) \partial_x + (dy/dt) \partial_y + (dz/dt) \partial_z) \]
\[ = \gamma d/dt \]

ProperTime Differential
\[ dr = (1/\gamma) d\tau \]

SRQM Diagram

The Tesseract, a 4D cube, symbolizes 4D SpaceTime

SR : Minkowski
Time • Space = 4D

(1+3)D = 4D

Trace[\( T^\mu \nu \)] = \( \eta_{\mu\nu} T^\mu \nu = T^\nu_\nu = T \]
\[ V \cdot V = V_\mu \eta^\mu \nu \cdot V^\nu_\nu = (V^\nu_\nu)^2 \]

= Lorentz Scalar Invariant
SRQM Diagram:

The Basis of Classical SR Physics

The Minkowski Metric ($\eta^{\mu\nu}$), Measurement

SR: Minkowski Metric

$\delta[R] = \partial^\mu R^\mu = \eta^{\mu\nu} V^\nu + H^{\mu\nu} \rightarrow$

Diag[1,1,-1,-1] = Diag[1,1,-3\delta^\mu,\delta^n]  
(in Cartesian form)  “Particle Physics” Convention

Generally, diagonal components $[\eta^{\mu\nu}] = 1/[\eta_{\mu\nu}]$, non-diagonal = 0; and $\nu^\mu = \delta^n$  

A 4-Vector SRQM Interpretation of QM

http://scirealm.org/SRQM.pdf

SciRealm.org  
John B. Wilson  
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SRQM Diagram: The Basis of Classical SR Physics

The Lorentz Transform \( \partial_v[R^\mu] = \partial R^\mu/\partial R^v = \Lambda^\mu_v \)

**The Trace Invariant of the various Lorentz Transforms leads to very interesting results: CPT Symmetry and Antimatter**
The Lorentz transformation can also be derived empirically. In order to achieve this, it's necessary to write down coordinate transformations that include experimentally testable parameters. For instance, let there be given a single "preferred" inertial frame \((t, x, y, z)\) in which the speed of light is constant, isotropic, and independent of the velocity of the source.

It is also assumed that Einstein synchronization and synchronization by slow clock transport are equivalent in this frame. Then assume another frame \((t', x', y', z')\) in relative motion, in which clocks and rods have the same internal constitution as in the preferred frame. The following relations, however, are left undefined:

- \(a(v)\): differences in time measurements,
- \(b(v)\): differences in measured longitudinal lengths,
- \(d(v)\): differences in measured transverse lengths,
- \(\varepsilon(v)\): depends on the clock synchronization procedure in the moving frame.

Then the transformation formula (assumed to be linear) between those frames are given by:

\[
\begin{align*}
\Delta t &= t' - vt \\
\Delta x &= x' - c(t' - vt) \\
\Delta y &= y' \\
\Delta z &= z' \\
\end{align*}
\]

The value of LightSpeed \((c)\) was empirically measured by Ole Rømer to be finite using the timing of Jovian moon eclipses.
The Basis of Classical SR Physics

**Dimension = 4D = (1+3)D**

Tensor Invariants include: {Trace, InnerProduct, Determinant, etc.}

- **4-Divergence**:\(\nabla\cdot\mathbf{R} = \text{Trace}[\text{Minkowski Metric}]\), and the InnerProduct[any of the Lorentz Transforms] give the Dimension of SR SpaceTime = 4D.

**Minkowski Metric**

- **Trace Invariant**\( \text{Tr}[\eta_{\mu\nu}] = \lambda_{\mu\nu} \lambda_{\nu\mu} = 4 \)
- **4-Position**\( \Delta R = (c\Delta t, \Delta r) \)
- **Outer Product**\( \text{dR} = (\text{d}t, \text{d}r)\)

**Lorentz Transform**

- **Inner Prod Invariant**\( \eta_{\mu\nu} \alpha_{\mu} \alpha_{\nu} = \eta_{\nu\mu} \alpha_{\nu} \alpha_{\mu} \)
- **Minkowski Metric**\( \lambda_{\mu\nu} \lambda_{\nu\mu} = 4 \)
- **Trace Invariant**\( \text{Tr}[\eta_{\mu\nu}] = (1 + 1 + 1 + 1) = 4 \)

**4-Tensor**

- **Trace Invariant**\( \text{Trace}[T_{\mu\nu}] = T_{\mu\nu} = T_{\nu\mu} = T_{00} + T_{01} + T_{10} + T_{11} \)

**SR 4-Tensor**

- \(T_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \)
- **SR 4-Vector**\( \mathbf{V} = (v_0, v_1, v_2, v_3) \)
- **SR 4-Scalar**\( s \)

**Conservation: Non-Divergence of Minkowski Metric**

\[
\nabla\cdot\eta_{\mu\nu} = 0
\]

**Relativity of Simultaneity:**

\[
\partial t = \gamma (c^2 \Delta t - \mathbf{u} \cdot \Delta r)
\]

**SRQM Diagram**

- **4-Divergence**\( \nabla\cdot\mathbf{R} = \text{Trace}[\text{Minkowski Metric}]\)
- **4-Position**\( \Delta R = (c\Delta t, \Delta r) \)
- **Lorentz Transform**\( \eta_{\mu\nu} \)
- **ProperTime Derivative**\( \gamma = \gamma (c^2 \Delta t - \mathbf{u} \cdot \Delta r) \)

**SR : Minkowski Time-Space is 4D**

\( (1+3)D = 4D \)

**Trace**\( [T_{\mu\nu}] = \eta_{\mu\nu} T_{\mu\nu} = T_{\mu\nu} = T \)

**V•V = \eta_{\mu\nu} V^\mu V^\nu = (v_0^2 - v_1^2 - v_2^2 - v_3^2) = \text{Lorentz Scalar Invariant}**

http://scirealm.org/SRQM.pdf
A Tensor Study

Lorentz Scalar (Dot) Product ($\eta_{\mu \nu} = \cdot$)

The Tensor Invariant Lorentz Scalar Product (LSP) is the SR 4D (Dot=\cdot) Product. It is used to make Invariant Lorentz Scalars from two 4-Vectors.

The LSP is used in just about every relation between any two interesting 4-Vectors. (\eta_{\mu \nu} \cdot a_{\mu} \cdot b_{\nu} = (a_{\mu} \cdot b_{\nu}) \cdot c_{\mu \nu})

$\eta_{\mu \nu} \cdot = \epsilon_{\mu \nu} \cdot \epsilon_{\mu \nu} \cdot \cdot$ (Cartesian basis)

(\eta_{\mu \nu} \cdot) is itself just the lowered-index form of the SR Minkowski Metric (\eta_{\mu \nu}), with individual components ($\eta_{\mu \mu} = 1$ [\eta_{\mu \nu}], else 0. In Cartesian basis, this gives (\eta_{\mu \nu} = \eta_{\mu \nu}) (Cartesian basis).

SRQM Diagram:

The Basis of Classical SR Physics

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
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http://scirealm.org/SRQM.pdf
4-Vector SRQM Interpretation of QM

4-Speed SRQM Interpretation of QM

SRQM Diagram: The Basis of Classical SR Physics

4-Velocity U, SpaceTime <Event> Motion

4-Velocity \( U = \gamma(c,u) = (\gamma c, \gamma u) = (U \cdot \dot{R}) = \gamma(\ddot{r} + u \cdot \nabla) \)

It is the SR 4-Vector that describes the motion of <Events> through SpaceTime.

(a) For an un-accelerated observer, the 4-Velocity \( U \) is a constant along the WorldLine at all points.

(b) For an accelerated observer, the 4-Velocity \( U \) is still tangent to the WorldLine at each point, but changes direction as the WorldLine bends thru SpaceTime.

The 4-UnitTemporal \( \dot{T} \) & 4-Velocity \( U \) are unlike most of the other SR 4-Vectors. They have 3 independent components, whereas the others usually have 4. This is due to the constraints placed by the LSP Tensor Invariants. \( \dot{T} \cdot T = +1 \) & \( \dot{U} \cdot U = c^2 \) have constant magnitudes, giving the Speed-of-Light (c) in SpaceTime.

Components:
- 3 independent + 0 independent \( \rightarrow \) 3 independent + 1 independent \( \rightarrow \) 4 independent

4-UnitTemporal \( \dot{T} = \gamma(1, \beta) \)

4-Velocity \( \dot{U} = \gamma(c, u) \)

4-Momentum \( \dot{P} = (mc, p) = (E/c, p) = m \dot{U} \)

They also usually have the Relativistic Gamma factor (\( \gamma \)) exposed in component form, whereas most of the other temporal 4-Vectors have it absorbed into the Lorentz 4-Scalar factor that goes into their components.

4-UnitTemporal \( \dot{T} = T = \gamma(1, \beta) = (\gamma c, \gamma u) = U/c \)

4-Velocity \( \dot{U} = U = \gamma(c, u) = (\gamma c, \gamma u) = cT \)

4-Momentum \( \dot{P} = P = (mc, p) = m \dot{U} = \gamma m(c, u) = (mc, m \dot{u}) = (mc, (E/c, p)) \)

\( \gamma = 1 / \sqrt{1 - \beta \cdot \beta} \), \( \beta = u/c \)

SR 4-Tensor
- (2,0)-Tensor \( T_{4,v}^{2} \)
- (1,1)-Tensor \( T_{4u}^{1} \) or \( T_{4u}^{2} \)

SR 4-Vector
- \( V = (v, \dot{v}) \)

SR 4-Scalar
- \( \gamma = \gamma(c, u) = \gamma(c, u) = \gamma_{\text{SR}} \)
- Lorentz Scalar

SRQM Diagram

Relativistic Gamma \( \gamma = 1 / \sqrt{1 - \beta \cdot \beta} \), \( \beta = u/c \)

4-UnitTemporal \( \dot{T} = \gamma(1, \beta) \)

4-Velocity \( \dot{U} = \gamma(c, u) \)

4-Momentum \( \dot{P} = (mc, p) = (E/c, p) = m \dot{U} \)

4-Displacement \( \dot{R} = (c \Delta t, \dot{r}) \)

4-Position \( R = (ct, \dot{r}) \)

Invariant Interval \( \dot{R} \cdot \dot{R} = (ct)^2 + \dot{r}^2 \)

4-Gradient \( \triangledown = \partial_{\mu} \nabla^{\mu} \)

4-Displacement Flow \( \dot{U} = 0 \)

ProperTime Derivative \( \dot{\psi} = (\gamma(c, u), \partial_{\mu} \nabla^{\mu}) \)

Relativity of Simultaneity/Stationary

SpaceTime Dimension

SRQM Diagram

4-UnitTemporal \( \dot{T} = \gamma(1, \beta) \)

4-Velocity \( \dot{U} = \gamma(c, u) \)

4-Momentum \( \dot{P} = (mc, p) = (E/c, p) = m \dot{U} \)

They also usually have the Relativistic Gamma factor (\( \gamma \)) exposed in component form, whereas most of the other temporal 4-Vectors have it absorbed into the Lorentz 4-Scalar factor that goes into their components.

4-UnitTemporal \( \dot{T} = T = \gamma(1, \beta) = (\gamma c, \gamma u) = U/c \)

4-Velocity \( \dot{U} = U = \gamma(c, u) = (\gamma c, \gamma u) = cT \)

4-Momentum \( \dot{P} = P = (mc, p) = m \dot{U} = \gamma m(c, u) = (mc, m \dot{u}) = (mc, (E/c, p)) \)

\( \gamma = 1 / \sqrt{1 - \beta \cdot \beta} \), \( \beta = u/c \)

SR 4-Tensor
- (2,0)-Tensor \( T_{4,v}^{2} \)
- (1,1)-Tensor \( T_{4u}^{1} \) or \( T_{4u}^{2} \)

SR 4-Vector
- \( V = (v, \dot{v}) \)

SR 4-Scalar
- \( \gamma = \gamma(c, u) = \gamma(c, u) = \gamma_{\text{SR}} \)
- Lorentz Scalar

SRQM Diagram

Relativistic Gamma \( \gamma = 1 / \sqrt{1 - \beta \cdot \beta} \), \( \beta = u/c \)

4-UnitTemporal \( \dot{T} = \gamma(1, \beta) \)

4-Velocity \( \dot{U} = \gamma(c, u) \)

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4-Gradient \( \triangledown = \partial_{\mu} \nabla^{\mu} \)

4-Displacement Flow \( \dot{U} = 0 \)

ProperTime Derivative \( \dot{\psi} = (\gamma(c, u), \partial_{\mu} \nabla^{\mu}) \)

Relativity of Simultaneity/Stationary
The newly made 4-Vectors thus have \( 3 + 1 = 4 \) independent components.

The Lorentz Scalar Product of the 4-Velocity leads to the Invariant [Magnitude] Speed-of-Light \( c \), one of the most important SR physical constants of physics. 

\[
\begin{align*}
U \cdot \mathbf{u} &= \gamma(c, u) = \gamma(\mathbf{c}, \mathbf{u}) = (U \cdot \mathbf{\hat{c}}) \mathbf{c} = (\gamma \mathbf{c}) \mathbf{c} = \gamma \mathbf{c}^2 \\
\text{Components: } &3 \text{ independent} \\
\end{align*}
\]

SR 4-Vector
\[
\begin{align*}
\text{SR 4-Vector} &= (1,0)-\text{Tensor V} = \mathbf{V} = (\mathbf{v}^0, \mathbf{v}) = (\mathbf{c}, \mathbf{u}) \\
\text{SR 4-Vector Lorentz} &= (0,1)-\text{Tensor S or } S_0 = \mathbf{V}_0 = (\mathbf{v}_0, \mathbf{v}) \\
\end{align*}
\]

SR 4-Vector Lorentz Transformations
\[
\begin{align*}
\mathbf{V} &= (\mathbf{v}^0, \mathbf{v}) = \gamma(\mathbf{c}, \mathbf{u}) = \gamma(\mathbf{c}, \mathbf{u}) = \gamma \mathbf{c}^2 \\
\text{SR 4-Vector} &= \text{SR 4-Vector Lorentz} \\
\end{align*}
\]
SRQM Diagram:
The Basis of Classical SR Physics

**Relativity of Simultaneity: Time-Delay**
(Simultaneity ↔ Same-Time Occurrence ↔ Δt=0)
(Time-Delay ↔ Different-Time Occurrence ↔ Δt≠0)

If Lorentz Scalar (U·ΔX = 0 = c²Δt), then the ProperTime derivative of Δt is zero.

ProperTime Derivative
\[ \frac{\partial t}{\partial \tau} = \frac{c}{\gamma} \frac{\partial x}{\partial \tau} \]

ProperTime Derivative of U
\[ U = \frac{c U}{\gamma} = \frac{c^2 x}{\gamma} \]

ProperTime Differential
\[ dt = \frac{dx}{c} + \frac{c}{\gamma} \frac{d\tau}{dt} \]

ProperTime Differential of U
\[ U = \frac{c U}{\gamma} = \frac{c^2 x}{\gamma} \]

ProperTime Derivative of U
\[ U = \frac{c U}{\gamma} = \frac{c^2 x}{\gamma} \]

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ProperTime Differential of U
\[ dt = \frac{dx}{c} + \frac{c}{\gamma} \frac{d\tau}{dt} \]
A Tensor Study

(1,1)-Tensor \( T \) in the alternate reference-frame. If this condition is met, in any Boosted Frame:

\[
\sqrt{\left( \gamma^2 \Delta t^2 - \mathbf{u} \cdot \Delta \mathbf{x} \right)} = c^2 \Delta t
\]

If \( \mathbf{u} \cdot \Delta \mathbf{x} > 0 \), then \( \gamma^2 = 1 \), which is RestFrame.

If \( \mathbf{u} \cdot \Delta \mathbf{x} < 0 \), then \( \gamma^2 = -1 \) and this is Not Stationary in the alternate reference-frame.

This can be shown on a Minkowski Diagram.
The derivation shows that the ProperTime Derivative \( \frac{d}{d\tau} \) is an Invariant Lorentz Scalar. Therefore, all observers must agree on its magnitude, regardless of their frame-of-reference. \( \frac{d}{d\tau} \) is used to derive some of the physical 4-Vectors: 4-Velocity, 4-Acceleration, 4-Force, 4-Torque, etc.

4-Position
\[ R = (c t, r) \]

4-Velocity
\[ U = \gamma(c, u) = (u^\mu, u^\nu) \]

4-Acceleration
\[ \dot{U} = \gamma(c, \dot{u}) = (\dot{u}^\mu, \dot{u}^\nu) \]

4-Force
\[ F = \gamma(E/c, f) = (F^\mu, F^\nu) \]

4-Torque
\[ \tau = R^\beta F^\alpha - R^\alpha F^\beta = R^\alpha F^\beta = (\mathbf{u} \wedge \mathbf{F}) \]

4-Tensor Anti-symmetric

SR 4-Vector
\[ (0,1)-\text{Vector} V = (\mathbf{v}, v) \]

SR 4-Vector
\[ (1,0)-\text{Tensor} V = (\mathbf{V}, V) \]

SR 4-Tensor
\[ (0,2)-\text{Tensor} T_{\mu\nu} = T_{\nu\mu} = (\mathbf{V} \cdot \mathbf{V}) \]

SR 4-Scalar
\[ (0,0)-\text{Tensor} S = S = \mathbf{V} \cdot \mathbf{V} \]

Relativistic Gamma
\[ \gamma = \frac{1}{\sqrt{1 - \beta \cdot \beta}} \]

\( \quad \beta = u/c \)
The Basis of Classical SR Physics
ProperTime Derivative in SR:
4-Tensors, 4-Vectors, and 4-Scalars
SRQM Diagram:
The Basis of Classical SR Physics
ProperTime Differential $(d\tau) \rightarrow$
Time Dilation & Length Contraction

There are several ways to derive Time Dilation.

- **ProperTime Differential (Lorentz 4-Scalar):** $d\tau = (1/\gamma)d\tau$
  
  $\gamma = \sqrt{1 - \frac{v^2}{c^2}}$

- **ProperLength Differential:** $dL = \gamma dL_{0}$
  
  $L = \frac{L_{0}}{\gamma}$

- **ProperTime Derivative:** $\frac{d\gamma}{d\tau} = \frac{\gamma - 1}{\gamma c^2}$

- **Relativity:** $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

The coordinate time $\Delta t$ measured by an observer is "dilated", compared to the ProperTime as measured by a clock moving with the object. This has the effect that moving objects appear to age more slowly than at-rest objects. The effect is reciprocal as well. Since velocity is relative, each observer will see the other as ageing more slowly, similarly to the effect that each will appear smaller to the other when seen at a distance.

Now multiply both sides by the moving-frame speed $v = |v|$ $\Delta t = \gamma v \Delta \tau$
$\Delta t = \text{distance } L_{0}$, the moving clock travels wrt. frame, which is a proper (fixed-to-frame) displacement length.
$L_{0} = \gamma L$
$L = \left(\frac{1}{\gamma}\right) L_{0}$ ← Length Contraction → {in spatial $v$ direction}

**SR 4-Tensor:**
- $(1 - 0)$-Tensor $T^{\mu}_{\nu}$
- $(1,1)$-Tensor $T^{\mu}_{\nu} = V = (v, v)$
- $(0,2)$-Tensor $T_{\nu}^{\mu} = \eta^{\mu\nu}$

**SR 4-Vector:**
- $(2,0)$-Vector $V^{\mu} = V = (c, v)$
- $(1,0)$-Vector $V^{\mu} = V = (c, v)$

**SR 4-Scalar:**
- $(0,0)$-Tensor $S$ or $S_{\mu} = \delta^{\mu}_{\nu}$
- Lorentz Scalar

**SRQM Diagram:**
- **4-Displacement:** $R = (\Delta t, \Delta \mathbf{r})$
- **4-Differential:** $d\mathbf{r} = (cdt, d\mathbf{r})$
- **4-Position:** $R = (ct, \mathbf{r})$

**SRQM Interpretation of QM:**
- **SRQM Diagram:**
- **SRQM Interpretation:**
- **SRQM Diagram Interpretation:**
- **SRQM:**
- **SRQM Interpretation:**
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- **SRQM Interpretation:**
The 4-Gradient is a 4D Vector function that can act on other 4-Vectors, Tensors, or Scalars. It is instrumental in creating the ProperTime Derivative, \( \dot{\mathbf{R}} = \frac{dR}{dt} \), which is the 4D version of the partial derivative function of calculus, one partial for each dimensional direction, just as the Del (\( \nabla \)) is the 3D version of the partial derivative function.

The 4-Gradient plays a major role in advanced physics, showing how SR waves are formed, creating the Hamilton-Jacobi equations, the Euler-Lagrange equations, Conservation Equations (\( \partial f/\partial t = 0 \)), Maxwell’s Equations, the Lorenz Gauge, the d’Alembertian, etc. It gives the dimensions of SpaceTime, the Minkowski Metric, and the Lorentz Transformations.

In QM, it provides the Schrödinger relations. \( \mathbf{P} = (\mathbf{E}/c, \mathbf{p}) \) = \( \mathbf{p} = \mathbf{p}(\partial/c, -\mathbf{V}) \).

The 4-Gradient is fundamental in connecting SR to QM.
The usual mathematical (complex) plane-wave solutions apply in SR:

\[ \text{Lorentz Scalar Product Invariant of the 4-Gradient gives the} \]
\[ \text{SR 4-Tensor} \]
\[ = A \cdot \partial = \mu J \]

Maxwell EM Wave Eqn

\[ 4\text{-CurrentDensity} \]
\[ J = J^\mu = (p, j) = p(\mathbf{c}, \mathbf{u}) = p_0 \mathbf{u} \]
\[ = q_0 \mathbf{u} = q \mathbf{N} \]

The Lorentz Scalar Product Invariant of the 4-Gradient gives the SR d'Alembertian Wave Equation, describing SR wave motion. It is seen, for example, in the SR Maxwell Equation for EM light waves.

\[ \partial \times \mathbf{A} - \partial_t \mathbf{A} = \mu_0 \mathbf{J} \]

Lorentz Gauge= Conservation of EM Potential: \( \partial A = 0 \)

Importantly, the d'Alembertian is fully from basic SR rules, with no quantum axioms required. However, it will be seen again in the Klein-Gordon RQM wave equation.

Its solution provides for the introduction of SR 4-WaveVector \( K \) which can also be given by the negative gradient of a Lorentz Scalar Phase \( \Phi \).

4-WaveVector \( K = (w/c)^2 \mathbf{U} = (w/c, k) \)

Where the mathematical (complex) plane-wave solutions apply in SR:

\[ f = (a)^{e^{i\omega t}} \phi = (\phi/c)^2 \mathbf{U} \]

\[ A = A^\mu = (\phi(c, \mathbf{u}) = (\mathbf{c}, \mathbf{u}) \]

\[ A_{EM} = A_{EM}^\mu = (\phi_{EM}/c, \mathbf{a}_{EM}) \]

The mathematical (complex) plane-wave solutions apply in SR:

\[ f = (a)e^{i\omega t} \phi = (\phi/c)^2 \mathbf{U} \]

\( 4\text{-CurrentDensity} \)
\[ J = J^\mu = (p, j) = p(\mathbf{c}, \mathbf{u}) = p_0 \mathbf{u} \]
\[ = q_0 \mathbf{u} = q \mathbf{N} \]

The usual mathematical (complex) plane-wave solutions apply in SR:

\[ f = (a)e^{i\omega t} \phi = (\phi/c)^2 \mathbf{U} \]

\[ 4\text{-CurrentDensity} \]
\[ J = J^\mu = (p, j) = p(\mathbf{c}, \mathbf{u}) = p_0 \mathbf{u} \]
\[ = q_0 \mathbf{u} = q \mathbf{N} \]

SRQM Diagram:
The Basis of Classical SR Physics

Invariant d’Alembertian Wave Equation \( \partial \Phi = 0 \)

\[ \text{SRQM Diagram} \]

SRQM Interpretation of QM

4-Vector SRQM Interpretation of QM

\[ \text{SRQM Diagram} \]

SR is the “natural” 4D arena for the description of waves, using the d'Alembertian

\[ \partial \Phi = (\partial/c)^2 \cdot \nabla \cdot \nabla \Phi = (\partial/c)^2 \Phi \]

\[ \text{Trace}(\mathbf{T}^\nu) = \eta^\nu_\nu \mathbf{T}^\nu = \eta^-_\nu \mathbf{T}^\nu = \mathbf{T} \]

\[ \mathbf{V} \cdot \mathbf{V} = \mathbf{V}^\nu \eta^\nu_\nu \mathbf{V} = \mathbf{V}^\nu \mathbf{V} = \mathbf{V}^\nu \mathbf{V} = (\mathbf{V}^\nu)^2 = \text{Lorentz Scalar Invariant} \]
Conservation of Charge, continuity eqn:

by the flow of that quantity in-to or out-of a local region.

Invariant Scalar equation, a continuity equation.

∂∙U = 0

Conservation of the 4-Velocity Flow

This leads to all the SR Conservation Laws.

Continuity of 4-Velocity Flow

All of the Physical Conservation Laws are in the form of a 4-Divergence ( ∂[ .. ] = 0 ), which is a Lorentz Invariant Scalar equation, a continuity equation.

These are local continuity equations which basically say that the temporal change of a quantity is balanced by the flow of that quantity in-to or out-of a local region.

Conservation of Charge, continuity eqn:
ρ,∂∙U = ρ,∂J = (∂ρ + ∇J) = 0

SR 4-Tensor
(2,0)-Tensor T\(_{\mu \nu}\)

SR 4-Vector
(1,0)-Tensor V\(_{\mu}\) = (v\(_{\mu}\), 0)

SR 4-CoVector:OneForm
(0,1)-Tensor V\(_{\nu}\) = (v\(_{\nu}\), 0)

SR 4-Scalar
(0,0)-Tensor S or S\(_{\mu\nu}\)

SR 4-Displacement
\(\Delta R = (c\Delta t, \Delta r)\)

SR 4-Gradient
\(\partial \cdot R\) = Lorentz Transform

SR 4-Divergence
\(\partial \cdot U\) = 0

Continuity of 4-Velocity Flow

\(\partial \cdot U = \partial \cdot \eta^{\mu\nu} = 0\)

SRQM Diagram:
The Basis of Classical SR Physics

Continuity of 4-Velocity Flow (\(\partial \cdot U = 0\))

\(\partial \cdot R = 4\)

SpaceTime Dimension

\(\partial \cdot \eta^{\mu\nu} = 0\)

ProperTime Derivative
\(\gamma (\partial t + (dx/dt)\Lambda + (dy/dt)\Lambda + (dz/dt)\Lambda)\)

ProperTime Differential
\(dt = (1/c^2) \Delta \tau\)

Conservation of 4-Velocity Flow

\(\partial \cdot U = 0\)

Any Lorentz Scalar:Rest Value
\(a_o\)

Conservation of (4-Vector A=\(a_o\)U)
\(\partial \cdot a_o U = a_o \partial \cdot U = 0\)

Trace[\(T^{\mu\nu}\)] = \(\eta^{\mu\nu} T^{\mu\nu} = T_{\nu\nu} = T\)

\(V \cdot V = \gamma (v^\mu v^\nu) = (v^\mu)^2 - (v^\nu)^2\)

= Lorentz Scalar Invariant

The Conservation Laws of SR quantities are all in the form of Continuity Equations:
Now focus on a few more of the main SR 4-Vectors.

- 4-Position \( R^\mu = (ct, r) \) (Location)
- 4-Velocity \( U^\mu = dr/d\gamma(c, u) \) (Motion)
- 4-Gradient \( \partial^\mu = \partial/\partial \gamma(c, -V) \) (Alteration)
- 4-Momentum \( p^\mu = (E/c, p) = (mc, p) = (mc, mu) \) = \((E/c^2)U = (m_0)U \)
- 4-WaveVector \( k^\mu = (\omega/c, k) = (\omega/c, \omega \hat{n}/v_{phase}) = (1/c\tau, \hat{n}/(c, -\hat{V})) \)
- 4-CurrentDensity: ChargeFlux \( J^\mu = (pc, j) = (pc, pu) = (\rho_\gamma(c, u) \)
- 4-(Dust)NumberFlux \( N^\mu = (n, c, n) = (n_0)(c, u) \) = \((n_0)U \)

These 4-Vectors give more of the main classical results of Special Relativity, including SR concepts like:

- SR Particles and Waves, Matter-Wave Dispersion
- Einstein’s \( E = mc^2 = \gamma m_0 c^2 = \gamma E_0 \), Rest Mass, Rest Energy
- Conservation of Charge (Q), Conservation of Particle Number (N), Continuity Equations

These concepts are illustrated in the SRQM Diagram:

**SRQM Diagram: The Basis of Classical SR Physics**

- <Event> Substantiation
- 4-Position \( \Delta R^\mu = (c\Delta t, \Delta r) \)
- 4-Gradient \( \partial^\mu = \partial/\partial \gamma(c, -\hat{V}) \)

**Motion of various Lorentz Scalars leads to the “Substantiation” of the various physical SR 4-Vectors.**

Lorentz 4-Scalar \( a_\mu = a^\mu \) (a^\mu, a) = \( a_\gamma(c, u) = a(c, u) = (ac, au) \)

**Trace** \( T^{\mu \nu} = \eta^{\mu \nu} T^{\mu \nu} = T^{\gamma \gamma} = T \)

\( V = V^\mu V^\nu = ((\gamma v)^2 - V^\mu V^\nu) = (\gamma v)^2 \)

= Lorentz Scalar Invariant

http://scirealm.org/SRQM.pdf
A Tensor Study of Physical 4-Vectors

The Basis of Classical SR Physics

4-Momentum, Einstein’s $E = mc^2$, $p=mu$

SRQM Diagram:

SR 4-Vector Interpretation of QM

SciRealm.org
John B. Wilson
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http://scirealm.org/4VectorSRQM.pdf
**SRQM Diagram:**

**The Basis of Classical SR Physics**

4-WaveVector, $|u \ast v_{\text{phase}}| = |v_{\text{group}} \ast v_{\text{phase}}| = c^2$

<table>
<thead>
<tr>
<th>4-Position $R = (ct, r)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-Gradient $\partial = (\partial / c, \nabla)$</td>
</tr>
<tr>
<td>4-Velocity $U = \gamma(c, u)$</td>
</tr>
</tbody>
</table>

4-WaveVector $K = (\omega/c, k) = (\omega_o/c^2)U = \gamma(\omega_o/c^2)(c, u)$

<table>
<thead>
<tr>
<th>Spatial part: $\omega = \gamma \omega_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temporal part: $k = \gamma(\omega_o/c^2)u = (\omega/c^2)u = \omega \nabla \nu_{\text{phase}}$</td>
</tr>
</tbody>
</table>

4-WaveVector $K_T = (\omega_T/c, k_T) = -\partial \Phi_{\text{phase,free}} = -\partial / c, \nabla \Phi_{\text{phase,free}}$

<table>
<thead>
<tr>
<th>Temporal part: $\omega = -\partial \Phi_{\text{phase,free}} : \omega_T = -\partial \Phi_{\text{phase}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spatial part: $k = +\nabla \Phi_{\text{phase,free}} : k_T = +\nabla \Phi_{\text{phase}}$</td>
</tr>
</tbody>
</table>

4-TotalWaveVector $K_T = (\omega_T/c, k_T) = -\partial \Phi_{\text{phase}} = -\partial (\gamma/c, \nabla)$

**4-TotalWaveVector** $K_T = (\omega_T/c, k_T) = -\partial \Phi_{\text{phase}} = -\partial (\gamma/c, \nabla)$

4-TotalWaveVector $K_T = (\omega_T/c, k_T) = -\partial \Phi_{\text{phase}} = -\partial (\gamma/c, \nabla)$

**Spatial part:** $k = \gamma(\omega_o/c^2)u = (\omega/c^2)u = \omega \nabla \nu_{\text{phase}}$

**Temporal part:** $\omega = \gamma \omega_o$

4-WaveVector $K = (\omega/c, k) = -\partial \Phi_{\text{phase,free}} = -\partial (\gamma/c, \nabla)$

4-Displacement $\Delta R = (c \Delta t, \Delta r)$

4-Position $R = (ct, r)$

4-Gradient $\partial = (\partial / c, \nabla)$

4-Velocity $U = \gamma(c, u)$

**SRQM Diagram:**

**Wave Phase Equation** $K_T = -\partial \Phi_{\text{phase}}$

**Proper Time Derivative** $U \cdot \gamma = \gamma \nu_{\text{group}} \ast \nu_{\text{phase}}$

4-Gradient $\partial = (\partial / c, \nabla) = \partial \nu_{\text{phase}}$

which matches:

$\Phi_{\text{phase}} = -\int \omega \text{d}t$ for a free particle

$\Phi_{\text{phase}} = -\int (\omega + V/\hbar) \text{d}t$ in a potential

Relativistic AngFreq $\omega$ vs Invariant Rest AngFreq $\omega_0$

$\omega = \gamma \omega_0$

**Matter-Wave Dispersion Relation**

$\omega^2 = (|k|^2 + \omega_o^2) + (\omega_o^2)$

**Trace** $[T^\gamma] = \eta_{\mu \nu} T^\gamma = T^\gamma = T^{\gamma \gamma}$

$V = V_{\gamma} = V_{\gamma \gamma} = (V_{\gamma \gamma}^0)^2 - V_{\gamma \gamma}$

$V_{\gamma \gamma} = (V_{\gamma \gamma}^0)^2$

$Lorentz$ $Scalar$ $Invariant$
**SRQM Diagram:**

The Basis of Classical SR Physics

4-CurrentDensity, Charge Conservation

- **4-Position** \( R = (ct, r) \)
- **4-Gradient** \( \partial = (\partial / c, -\nabla) \)
- **4-Velocity** \( U = \gamma(c, u) \)

**4-CurrentDensity** \( J = (pc, j) = \rho_o U = \gamma \rho_o (c, u) = \rho(c, u) \)

**4-ChargeFlux** \( J \)

**Temporal part:** \( \rho = \gamma \rho_o \)

**Spatial part:** \( j = \gamma \rho_o u = \rho u \)

**Conservation of Charge (Q)**

\[ \partial \cdot J = (\partial / c, -\nabla) \cdot (pc, j) = (\partial \rho + \nabla \cdot j) = 0 \]

**Continuity Equation:** Noether's Theorem

The temporal change in charge density is balanced by the spatial change in current density. Charge is neither created nor destroyed. It just moves around as charge currents...

**Relativistic Charge Density** \( \rho \) vs **Invariant Rest Charge Density** \( \rho_o \)

\[ \rho = \gamma \rho_o \]

**SRQM Diagram**

- **Conservation of Charge** \( \partial J = 0 \)
- **4-Displacement** \( \Delta R = (c\Delta t, \Delta r) \)
- **4-Gradient** \( \partial = (\partial / c, -\nabla) \)
- **4-Position** \( R = (ct, r) \)
- **4-CurrentDensity** \( J = (pc, j) = (pc, u) = \rho_o U \)

**4-Scalar** \( \rho^2 = (|j|^2/c^2 + \rho_o^2) = (|j|/c)^2 + (\rho_o)^2 \)

**SR 4-Tensor**
- \((2,0)\)-Tensor \( T^{ix} \)
- \((1,1)\)-Tensor \( T^i = (v^i, v) \)
- \((0,2)\)-Tensor \( T_{ij} \)

**SR 4-Vector**
- \((1,0)\)-Tensor \( V^i = (v^i, V) \)

**SR 4-Scalar**
- \((0,0)\)-Tensor \( S \) or \( S_0 \)

Lorentz Scalar

**Rest Volume**

\( V_o = \int d^3x = \int dA \)

-emphasizing linear contraction along direction \( dr \)

---

Interpretation

John B. Wilson
SciRealm.org

http://scirealm.org/SRQM.pdf
SRQM Diagram: 
The Basis of Classical SR Physics 
4-(Dust)NumberFlux, Particle # Conservation

4-Position \( R = (ct, r) \)
4-Gradient \( \partial = (\partial t/c, -\nabla) \)
4-Velocity \( U = \gamma (c, u) \)

4-NumberFlux \( N = (nc, n) = n_o U = \gamma n_o (c, u) = n(c, u) \)

Temporal part: \( n = \gamma n_o \) 
{number-density}

Spatial part: \( n = \gamma n_o u = nu \) 
{3-number-flux}

Conservation of Particle # \((N)\) 
\( \partial \cdot N = 0 \)
\( (\partial t/c, -\nabla) \cdot (nc, n) = (\partial_t n + \nabla \cdot n) = 0 \)

Continuity Equation: Noether's Theorem
The temporal change in number density is balanced by the spatial change in number-flux. 
Particle # is neither created nor destroyed. It just moves around as number currents...

\( n^2 = (|n|^2/c^2 + n_o^2) \)
\( n_o^2 = n^2/\beta^2 + n_o^2 \)
\( n_o^2 = n^2/(1 - |\beta|^2) \)
\( n = \gamma n_o \)

Relativistic NumberDensity\((n)\) vs Invariant Rest NumberDensity\((n_o)\)

SR 4-Tensor
\( (2,0)\)-Tensor \( T^{\mu \nu} \)
\( (1,1)\)-Tensor \( T^\mu, T^\nu \)
\( (0,2)\)-Tensor \( T_{\mu \nu} \)

SR 4-Vector
\( (1,0)\)-Tensor \( V^\mu = V = (\vec{v}, v) \)
SR 4-CoVector: OneForm
\( (0,1)\)-Tensor \( V_\mu = (v_\nu) \)

SR 4-Scalar
\( (0,0)\)-Tensor \( S \) or \( S_o \)
Lorentz Scalar

Trace\([T^\mu \nu] = \eta_\mu \eta_\nu T^{\mu \nu} = T^o = T \)
\( V \cdot V = V \eta_\mu V^\mu = (|v|^2) - 2v \cdot \vec{v} = (\vec{v}^2) \)

Lorentz Scalar Invariant

Emphasizing linear contraction along direction \( dr \)

SciRealm.org
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http://scirealm.org/SRQM.pdf
The main idea that makes a generic 4-Vector into an SR 4-Vector is that it must transform correctly according to an SR Lorentz Transformation $\Lambda^\mu_\nu = \partial_\nu [X^\mu]$, which is basically any linear, unitary or antiunitary, transform (Determinant[\Lambda^\mu_\nu] = \pm 1) which leaves the Invariant Interval unchanged.

The SR continuous transforms (variable with some parameter) have $\text{Det} = +1, \text{Proper}$ and include:
- "Rotation" (a mixing of space-space coordinates) and "(Velocity) Boost" (a mixing of time-space coordinates).

The SR discrete transforms can be $\{\text{Det} = +1, \text{Proper}\}$ or $\{\text{Det} = -1, \text{Improper}\}$ and include:
- "Space Parity-Inversion" (reversal of the all space coordinates), “Time-Reversal” (reversal of the temporal coordinate),
- "Identity" {no change}, various single dimension “Flips”, “Fixed Rotations”, and combinations of all of these discrete transforms.

Continuous: Boost depends on variable parameter $\beta$, with $\gamma = 1/\sqrt{1-\beta^2}$

$\Lambda^\mu_\nu = \partial_\nu [X^\mu]$

Discrete: Parity has no variable parameters

Det[$P^\mu_\nu$] = -1, Improper

Trace[$T^\mu_\nu$] = $\eta^\mu_\nu T^\mu_\nu = T_{\nu \nu} = T$

$V V = V^\nu (v^\nu) = (\nu^\nu)^2$ = Lorentz Scalar Invariant

SR: Lorentz Transform

$\partial_\nu [R^\mu] = \partial R^\mu / \partial R^\nu = \Lambda^\mu_\nu$

$\Lambda^\mu_\nu = (\Lambda^\nu)^\mu : \Lambda^\nu_\mu A^\nu = \eta^\nu_\nu = \delta^\nu_\nu$

$\eta_\mu_\nu \Lambda^\mu_\nu = \eta_\mu_\nu$

Det[$\Lambda^\mu_\nu$] = \pm 1

$\text{Tr}[\Lambda^\mu_\nu] = -\infty, +\infty$

Lorentz Transform Type

Proper: preserves orientation of basis

Improper: reverses orientation of basis

4-Vector SRQM Interpretation of QM
Lorentz Transforms $\Lambda_{\mu'\nu} = \partial_{\nu}[X^{\mu'}]$

Proper Lorentz Transforms (Det=+1): Continuous: (Boost) vs (Rotation)

$\beta = v/c$: dimensionless Velocity Beta Factor \{ $\beta=0..1$, with speed-of-light ($c$) at ($\beta=1$) \}

$\gamma = 1/\sqrt{1-\beta^2} = 1/\sqrt{1-\beta \cdot \beta}$: dimensionless Lorentz Relativistic Gamma Factor \{ $\gamma=1..\infty$ \}

Typical Lorentz Boost Transform (symmetric): for a linear-velocity frame-shift ($x,t$)-Boost in the $x$-direction:

$\Lambda^{\mu'}_{\nu} \rightarrow B^{\mu'}_{\nu}, \{ [x] = e^{\gamma \beta x J} \}

\begin{align*}
\gamma - \beta y & \begin{pmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 2 & 0 \\
\end{pmatrix}
\end{align*}

Determinant = +1

$\{ \cos^2 + \sin^2 = 1 \}\}

$\{ \gamma^2 - \beta^2 y^2 = 1 \}

\{ \cosh^2 - \sinh^2 = 1 \}

$\Lambda' = (a', a', a', a') = B^{\nu}_{\mu'}A^{\mu'} = (\gamma a' - \gamma' a', -\gamma' a' + \gamma a, a', a')$

Typical Lorentz Rotation Transformation (non-symmetric): for an angular-displacement frame-shift ($x,y$)-Rotation about the $\hat{z}$-direction:

$\Lambda^{\mu'}_{\nu} \rightarrow R^{\mu'}_{\nu}, \{ \theta = e^{i \theta J} \}

\begin{align*}
1 & \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\end{align*}

Determinant = +1

$\{ \cos^2 + \sin^2 = 1 \}\}

$\{ \gamma^2 - \beta^2 y^2 = 1 \}

$\{ \cosh^2 - \sinh^2 = 1 \}

$\Lambda' = (a', a', a', a') = R^{\nu}_{\mu'}A^{\mu'} = (a, \cos[\theta]a' - \sin[\theta]a', \sin[\theta]a' + \cos[\theta]a', a')$

The Lorentz Rotation $R^{\nu}_{\mu'}$ is a 4D rotation matrix. It simply adds the time component, which remains unchanged (1), to the standard 3D rotation matrix.
Lorentz Transforms $\Lambda^{\mu'}_{\nu} = \partial_{\nu}[X^{\mu'}]$

Proper Lorentz Transforms (Det=+1): (Boost) vs (Rotation) vs (Identity)

**General Lorentz Boost Transform** (symmetric, continuous): for a linear-velocity frame-shift (Boost) in the $v/c=\beta=(\beta^0,\beta^1,\beta^2)$-direction:
$$\Lambda^{\mu'}_{\nu} \rightarrow B^{\mu'}_{\nu} = \gamma \left[ \begin{array}{cc} 1 & \gamma \beta^0 \\ \gamma^{-1} \beta^0 & 1 \end{array} \right] \Lambda^{\mu'}_{\nu} = \Lambda^{\mu'}_{\nu} B = 4$$

**Lorentz Identity Transform**
$$\Lambda^{\mu'}_{\nu} \rightarrow \eta^{\mu'}_{\nu} = \delta^{\mu'}_{\nu} = I_{(4)}$$

**Lorentz Rotation Transform**
$$\Lambda^{\mu'}_{\nu} \rightarrow R^{\mu'}_{\nu}$$

**Rotated 4-Vector**
$$A^I = A^\mu = (a^0, a^I)$$

**Boosted 4-Vector**
$$A^I = A^\mu = (a^0, a^I)$$

**Identical 4-Vector**
$$A^I = A^\mu = (a^0, a^I)$$

**Lorentz Boost Transform** (Det=+1)
$$\text{Tr}[B^{\mu'}_{\nu}] = (4..\infty)$$
$$\text{Det}[B^{\mu'}_{\nu}] = +1$$

**Lorentz Rotation Transform** (Det=+1)
$$\text{Tr}[R^{\mu'}_{\nu}] = (0..4)$$
$$\text{Det}[R^{\mu'}_{\nu}] = +1$$

**Lorentz Identity Transform**
$$\Lambda^{\mu'}_{\nu} \rightarrow B^{\mu'}_{\nu}$$

The Lorentz Identity Transform is the limit of both the Rotation and Boost Transforms when the respective "rotation angle" is 0

**Lorentz Identity Transform (symmetric,"discrete":continuous):**
for a non-frame-shift (Identity) in any direction:
$$\Lambda^{\mu'}_{\nu} \rightarrow \eta^{\mu'}_{\nu} = \delta^{\mu'}_{\nu} = \text{Diag}[1, \delta] = I_{(4)}$$

**Lorentz Rotation Transform (non-symmetric,continuous):**
for an angular-displacement frame-shift (Rotation) angle $\theta$ about the $n=(n^1, n^2, n^3)$-direction:
$$\Lambda^{\mu'}_{\nu} \rightarrow R^{\mu'}_{\nu} = \Lambda^{\mu'}_{\nu} = \left( \begin{array}{cc} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{array} \right) \Lambda^{\mu'}_{\nu}$$

**Matrix symmetry:** $B$ is symmetric (equals transpose, $B=B^T$), while $R$ is nonsymmetric but orthogonal (transpose equals inverse, $R^T = R^{-1}$)

**SR: Lorentz Transform**
$$\partial_{\nu}[R^{\mu'}_{\nu}] = \partial R^{\mu'}_{\nu} = \Lambda^{\mu'}_{\nu}$$

**Trace** $\Lambda^{\mu'}_{\nu} = \eta_{\mu\nu} \Lambda^{\mu'}_{\nu} = T^{\mu'}_{\nu} = T$
$$V \cdot V = V^\mu \eta_{\mu\nu} V^\nu = (V^0)^2 - (V^I)^2 = (V^0)^2$$

$V^0 \cdot V^I$ is Lorentz Scalar Invariant

**Trace of Lorentz Transformation**
$$\text{Tr}[\Lambda^{\mu'}_{\nu}] = \eta_{\mu\nu} \Lambda^{\mu'}_{\nu} = \text{Det}[\Lambda^{\mu'}_{\nu}] = \pm 1$$

**Lorentz Boost Transformation**
$$\text{Tr}[B^{\mu'}_{\nu}] = (4..\infty)$$
$$\text{Det}[B^{\mu'}_{\nu}] = +1$$

**Lorentz Rotation Transformation**
$$\text{Tr}[R^{\mu'}_{\nu}] = (0..4)$$
$$\text{Det}[R^{\mu'}_{\nu}] = +1$$

**Lorentz Identity Transform**
$$\text{Tr}[I_{(4)}] = 4$$

**Lorentz Symmetry**
$$\text{Det}[\Lambda^{\mu'}_{\nu}] = \pm 1$$

**Lorentz Relativistic Gamma Factor**
$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

**Identity Transformation for zero relative motion:** Boost/rotation:
$$B(0) = R(0) = I_{(4)}$$

**Proper Transformation** = positive unit determinant:
$$\text{det}[B] = \text{det}[R] = \text{det}[\eta] = +1.$$
Lorentz Transforms $\Lambda^\mu_\nu = \partial_\nu [X^\mu]$

Discrete (non-continuous) (Parity-Inversion) vs (Time-Reversal) vs (Identity)

- **General Lorentz Parity-Inversion (Space-Reversal) Transform:** $\Lambda^\mu_\nu \rightarrow P^\mu_\nu$ (Improper, symmetric, discrete)
  
  $\Lambda^\mu_\nu = \begin{bmatrix} 1 & 0 \\ 0 & -\delta \end{bmatrix}$

- **General Lorentz Time-Reversal Transform:** $\Lambda^\nu_\mu \rightarrow T^\nu_\mu$ (Improper, symmetric, discrete)
  
  $\Lambda^\nu_\mu = \begin{bmatrix} -1 & 0 \\ 0 & \delta \end{bmatrix}$

- **General Lorentz Identity Transform:** $\Lambda^\mu_\nu \rightarrow I_{(4)}$ (Proper, symmetric, discrete)
  
  $\Lambda^\mu_\nu = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

**SR: Lorentz Transform**

$\delta[R^\mu_\nu] = \partial^\mu_\nu R^\nu_\mu = \Lambda^\mu_\nu$

$\Lambda^\mu_\nu = (\Lambda^{-1})^\nu_\mu : \Lambda^\mu_\nu \Lambda^\nu_\rho = \delta^\mu_\rho$

$\eta_{\mu\nu}\Lambda^\mu_\rho \Lambda^\rho_\nu = \eta_{\alpha\beta}$

$\text{Det}[\Lambda^\mu_\nu] = +1$

$\text{Tr}[\Lambda^\mu_\nu] = \{\pm 1\} = \{+1\}$

Both the Parity-Inversion (P) and Time-Reversal (T) have a Determinant of -1, which is an improper transform. However, combinations (PP), (TT), (PT) have overall Determinant of +1, which is proper.

**SR Time Reversal** neglects spin and charge. When included, there is also a Charge-Conjugation (C) transform. Then one gets (CC), (PP), ((PT){PT}), & permutations of (CPT) transforms all leading back to the Identity ($I_{(4)}$).

**4-Vector SRQM Interpretation of QM**

*Note that the Trace of Discrete Lorentz Transforms goes in steps from (-4,-2,2,4). As we will see in a bit, this is a major hint for SR antimatter and CPT Symmetry.*
SRQM Lorentz Transforms $\Lambda_{\nu'}^{\mu'} = \partial_{\nu}[X^{\mu}]$

Discrete & Fixed Rotation → Particle Exchange

Lorentz Coordinate-Flip Transforms

Discrete & Fixed Rotation

SR 4-Vector

$T_{\alpha}^{\nu}$

SR 4-Scalar

$\Lambda_{\nu}^{\nu'} = \Lambda_{\nu}^{\mu'}\Lambda_{\mu}^{\nu'} = \eta_{\nu\mu} \Lambda_{\nu}^{\mu'} = \eta_{\nu\mu} \Lambda_{\nu}^{\mu'}$

$\text{Det}[\Lambda_{\nu}^{\nu'}] = -1$

$Lorentz$ and $\text{Parity}$ Transforms

$\text{SR: Lorentz Transform}$

$\partial_{\nu}[R_{\nu'}^{\nu}] = \partial_{\nu}R_{\nu'/\nu}$

$\Lambda_{\nu'}^{\nu} = (\Lambda^{(\pi)})_{\nu'}^{\nu} : \Lambda_{\nu}^{\nu'}\Lambda_{\nu'}^{\nu} = \eta_{\nu\mu} \partial_{\nu}R_{\nu'/\nu}$

$\text{Tr}[\Lambda_{\nu}^{\nu'}] = \Lambda_{\nu}^{\nu'} = \text{Lorentz Transform Type}$

Any single Lorentz Flip Transform is Improper, with a Determinant of $-1$.

However, pairwise combinations are Proper, with a Determinant of $+1$.

All single flips have Trace of $2$.

The combination of any two Spatial Flips is the equivalent of a Spatial Rotation by $(\pi)$ about the associated rotational axis.

sin$(\pi) = 0$, cos$(\pi) = -1$

Since this is a Proper transform, it is also the equivalent of a particle location exchange.

The combination of all three Spatial Flips, Flip-xyz, gives the Lorentz Parity Transform, which is again Improper, with Trace of $-2$.

The Flip-t is the standard Lorentz Time-Reversal, Improper.

$Lorentz$ Transform $\partial_{\nu}[R_{\nu'}^{\nu}] = \Lambda_{\nu'}^{\nu}$

All Lorentz Trans.

$\Lambda_{\nu}^{\nu'} = 4 = \Lambda_{\nu}^{\nu'}$

$\text{Trace}[T_{\nu'}] = \eta_{\nu'\nu}^{\nu'} = \Lambda_{\nu}^{\nu'} = T_{\nu'} = T$

$V \cdot V = V \Lambda_{\nu}^{\nu'} = [(V')^{\nu} \cdot V](\nu')^{\nu}$

$= \text{Lorentz Scalar Invariant}$
SRQM Lorentz Transforms $\Lambda^{\mu'}_{\nu} = \partial_{\nu}[\mathcal{X}^{\mu}]$

Lorentz Transform Connection Map
Examine all possible combinations of Discrete Lorentz Transformations which are Linear (Determinant of $\pm 1$).

A lot of the standard SR texts only mention (P)arity-Inverse and (T)ime-Reversal. However, there are many others, including (F)lips and (R)otations of a fixed amount. However, the (T)imeReversal and Combo(P)arity(T)ime take one into a separate section of the chart. Taking into account all possible discrete Lorentz Transformations fills in the rest of the chart. The resulting interpretation is that there is CPT Symmetry (Charge:Parity:Time) and Dual Time-Space (with reversed timeflow). In other words, one can go from the Identity Transform (all $+1$) to the Negative Identity Transform (all $-1$) by doing a Combo PT Lorentz Transform or by Negating the Charge (Matter→Antimatter). The Feynman-Stueckelberg CPT Interpretation (Antimatter moving spacetime-backward = NormalMatter moving spacetime-forward) aligns with this as a Dual-Universe "Antimatter" Side.

This is similar to Dirac's prediction of Antimatter, but without the formal need of Quantum Mechanics, or Spin. In fact, it is more general than Dirac's work, which was about the electron. This is from general Lorentz Transforms for any kind of particle: event.

<table>
<thead>
<tr>
<th>Det</th>
<th>Trace: Determinant</th>
<th>CPT, Big-Bang, Matter ↔ AntiMatter, Arrow(s)-of-Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pm 1$</td>
<td>$\det[\Lambda^\nu_\nu] = \pm 1$</td>
<td>$\Lambda^\nu_\nu = (\Lambda^\nu_\nu)^\dagger = \eta^\mu_\mu \Lambda^\mu_\nu = \eta^\nu_\nu = \delta^\nu_\nu$</td>
</tr>
<tr>
<td>$\pm 2$</td>
<td>$\det[\Lambda^\nu_\nu] = \pm 2$</td>
<td>$\Lambda^\nu_\nu = \pm 4 = \Lambda^\mu_\nu \Lambda^\nu_\mu$</td>
</tr>
<tr>
<td>$\pm 4$</td>
<td>$\det[\Lambda^\nu_\nu] = \pm 4$</td>
<td>$\Lambda^\nu_\nu = \pm 4 = \Lambda^\mu_\nu \Lambda^\nu_\mu$</td>
</tr>
</tbody>
</table>

| $\pm 1$ | $\det[\Lambda^\nu_\nu] = \pm 1$ | $\Lambda^\nu_\nu = \pm 1$ |
| $\pm 2$ | $\det[\Lambda^\nu_\nu] = \pm 2$ | $\Lambda^\nu_\nu = \pm 2$ |
| $\pm 4$ | $\det[\Lambda^\nu_\nu] = \pm 4$ | $\Lambda^\nu_\nu = \pm 4$ |

SRQM Lorentz Transforms $\Lambda^\mu_\nu = \partial_\nu[ X^\mu ]$

Lorentz Transform Connection Map – Discrete Transforms

CPT, Big-Bang, [Matter ↔ AntiMatter], Arrow(s)-of-Time

Discrete NormalMatter (NM) Lorentz Transform Type

Table:

<table>
<thead>
<tr>
<th>$t$</th>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
</tr>
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</tr>
</tbody>
</table>
SRQM Lorentz Transforms \( \Lambda_{\mu'\nu} = \partial_{\nu}[X^{\mu}] \)

Lorentz Transform Connection Map – Discrete Transforms
CPT, Big-Bang, [Matter↔AntiMatter], Arrow(s)-of-Time

I ran across another variation of YinYang symbol, also known as the T'ai chi symbol, on the internet. I like the \(1+3=4\)-level symmetries. There are 8 total circles, with an overall even balance of \{white:black\}, \{+:-\}, so 4D in the two dual realms.

It also reminds me a bit of a Penrose Diagram, which is an extension of the Minkowski Diagram.
SRQM Lorentz Transforms $\Lambda^\mu_\nu = \partial_\nu [X^\mu']$

**Lorentz Transform Connection Map – Trace Identification**

CPT, Big-Bang, [Matter ↔ AntiMatter], Arrow(s)-of-Time

All Lorentz Transforms have Tensor Invariants: Determinant = ±1 and InnerProduct = 4. However, one can use the Tensor Invariant Trace to Identify CPT Symmetry & AntiMatter

<table>
<thead>
<tr>
<th>Trace : Determinant</th>
<th>Det = +1 Proper</th>
<th>Det = -1 Improper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tr[ NM-Rotate ] = {0...+4}</td>
<td>Tr[NM-Identity] = +4</td>
<td>Tr[NM-Boost] = {+4...+∞}</td>
</tr>
<tr>
<td>Tr[ AM-Rotate ] = {0...-4}</td>
<td>Tr[AM-Identity] = -4</td>
<td>Tr[AM-Boost] = {-4.....-∞}</td>
</tr>
</tbody>
</table>

**Discrete NormalMatter (NM) Lorentz Transform Type**

- **Minkowski-Identity**: AM-Flip-xyz=AM-ComboPT
- **Flip-t=TimeReversal, Flip-x, Flip-y, Flip-z** AM-Flip-xyz=AM-ParityInverse
- **Flip-xy=Rotate-xy(π)**, **Flip-xz=Rotate-xz(π)**, **Flip-yz=Rotate-yz(π)**

**AM-Minkowski-Identity**: Flip-xyz=ComboPT

**Discrete AntiMatter (AM) Lorentz Transform Type**

<table>
<thead>
<tr>
<th>Trace : Determinant</th>
<th>Det = +1 Proper</th>
<th>Det = -1 Improper</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM-Flip-xy=AM-Rotate-xy(π), AM-Flip-xz=AM-Rotate-xz(π), AM-Flip-yz=AM-Rotate-yz(π)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AM-Flip-xyz=ParityInverse AM-Flip-t=AM-TimeReversal, AM-Flip-x, AM-Flip-y, AM-Flip-z</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Trace : Determinant**

- **Tr = +4** : **Det = +1 Proper**
- **Tr = +2** : **Det = -1 Improper**
- **Tr = 0** : **Det = +1 Proper**
- **Tr = -2** : **Det = -1 Improper**
- **Tr = -4** : **Det = +1 Proper**

**Two interesting properties of (1,1)-Tensors, of which the Lorentz Transform is an example:**

- **Trace** = **Sum (∑)** of EigenValues : **Determinant** = **Product (∏)** of EigenValues

- Create an Anti-Transform which has all EigenValue Tensor Invariants negated.

- $\Sigma[-(EV's)] = -\Sigma[EV's]$ : The Anti-Transform has negative Trace of the Transform.
- $\Pi[-(EV's)] = (-1)^4 \Pi[EV's] = \Pi[EV's]$ : The Anti-Transform has equal Determinant.

The Trace Invariant identifies a “Dual” Negative-Side for all Lorentz Transforms.
SRQM Lorentz Transforms $\Lambda^\mu_\nu = \partial_\nu[X^\mu]$

Lorentz Transform Connection Map - Interpretations

CPT, Big-Bang, [Matter↔AntiMatter], Arrow(s)-of-Time

Based on the Lorentz Transform properties of the last few pages, here is interesting observation about Lorentz Transforms:

They all have Determinant of (±1), and Inner Product of (+4 = 4D), but the Trace varies depending on the particular Transform.

The Trace of the Identity is at (+4). Assume this applies to normal matter particles.
The Trace of normal-matter particle Rotations varies continuously from (0..+4).
The Trace of the normal-matter particle Boosts varies continuously from (+4..+Infinity (+∞)).
So, one can think of Trace = (+4) being the connection point between normal-matter Rotations and Boosts.

Now, various Flip Transforms (inc. the Time Reversal and Parity Transforms, and their combination as PT transform), take the Trace in discrete steps from (-4,-2,0,+2,+4). Applying a bit of symmetry:

The Trace of the Negative Identity is at (-4). Assume this applies to anti-matter particles.
The Trace of anti-matter particle Rotations varies continuously from (0..-4).
The Trace of the anti-matter particle Boosts varies continuously from (-4..-Infinity (-∞)).
So, one can think of Trace = (-4) being the connection point between anti-matter Rotations and Boosts.

This observation would be in agreement with the CPT Theorem:(Feynman-Stueckelberg) idea that (normal/anti)-matter particles moving backward in SpaceTime are CPT symmetrically equivalent to (anti/normal)-matter particles moving forward in SpaceTime.

Now, scale this up to Universe size: The Baryon Asymmetry problem (aka. The Matter-AntiMatter Asymmetry Problem).

If the Universe was created as a huge chunk of energy, and matter-creating energy is always transformed into matter-antimatter mirrored pairs, then

Answer: It is temporally on the “Other/Dual-Side” of the Big-Bang! The antimatter created at the Big-Bang is travelling in the negative-time (-t) direction from the Big-Bang creation point, and the normal matter is travelling in the positive-time direction (+t).

Universal CPT Symmetry. So, what happened “before” the Big-Bang? It “is” the AntiMatter Dual to our normal matter universe!

Pair-production is creation of AM-NM mirrored pairs within SpaceTime. The Big-Bang is the creation of Space-Time itself.

This also resolves the Arrow-of-Time Problem. If all known physical microscopic processes are time-symmetric, why is the flow of Time experienced as uni-directional?? (see Wikipedia “CPT Symmetry”; CP Violation”; Andrei Sakharov)

Answer: Time flow on This-Side of the Universe is (+t) direction, while time flow on the Dual-Side of the Universe is (-t) direction.

The math all works out. Time flow is bi-directional, but on opposite sides of the BB/Origin-Singularity. Universal CPT Symmetry!!

This gives total CPT Symmetry to all of the possible Lorentz Transforms (AM=AntiMatter, NM=NormalMatter):

Trace Various (AM_Flips) : Trace Various (NM_Flips)
-Infinity...(AM_Boosts)...(AM_Identity=4)...(AM_Rotations)...0...(NM_Rotations)...(+4 NM_Identity)...(NM_Boosts)...+Infinity

This solves the: Baryon (Matter-AntiMatter) Asymmetry Problem & Arrow(s)-of-Time Problem ( +t / -t )
This idea of Universal CPT Symmetry also gives a Universal Dimensional Symmetry as well.

Consider the well-known “balloon” analogy of the universe expansion. The “spatial” coordinates are on the surface of the balloon, and the expansion is in the (+t) direction. There is symmetry in the (+/-) directions of the spatial coordinates, but the time flow is always uni-directional, (+t), as the balloon gets bigger—inflates.

By allowing a “Dual-Side”, it provides a universal dimensional symmetry. One now has (+/-) symmetry for the temporal directions.

The “center” of the Universe is, literally, the Big Bang Singularity. It is the “center = zero = origin” point of both time and space directions. There are some people who prefer to say the BB is after inflation, but I am simply referring the “Origin-Singularity”.

The expansion gives time-flow always AWAY FROM the Big Bang singularity in both the Normal-Side (+t) and the Dual-Side (-t). All spatial coordinates expand in both the (+/-) directions on both temporal sides of the singularity.

Note that this gives an unusual interpretation of what came “before” the Big Bang, The “past” on either side extends only to the BB singularity, not beyond. Time flow is always away from this creation singularity.

This is also in accord with known black hole physics, in that all matter entering a BH event horizon ends at the BH singularity. Time and space coordinates both come to a stop at either type of singularity, from the point of view of an observer that is in the spacetime but not at one of these singularities.

So, the Big Bang is a “starting” singularity, and black holes are “ending” singularities. This also provides for idea of “white holes” actually just being black holes on the Dual-Side. White hole = time-reversed black hole. Always confusion about stuff coming out. This way, the mass is still attractive. Time-flow is simply reversed on the alternate side so stuff still goes INTO the hole…which makes way more sense than stuff that can only come out of the “massive→attractive” white-hole.

So, Universal CPT Symmetry = Universal Dimensional Symmetry.

And, going even further, I suspect this is the reason there is a duality in Metric conventions. In other words, physicists have wondered why one can use Metric signature (+, -, -, -) or (-, +, +, +).

I submit that one of these metrics applies to the Normal Matter side, while the other complementarily applies to the Dual side. This would allow correct causality conditions to apply on either side.

Again, this is similar to the Dirac prediction of antimatter based on a duality of possible solutions.

This gives total CPT Symmetry to all of the possible Lorentz Transforms (AM=AntiMatter , NM=NormalMatter):

- Trace Various (AM_Flips) : Trace Various (NM_Flips)
- (AM_Identity=-4)(AM_Rotations)…(NM_Rotations)...(+4 NM_Identity)...(NM_Boosts)...+Infinity

This solves the:
Baryon (Matter→AntiMatter) Asymmetry Problem & Arrow(s)-of-Time Problem (+t / -t)
A Klein geometry is a pair \((G, H)\) where \(G\) is a Lie group and \(H\) is a closed Lie subgroup of \(G\) such that the (left) coset space \(X:= G/H\) is connected.

\(G\) acts transitively on the homogeneous space \(X\).

We may think of \(H \triangleleft G\) as the stabilizer subgroup of a point in \(X\).

\(d\) is the dimension of the SpaceTime, which is 4D for our universe.
### Classical Transforms: Venn Diagram

**Full Galilean = Galilean + Translations**

<table>
<thead>
<tr>
<th>Transformations</th>
<th>Discrete</th>
<th>Continuous</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Galilean Transform</strong> $G^\mu_\nu$</td>
<td>4-Tensor {mixed type-(1,1)}</td>
<td></td>
</tr>
<tr>
<td>Time-reversal $G^\mu_\nu \rightarrow T^\mu_\nu$</td>
<td>(0)</td>
<td></td>
</tr>
<tr>
<td>SpatialFlipCombinations $G^\mu_\nu \rightarrow P^\mu_\nu$</td>
<td>(0)</td>
<td></td>
</tr>
<tr>
<td>Parity-Inversion $G^\mu_\nu \rightarrow P^\mu_\nu$</td>
<td>(0)</td>
<td></td>
</tr>
<tr>
<td>Identity $G^\mu_\nu = \eta^\mu_\nu=\delta^\mu_\nu$</td>
<td>(0)</td>
<td></td>
</tr>
<tr>
<td>Rotation $G^\mu_\nu \rightarrow R^\mu_\nu$</td>
<td>(3)</td>
<td></td>
</tr>
<tr>
<td>Motion:Shear $G^\mu_\nu \rightarrow S^\mu_\nu$</td>
<td>(3)</td>
<td></td>
</tr>
<tr>
<td>4-Zero $\Delta X^\mu \rightarrow (0,0)$</td>
<td>(0)</td>
<td>no motion</td>
</tr>
<tr>
<td>Isotropy (same all directions)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Translation Transform $\Delta X^\mu$</th>
<th>Discrete</th>
<th>Continuous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temporal $\Delta X^\mu \rightarrow (\Delta t,0)$</td>
<td>(1)</td>
<td>$\Delta t$</td>
</tr>
<tr>
<td>Spatial $\Delta X^\mu \rightarrow (0,\Delta x)$</td>
<td>(3)</td>
<td>$\Delta x \mid \Delta y \mid \Delta z$</td>
</tr>
<tr>
<td>Homogeneity (same all points)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Galilean Transformation Group** aka. Inhomogeneous Galilean Transformation

Lie group of all affine isometries of Classical: Euclidean Time + Space (preserve quadratic form $\delta_{\mu\nu}$)

General Linear, Affine Transform $X^\mu_{\nu}' = G^\mu_{\nu}'X^\nu + \Delta X^\mu_{\nu}$ with $\text{Det}[\Lambda]^\nu_{\nu} = \pm 1$

$(6+4=10)$
### Transformations

(# of independent parameters = # continuous symmetries = # Lie Dimensions)

**Poincaré Transformation Group** aka. Inhomogeneous Lorentz Transformation

Lie group of all affine isometries of SR:Minkowski **Time-Space** (preserve quadratic form $\eta_{\mu\nu}$)

General Linear, Affine Transform $X'_{\mu} = \Lambda_{\mu'\nu} X_{\nu} + \Delta X_{\mu'}$ with $\text{Det}[\Lambda_{\mu'\nu}] = \pm 1$

$(6+4=10)$

**Translation Transform**

$\Delta X_{\mu'}$  
$(1+3=4)$  
4-Vector

**Lorentz Transform**

$\Lambda_{\mu'\nu}$  
$(3+3=6)$  
4-Tensor {mixed type-(1,1)}

---

#### Lorentz Transform

- **Time-reversal**  
  $\Lambda_{\mu'\nu} \rightarrow T_{\mu'\nu}$  
  $(0)$  
  $t \rightarrow -t^*$  
  time parity anti-unitary

- **Parity-Inversion**  
  $\Lambda_{\mu'\nu} \rightarrow P_{\mu'\nu}$  
  $(0)$  
  $r \rightarrow -r$  
  space parity unitary

- **Charge-Conjugation**  
  $\Lambda_{\mu'\nu} \rightarrow C_{\mu'\nu}$  
  $(0)$  
  $R \rightarrow -R^*$, $q \rightarrow -q$  
  charge parity anti-unitary

- **SpatialFlipCombos**  
  $\Lambda_{\mu'\nu} \rightarrow F_{\mu'\nu}$  
  $(0)$  
  $\{x|y|z\} \rightarrow -\{x|y|z\}$  
  unitary

- **Identity**  
  $I_{(4)}$  
  $(0)$  
  no mixing unitary

- **Rotation**  
  $\Lambda_{\mu'\nu} \rightarrow R_{\mu'\nu}$  
  $(3)$  
  $x:y | x:z | y:z$

- **Boost**  
  $\Lambda_{\mu'\nu} \rightarrow B_{\mu'\nu}$  
  $(3)$  
  $tx | ty | tz$

---

#### Translation Transform

- **Temporal**  
  $\Delta X_{\mu'} \rightarrow (c\Delta t, 0)$  
  $(0)$  
  $\Delta t$

- **Spatial**  
  $\Delta X_{\mu'} \rightarrow (0, \Delta x)$  
  $(3)$  
  $\Delta x | \Delta y | \Delta z$

---

#### Discrete vs. Continuous

- **Discrete**
  - Time-reversal  
  - Parity-Inversion  
  - Charge-Conjugation

- **Continuous**
  - SpatialFlipCombos  
  - Identity  
  - Rotation  
  - Boost

- **Homogeneity**
  - (same all points)

---

**SRQM Transforms: Venn Diagram**

**Poincaré = Lorentz + Translations**

$(10)$  
$(6)$  
$(4)$

---

**Feynman-Stueckelberg Interpretation**

Amusingly, Inhomogeneous Lorentz adds homogeneity.

---

**SciRealm.org**  
John B. Wilson  
SciRealm@aol.com  
http://scirealm.org/SRQM.pdf

---

[(R→-R*)] or [(t→-t*) & (r→-r)] imply q→-q
Feynman-Stueckelberg Interpretation

---

### SRQM Transforms: Venn Diagram

#### Lorentz Transform

- **Time-reversal**  
  $\Lambda_{\mu'\nu} \rightarrow T_{\mu'\nu}$  
  $(0)$  
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- **Charge-Conjugation**  
  $\Lambda_{\mu'\nu} \rightarrow C_{\mu'\nu}$  
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  $R \rightarrow -R^*$, $q \rightarrow -q$  
  charge parity anti-unitary

- **SpatialFlipCombos**  
  $\Lambda_{\mu'\nu} \rightarrow F_{\mu'\nu}$  
  $(0)$  
  $\{x|y|z\} \rightarrow -\{x|y|z\}$  
  unitary

- **Identity**  
  $I_{(4)}$  
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  no mixing unitary

- **Rotation**  
  $\Lambda_{\mu'\nu} \rightarrow R_{\mu'\nu}$  
  $(3)$  
  $x:y | x:z | y:z$

- **Boost**  
  $\Lambda_{\mu'\nu} \rightarrow B_{\mu'\nu}$  
  $(3)$  
  $tx | ty | tz$

#### Translation Transform

- **Temporal**  
  $\Delta X_{\mu'} \rightarrow (c\Delta t, 0)$  
  $(0)$  
  $\Delta t$

- **Spatial**  
  $\Delta X_{\mu'} \rightarrow (0, \Delta x)$  
  $(3)$  
  $\Delta x | \Delta y | \Delta z$

---

**Discrete vs. Continuous**

- **Discrete**
  - Time-reversal  
  - Parity-Inversion  
  - Charge-Conjugation

- **Continuous**
  - SpatialFlipCombos  
  - Identity  
  - Rotation  
  - Boost

---

**Homogeneity**

- (same all points)

---

**SRQM Transforms: Venn Diagram**

**Poincaré = Lorentz + Translations**

$(10)$  
$(6)$  
$(4)$

---

**Feynman-Stueckelberg Interpretation**

Amusingly, Inhomogeneous Lorentz adds homogeneity.
SRQM Study:
Lie Groups and Generators

Lie Groups

de Sitter Group SO(1,4)
de Sitter invariant relativity
(\text{?maybe?})

Poincaré Group ISO(1,3)
\{ r << r_{ds} = \text{de Sitter Radius} \}
\quad r_{ds} = \sqrt{3/\Lambda} = L_1/\sqrt{|\Omega_3|}

SR & GR Physics
\text{(** currently thought correct **)}

\Lambda^{\nu}_{\mu'} \rightarrow B^{\nu}_{\mu'} = \text{de Sitter Group SO(1,4)}
\quad \text{Lorentz Boost}

Galilei Group
\{ |v| << c \}
\quad \text{Classical Physics}

\Gamma^{\nu}_{\mu'} \rightarrow S^{\nu}_{\mu'} = \text{Motion:Shear}

SRQ: Lorentz Boost
\quad \text{ct}' = (\gamma) c t - (\beta)(\gamma) x
\quad \text{y}' = y
\quad \text{x}' = - (\beta)(\gamma) c t + (\gamma) x
\quad \text{z}' = z

Classical: Galilean Motion Shear
\quad t' = t
\quad \text{x}' = x - \beta (t)c
\quad \text{y}' = y
\quad \text{z}' = z

Lorentz Matrices can be generated by a matrix M with \text{Tr}[M]=0 which gives:
\quad \{ \Lambda = e^{\mu \nu} M = e^{\mu \nu} (+\theta J - \xi K) \}
\quad \{ \Lambda' = (e^{\nu \mu})^\dagger \} = e^{\nu \mu} M^\dagger \}
\quad \{ \Lambda^{-1} = e^{\nu \mu} M^{-1} \} = e \mu \nu - M \}

SR:Lorentz Transform
\quad \delta_{[R \nu]} = \partial R_{\nu}/\partial R^{\nu} = \Lambda^{\nu}_{\nu}

\quad \Lambda^{\nu}_{\nu} = (\Lambda^{-1})_{\mu}^{\nu} : \Lambda^{\nu}_{\nu} \Lambda^{\mu \nu} = \eta_{\nu \nu} = \delta^{\nu}_{\nu}
\quad \eta_{\nu \nu} \Lambda^{\mu \nu} \Lambda^{\nu \nu} = \eta_{\nu \nu}

Det[\Lambda^{\nu}_{\nu}] = \pm 1

\Lambda^{\mu \nu} = 4

Rotations \text{J} = -\varepsilon_{\mu \nu \nu \nu} M^{\mu \nu} / 2, \text{Boosts} \text{K} = M_0
Review of SR Transforms

10 Poincaré Symmetries, 10 Conservation Laws

10 Generators: Noether’s Theorem
Review of SR Transforms
Poincaré Algebra & Generators
Casimir Invariants

The (10) one-parameter groups can be expressed directly as exponentials of the generators:

\[ U[I, (a, 0)] = e^{ia\hat{a} \cdot \hat{H}} = e^{ia^\mu \hat{p}_\mu} \]:
1. Hamiltonian (Energy) = Temporal Momentum \( \hat{H} = \hat{P}_0 \)
2. Linear Momentum \( \hat{p}_0 \)
3. Poincaré Algebra has 2 Casimir Invariants = Operators that commute with all of the Poincaré Generators
4. Angular Momentum \( \hat{J} \)
5. Lorentz Boost \( K \)

These are the commutators of the Poincaré Algebra:

\[ [\hat{X}, \hat{P}] = i\hbar q(\hat{X}) \] if interacting with EM field; otherwise = 0 for free particles
\[ M^{\mu\nu} = (\hat{X}^{\mu}\hat{P}_\nu - \hat{X}_\nu\hat{P}_\mu) \]
\[ [M^{\mu\nu}, M^{\rho\sigma}] = i\hbar(\eta^{\rho\nu}M^{\mu\sigma} + \eta^{\rho\sigma}M^{\mu\nu} + \eta^{\mu\nu}M^{\rho\sigma} + \eta^{\mu\sigma}M^{\rho\nu}) \]

Component form:

Rotations \( J = -i\epsilon_{\mu\nu\rho\sigma}M^{\mu\nu}/2 \), Boosts \( K = M_{\lambda} \)

\[ \left[ J_{\mu}, P_\lambda \right] = i\epsilon_{\mu\nu\lambda}P_\nu \]
\[ \left[ K_\mu, P_\lambda \right] = 0 \]
\[ \left[ K_\mu, J_\nu \right] = i\epsilon_{\mu\lambda\kappa}K_\lambda \]
\[ \left[ K_{\mu\nu}, K_{\lambda} \right] = i\epsilon_{\mu\nu\lambda\kappa}K_\kappa \]
\[ \left[ J_{\mu\nu}, J_\kappa \right] = i\epsilon_{\mu\nu\lambda\kappa}J_\lambda \]

The Poincaré Algebra is the Lie Algebra of the Poincaré Group:

Total of \( 1+3+3+3 = (1+3)+(3+3) = 4+6 = 10 \) Invariances from Poincaré Symmetry

Covariant form:

\[ M^{\mu\nu} = X^\mu P^\nu - X^\nu P^\mu \]
\[ P^\mu = P \]

M = Generator of Lorentz Transformations (6) = \{ Rotations (3) + Boosts (3) \}
P = Generator of Translation Transformations (4) = \{ Time-Move (1) + Space-Move (3) \}

Rotations \( J = -i\epsilon_{\mu\nu\rho\sigma}M^{\mu\nu}/2 \), Boosts \( K = M_{\lambda} \)

The set of all Lorentz Generators \( V = \{ \zeta \cdot K + \theta \cdot J \} \) forms a vector space over the real numbers.

The generators \( \{ J_x, J_y, J_z, K_x, K_y, K_z \} \) form a basis set of \( V \). The components of the axis-angle vector and rapidity vector \( \{ \theta_\mu, \theta_\nu, \zeta \} \) are the coordinates of a Lorentz generator wrt. this basis.

Very importantly, the Poincaré group has Casimir Invariant Eigenvalues = \{ Mass m, Spin j \}, hence Mass *and* Spin are purely SR phenomena, no QM axioms required!

This Representation of the Poincaré Group or Representation of the Lorentz Group is known as Wigner’s Classification in Representation Theory of Particle Physics

Poincaré Algebra has 2 Casimir Invariants = Operators that commute with all the Poincaré Generators

These are \( \{ P^2 = P_\mu P_\mu = (m_c)^2, W^2 = W_\mu W_\mu = -(m_c)^2(j + 1) \} \), with \( W^2 = (1/2)\epsilon^{\mu\nu\rho\sigma}J_\mu P_\rho P_\sigma \) as the Pauli-Lubanski Pseudovector

\[ [P^2, P_\lambda] = [P^2, J] = [P^2, K] = 0 \]: Hence the 4-Momentum Magnitude squared commutes with all Poincaré Generators
\[ [W^2, P_\lambda] = [W^2, J] = [W^2, K] = 0 \]: Hence the 4-SpinMomentum Magnitude squared commutes with all Poincaré Generators

\[ \text{Trace}[T^{\nu\mu}] = \eta_{\nu\mu} T^{\nu\mu} = T_{\nu\mu} = T \]
\[ V_\nu V_\nu = V^\mu V^\mu = (V_\nu V_\nu) V^{\nu\mu} = (V_\nu V_\nu)^2 \]

= Lorentz Scalar Invariant
SRQM Study:
10 Poincaré Symmetry Invariances
Noether’s Theorem: 10 SR Conservation Laws

\( \mathbf{A}^\text{Tensor Study} \) of Physical 4-Vectors

\( \mathbf{4} \)-Vector SRQM Interpretation of QM

\( \Phi \) of QM

\( \text{SR Waves:} \)

\( \Psi = ae^{-i(\mathbf{K} \cdot \mathbf{x})} \), \( \Psi_\gamma = ae^{+i(\mathbf{K} \cdot \mathbf{x})} \), \( \Psi_\sigma = ae^{-i(\mathbf{K} \cdot \mathbf{x})} \), \( \Psi_\delta = ae^{+i(\mathbf{K} \cdot \mathbf{x})} \)

\( \mathbf{4} \)-Gradient\n
\( \partial = \partial_x = (\partial_x/c)^2 - \nabla \cdot \nabla = (\partial_x/c)^2 \)

Time Translation:\n
Let \( \mathbf{X}_\gamma = (ct+\Delta t, \mathbf{x}) \), then \( \partial \mathbf{X} = (\partial_x/c, -\nabla)(ct+\Delta t, \mathbf{x}) = \text{Diag}[1, -1] = \partial \mathbf{x} = \eta^{\mu \nu} \)

so \( \partial \mathbf{x}_\gamma = \partial \mathbf{x} \) and \( \partial \mathbf{K} = [0] \)

\( (\partial \partial)[K \times \mathbf{x}_\gamma] = \partial(\partial[K \times \mathbf{x}_\gamma]) = -\partial(\partial[K \times \mathbf{x}]) + \partial(K \times \partial \mathbf{x}) = 0 + \mathbf{K} \cdot \partial \mathbf{x} = \partial(K \times \mathbf{K} \cdot \partial \mathbf{x}) = \partial(\partial[K \times \mathbf{x}]) = (\partial \partial)(K \times \mathbf{x}) \):

Lorentz Space-Space Rotation:\n
Let \( \mathbf{X}_R = (ct, \mathbf{R} \mathbf{x}) \), then \( \partial \mathbf{x}_R = (\partial_x/c, -\nabla)(ct, \mathbf{R} \mathbf{x}) = \text{Diag}[1, -1] = \partial \mathbf{x} = \eta^{\mu \nu} \)

so \( \partial \mathbf{x}_\gamma = \partial \mathbf{x} \) and \( \partial \mathbf{K} = [0] \)

\( (\partial \partial)[K \times \mathbf{x}_R] = \partial(\partial[K \times \mathbf{x}_R]) = -\partial(\partial[K \times \mathbf{x}]) + \partial(K \times \partial \mathbf{x}) = 0 + \mathbf{K} \cdot \partial \mathbf{x} = \partial(K \times \mathbf{K} \cdot \partial \mathbf{x}) = \partial(\partial[K \times \mathbf{x}]) = (\partial \partial)(K \times \mathbf{x}) \):

Lorentz Time-Space Boost:\n
Let \( \mathbf{X}_B = \gamma(\mathbf{ct}, -\mathbf{ct} + \mathbf{x}) \), where \( \gamma(\mathbf{ct}, -\mathbf{ct} + \mathbf{x}) = \Lambda^{\mu \nu} \mathbf{X} = \Lambda^{\mu \nu} = \Lambda^{00}(0) = 0 = \partial \mathbf{K} = \text{Divergence of } K = 0 \), as expected

\( (\partial \partial) = \Lambda^{\mu \nu} = \Lambda^{00}(0) = 0 = \partial \mathbf{K} = \text{Divergence of } K = 0 \), as expected

\( (\partial \partial)[K \times \mathbf{x}_B] = \partial(\partial[K \times \mathbf{x}_B]) = -\partial(\partial[K \times \mathbf{x}]) + \partial(K \times \partial \mathbf{x}) = 0 + \mathbf{K} \cdot \partial \mathbf{x} = \partial(K \times \mathbf{K} \cdot \partial \mathbf{x}) = \partial(\partial[K \times \mathbf{x}]) = (\partial \partial)(K \times \mathbf{x}) \):

SR Waves: (1) Conservation of Energy = (Temporal) 1-momentum

\( \mathbf{V} = (E/c, \mathbf{p}) \)

\( \mathbf{p} = (0, \mathbf{p}) \)

SR 4-Vector

\( \mathbf{4} \)-Vector SRQM Interpretation of QM

\( \Phi \) of QM

See Wikipedia: Relativistic Angular Momentum

\( \text{SR Waves:} \)

\( \Psi = ae^{-i(\mathbf{K} \cdot \mathbf{x})} \), \( \Psi_\gamma = ae^{+i(\mathbf{K} \cdot \mathbf{x})} \), \( \Psi_\sigma = ae^{-i(\mathbf{K} \cdot \mathbf{x})} \), \( \Psi_\delta = ae^{+i(\mathbf{K} \cdot \mathbf{x})} \)

\( (\partial \partial)[K \times \mathbf{x}_\gamma] = \partial(\partial[K \times \mathbf{x}_\gamma]) = -\partial(\partial[K \times \mathbf{x}]) + \partial(K \times \partial \mathbf{x}) = 0 + \mathbf{K} \cdot \partial \mathbf{x} = \partial(K \times \mathbf{K} \cdot \partial \mathbf{x}) = \partial(\partial[K \times \mathbf{x}]) = (\partial \partial)(K \times \mathbf{x}) \):

Wave Equation Invariant under all Poincaré transforms

Total of (1+3+3+3 = 10) Invariances from Poincaré Symmetry
SRQM Study: 4-Vector Operations

Lorentz Scalar Product $A \cdot B = A_\mu B^\mu$

Exterior Product $A^B = A^\mu B^\nu - A^\nu B^\mu$

4-Gradient

\[ \partial = (\partial/c, -\vec{\nabla}) = \partial R_\mu \]

\[ \partial^2 = (\partial/c)^2 - \nabla \cdot \nabla \]

d'Alembertian

Minkowski Metric

\[ \partial[R] = \partial_\mu [R^\nu] = \eta^{\mu\nu} \]

Lorentz Transform

\[ \partial[R^\mu] = \Lambda_{\mu\nu} R^\nu = 4 \text{ Dimension} \]

4-Position

\[ R = (ct, \vec{r}) \]

4-Momentum

\[ P = (mc, \vec{p}) = (E/c, \vec{p}) \]

4-Gradient

\[ \partial^\mu = \partial^\mu \partial - \partial^\nu \partial \]

\[ \partial^0 = \partial^0 \partial - \partial^\nu \partial \]

Action Scalar

\[ \partial^A = \partial^\mu A^\mu \]

Faraday EM 4-Tensor

\[ F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \]

4-Angular-Momentum Tensor

\[ M^{\mu\nu} = R^P = R^\mu P^\nu - R^\nu P^\mu \]

4-Angular-Momentum 4-Tensor

\[ \partial \cdot A^\mu = \partial_\mu A^\mu = 0 \]

Lorenz Gauge, a conservation of 4-EMVectorPotential

\[ \partial \cdot P = \partial_\mu P^\nu = -S_{\text{action, free}} \]

Action Scalar

\[ \partial \cdot F^\mu = \partial_\mu F^\mu = ???: \text{ probably something important} \]

There are at least three 4-Vector relations which use the Exterior (Wedge=\^) Product.

Likewise, each of these has a physical (Dot=\cdot) Product relation as well.

SR 4-Vector

(2,0)-Tensor $T^{\mu\nu}_{\tau\rho}$

(1,1)-Tensor $V^\mu = V = (v^0, \vec{v})$

(0,2)-Tensor $T^{\mu\nu}_{\rho\sigma}$

SR 4-Divergence

(1,0)-Tensor $V^\rho = \nabla \cdot V$

SR 4-Curl

(0,1)-Tensor $V_\mu = \nabla \times V$

Lorentz Scalar

\[ \eta^{\mu\nu} = \text{Lorentz Scalar Invariant} \]

SR $\rightarrow$ QM

Physics

4-Vector SRQM Interpretation of QM

\[ \phi / c^2 = \text{Electric-Magnetic} \]

\[ \rho / c^2 = \text{Energy-Mass} \]

\[ \mu_0 J = \text{Maxwell EM Wave Eqn} \]

\[ \varepsilon_0 c^2 = \text{Maxwell EM Wave Eqn} \]

\[ \mu_0 = \text{Maxwell EM Wave Eqn} \]

\[ \varepsilon_0 c^2 = \text{Maxwell EM Wave Eqn} \]

\[ \eta_{\mu\nu} T^{\mu\nu} = T_{\mu\nu} = T \]

\[ V \cdot V = V^\rho V^\rho = (v^0)^2 - \vec{v} \cdot \vec{v} = (v^0)^2 \]

= Lorentz Scalar Invariant

\[ \text{Trace}[T^{\mu\nu}] = \eta^{\mu\nu} T^{\mu\nu} = T_{\mu\nu} = T \]

\[ V \cdot V = V^\rho V^\rho = (v^0)^2 - \vec{v} \cdot \vec{v} = (v^0)^2 \]

= Lorentz Scalar Invariant
SRQM Study: 4-Momentum → 4-Force
4-AngularMomentum → 4-Torque

Linear:
4-Force is the ProperTime Derivative of 4-Momentum.

Angular:
4-Torque is the ProperTime Derivative of 4-AngularMomentum.

\[
\frac{d}{d\tau}[\mathbf{M}^{\mu\nu}] = \frac{d}{d\tau}[\mathbf{X}^\mu\mathbf{P}]
\]
\[
= \frac{d}{d\tau}[\mathbf{X}^\mu\mathbf{P} - \mathbf{X}^\nu\mathbf{P}]
\]
\[
= [\mathbf{U}^\mu\mathbf{P} - \mathbf{U}^\nu\mathbf{P}]
\]
\[
= [\mathbf{X}^\mu\mathbf{U} - \mathbf{X}^\nu\mathbf{U}]
\]
\[
= [\mathbf{m}_0(\mathbf{U}^\mu\mathbf{U} - \mathbf{U}^\nu\mathbf{U}) + \mathbf{X}^\nu\mathbf{F}]
\]
\[
= [\mathbf{m}_0(\mathbf{U}^\mu\mathbf{U} - \mathbf{U}^\nu\mathbf{U}) + \mathbf{x}^\nu\mathbf{F}]
\]
\[
= [\mathbf{X}^\mu\mathbf{F} - \mathbf{X}^\nu\mathbf{F}]
\]
\[
\frac{d}{d\tau}[\mathbf{M}^{\mu\nu}] = T^{\mu\nu} = [\mathbf{X}^\mu\mathbf{F} - \mathbf{X}^\nu\mathbf{F}] = \mathbf{X}^\mu\mathbf{F}
\]
SR 4-Vectors & 4-Tensors

Lorentz Scalar Product & Tensor Trace

Invariants: Similarities

All {4-Vectors:4-Tensors} have an associated {Lorentz Scalar Product:Trace}

Each 4-Vector has a “magnitude” given by taking the Lorentz Scalar Product of itself.
\[ V \cdot V = V_\mu \eta_{\mu\nu} V^\nu = (v_0^2 + v_1^2 + v_2^2 + v_3^2) = (v_0^2 - v \cdot v) = (v_0^2) \]
The absolute magnitude of \( V \) is \( \sqrt{|V \cdot V|} \)

Each 4-Tensor has a “magnitude” given by taking the Tensor Trace of itself.
\[ \text{Trace}[T_{\mu\nu}] = T_{\mu\mu} = T^{00} - T^{11} - T^{22} - T^{33} = T \]
Note that the Trace runs down the diagonal of the 4-Tensor.

Notice the similarities. In both cases there is a tensor contraction with the Minkowski Metric Tensor \( \eta_{\mu\nu} \rightarrow \text{Diag}[+1,-1,-1,-1] \) (Cartesian basis)

ex. \( P \cdot P = (E/c)^2 - p \cdot p = (E_0/c)^2 = (m_0 c)^2 \)
which says that the “magnitude” of the 4-Momentum is the RestEnergy/c = RestMass*c

ex. \( \text{Trace}[\eta_{\mu\nu}] = (\eta^{00} - \eta^{11} - \eta^{22} - \eta^{33}) = 1 -(-1) -(-1) -(-1) = 1+1+1+1 = 4 \)
which says that the “magnitude” of the Minkowski Metric = SpaceTime Dimension = 4

\[ \text{Lorentz Scalar Invariant} \]
\[ V \cdot V = V^\mu \eta_{\mu\nu} V^\nu = (v_0^2 - v \cdot v) = (v_0^2) \]

\[ \text{4-Vector} \]
\[ V^\mu = (v^0, v^1, v^2, v^3) \]

\[ \text{Trace Tensor Invariant} \]
\[ \text{Trace}[T_{\mu\nu}] = T_{\mu\mu} = T^{00} - T^{11} - T^{22} - T^{33} = T \]

\[ \text{4-Tensor} \]
\[ T_{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix} \]

\[ P \cdot P = (m_0 c)^2 = (E_0/c)^2 \]

\[ \text{4-Momentum} \]
\[ P = (m_0, p) = (E_0/c, p) \]

\[ \text{Minkowski Metric} \]
\[ \delta[R] = \eta_{\mu\nu} \rightarrow \text{Diag}[+1,-1,-1,-1] \]

\[ \text{Trace}[T_{\mu\nu}] = \text{Trace}[T_{\mu\nu}] = T_{\mu\mu} = T \]

\[ V \cdot V = V^\mu \eta_{\mu\nu} V^\nu = (v_0^2 - v \cdot v) = (v_0^2) \]

\[ = \text{Lorentz Scalar Invariant} \]
SR 4-Vectors & 4-Tensors

More 4-Vector-based Invariants

Some other SR Invariants include:

- 4-Vector
  \[ V^\mu = (v_0, v) \]
  \[ V \cdot V = V^\mu \eta_{\mu \nu} V^\nu = (v_0^2 - v \cdot v) = \left( v_0 \right)^2 \]

- Lorentz Scalar Invariant
  \[ \gamma dV = \gamma dx \cdot dy \cdot dz = \gamma d^3x \]

- 4-Momentum
  \[ P = (mc, p) = (E/c, p) \]
  \[ P \cdot P = (mc)^2 = \frac{1}{c^2} \left( m \gamma c \right)^2 = \left( \frac{E_0}{c} \right)^2 \]

- Phase Space Invariant
  \[ d^3p/E \]

- Particle ID
  \[ N = (-V_0/c) \int dT \cdot N \]
  \[ \rightarrow n_o V_o \]

- EM Charge
  \[ Q = (-V_0/c) \int dT \cdot J \]
  \[ \rightarrow \rho_o V_o \]

- Rest Volume
  \[ V_o = \int \gamma dV = \int \gamma d^3x \]
  \[ = -cN/\int dT \cdot N \]

- Trace
  \[ \text{Trace}[T_{\mu \nu}] = \eta_{\mu \nu} T_{\mu \nu} = T_{\mu \mu} = T \]
  \[ V \cdot V = V^\mu \eta_{\mu \nu} V^\nu = \left( v_0^2 \right) - v \cdot v = \left( v_0^2 \right)^2 \]

Lorentz Scalar Invariant
Some 4-Vectors have an alternate form of Tensor Invariant: \( \frac{dv'}{v} = \frac{dv}{v_0} \) or \( \frac{d^3v'}{v} = \frac{d^3v}{v_0} \) in addition to the standard Lorentz Invariant \( V \cdot V = V^\mu V_\mu = (v_0^2 - v \cdot v) = (v_0^2) \).

If \( V \cdot V = (\text{constant}) \), with \( V = (v_0, v) \), then \( d(V \cdot V) = 2(V \cdot dv) = d(\text{constant}) = 0 \) hence \( (V \cdot dv) = 0 = v^2dv_0 - v \cdot dv \)

Generally, with \( \Lambda = \Lambda^\mu_\nu = \text{Lorentz Boost Transform in the } \beta \text{-direction} \)
\( V' = \Lambda V \) : from which the temporal component \( v'_0 = (\gamma v_0 - \gamma \beta \cdot v) \)
\( dv' = \Lambda dv \) : from which the spatial component \( dv' = (\gamma dv - \gamma \beta dv_0) \)

Combining:
\( dv' = (\gamma dv - \gamma \beta (v_0 \cdot dv/v_0)) \)
\( dv' = (1/v_0)^2(\gamma v_0 dv_0 - \gamma \beta \cdot dv) \)
\( dv' = (1/v_0)^2(\gamma v_0 dv_0 - \gamma \beta \cdot dv)_0 \)
\( dv' = (\gamma v_0 - \gamma \beta \cdot v)(1/v_0)^2 dv \)
\( dv' = (v/v_0)dv \)
\( dv'/v_0 = dv/v_0 \) : Invariant of \( V = (v', v) \) for \( V \cdot V = (\text{constant}) \)

So, for example:
\( P \cdot P = (mc)^2 = (\text{constant}) \)

Thus, \( dp'/c = dp/\gamma c = \text{Invariant} \)
Or: \( dp'/E' = dp/E \to d^3p/E = dp^*dp/\gamma c \) is Invariant, usually seen as \( \int F(\text{various invariants}) d^3p/E = \text{Invariant} \)

An alternate approach is:
\[ \int d^4p \delta[p^2-(mc)^2] = \int d^4p (1/2)[\delta[p+m_0 c] + \delta[p-m_0 c]] = cd^3p/2E = \text{Invariant} \]
SR 4-Vectors & 4-Tensors

More 4-Vector-based Invariants

Phase Space Integration

Invariant $d^4\mathbf{X} = -(V_o) dT \cdot d\mathbf{X} = -(dV_o) T \cdot d\mathbf{X} = cdt \, d^3\mathbf{x} = cdt \, dx \cdot dy \cdot dz$

The 4D Position coords that are integrated to give a 4D volume: SI units $[m^4]$

4-Differential $d\mathbf{x} = (cdt, dx)$; $d\mathbf{R} = (cdt, dr)$;

4-UnitTemporal $T = \gamma(1, \beta) = (1, \beta)$

4-UnitTemporal Differential $dT = d[(1, \beta)] = (d[\gamma], d[\gamma \beta])$

$V = |dV| = |dx| \, |dy| \, |dz| = \left| \int dx \, dy \, dz \right| = d^3\mathbf{x}$

$V = V_{ij} = 3D$ Spatial Volume: SI units $[m^3]$

$dV = d^3\mathbf{x} = 3D$ Spatial Volume Element

$\gamma = V_o/V$

$dy = -(V_o/V^2) dV$

$-(V_o) dT \cdot d\mathbf{X} = $ Invariant, because (Rest Scalar $\ast$ Lorentz Scalar Product) = Invariant

$= -(V_o) (d[\gamma], d[\gamma \beta]) \cdot (cdt, dx)$

$= -(V_o) (d[\gamma] d[\gamma \beta] - d[\gamma \beta] dx)$

$= -(V_o) (-c(\gamma/V) d\mathbf{v} cdt - d(\gamma \beta) dx) \text{ by taking the usual rest-case}$

$= -(V_o) (-c(V_o \gamma) d\mathbf{v} cdt)$

$= -(V_o) (-1/V_o) cdt d\mathbf{v}$

$= -cdt d\mathbf{v}$

$cdt = c(d\mathbf{v} \cdot dy \cdot dz) = c d^3\mathbf{x}$

$d^4\mathbf{x} = $ Invariant

And, this makes sense.

$T$ is a temporal 4-Vector with fixed magnitude: $T \cdot T = 1$. $d(T \cdot T) = d(1) = 0 = 2(dT \cdot T)$

Since $(dT \cdot T) = 0$, $dT$ must orthogonal to $T$ and thus must be a spatial 4-Vector.

If $d\mathbf{x}$ is also spatial, then the Lorentz scalar product $\{ (dT \cdot d\mathbf{X}) = -\text{magnitude} \}$ will be negative with this choice of Minkowski Metric.

Thus, multiplying by $-(V_o)$ gives a positive volume element $(cdt \, dx \, dy \, dz = d^3\mathbf{x})$

It is sort of quirky though, that the temporal $cdt$ comes from the $d\mathbf{X}$ part, and the spatial $d^3\mathbf{x}$ comes from the $dT$ part.
4-CurrentDensity $J = (pc, j)$
4-NumberFlux $N = (nc, n)$
4-UnitTemporal $T = \gamma(1, 0)$
4-UnitTemporalDifferential $dT = (d[\gamma], d[\gamma])$

$$V = V_{\gamma}$$
$$d\gamma = -(V_{\gamma})^{2} dV$$

$$(V/c)dT \cdot J = \text{Invariant, because (Rest Scalar * Lorentz Scalar Product) = Invariant}$$

$$= (-V/c)(d[\gamma], d[\gamma]) + (pc, j)$$
$$= (-V/c)(d[\gamma], d[\gamma]) - (pc, j)$$
$$= (V/c)(d[\gamma], d[\gamma]) - (pc, j)$$
$$= (V/c)(d[\gamma], d[\gamma]) - (pc, j)$$
$$= (V/c)(d[\gamma], d[\gamma]) - (pc, j)$$
$$= (V/c)(d[\gamma], d[\gamma]) - (pc, j)$$
$$= (V/c)(d[\gamma], d[\gamma]) - (pc, j)$$
$$= (V/c)(d[\gamma], d[\gamma]) - (pc, j)$$
$$= (V/c)(d[\gamma], d[\gamma]) - (pc, j)$$
$$= (V/c)(d[\gamma], d[\gamma]) - (pc, j)$$

Total Charge $Q = \int \rho d^3x = \int p d^3x = \text{Lorentz Scalar Invariant}$
Total Particle # $N = \int n d^3x = \text{Lorentz Scalar Invariant}$
Total RestVolume $V_o = \int d^3x = \text{Lorentz Scalar Invariant}$

This also gives an alternate way to define the RestVolume Invariant $V_o$.

$$(V/c)dT \cdot N = nd^3x$$

$$N = \int nd^3x = \int \left(-\frac{V}{c}\right)dT \cdot N$$
$$cN/V_o = \int dT \cdot N$$
$$V_o = cN/dT \cdot N$$

SR 4-Tensor

$(2, 0)$-Tensor $T^{\mu}_{\nu}$
$(1, 1)$-Tensor $T^\mu_{\nu}$, or $T_{\nu}^\mu$
$(0, 2)$-Tensor $T_{\mu \nu}$

SR 4-Vector

$V = (V^0, V)$
$\dot{V} = (\dot{V}^0, \dot{V})$

SR 4-CoVector: OneForm

$(0, 1)$-Tensor $V_{\nu} = (V_{\nu 0})$

SR 4-Scalar

$(0, 0)$-Tensor $S$ or $S_0$
Lorentz Scalar

$$\text{Trace}[T^{\mu}_{\nu}] = \eta_{\mu \nu} T^{\mu}_{\nu} = T_{\mu \mu} = T$$

$$V \cdot V = V^0 \eta_{\mu \nu} V^\nu = (V^0)^2 - V^\nu V^\nu = (V^0)^2$$

= Lorentz Scalar Invariant

Phase Space Integration
SR 4-Vectors & 4-Tensors
More 4-Vector-based Invariants
Phase Space Integration

\[\text{4-DifferentialMomentum } \, d\mathbf{P} = (d\mathbf{E}/c) \, dp \]
\[\text{4-DifferentialWaveVector } \, d\mathbf{K} = (d\omega/c) \, d\mathbf{k} \]

The 4D Momentum coords that are integrated to give a 4D Momentum Volume: SI Units \[(kg \cdot m/s)^4\]

The 4D WaveVector coords that are integrated to give a 4D WaveVector Volume: SI Units \[(1/m)^4\]

Likewise, \(d^4\mathbf{K}\) is Invariant

\[\int \text{d}^4\mathbf{P} = \int (d\mathbf{E}/c) \, dp \cdot dp' \cdot dp'' \cdot dp'''\]
\[\int \text{d}^4\mathbf{K} = \int (d\omega/c) \, dk \cdot dk' \cdot dk'' \cdot dk'''\]

\[\text{4-UnitTemporalDifferential } dT = (d[\gamma], d[\gamma\beta])\]
\[\text{4-MomentumDifferential } d\mathbf{P} = d\mathbf{p}'' = (d\mathbf{E}/c, dp)\]

\[\text{Phase Space Tensor Invariant } \int \text{d}^4\mathbf{P} \]
\[\text{Phase Space Tensor Invariant } \int \text{d}^4\mathbf{K} \]

\[\text{Trace}[\mathbf{T}^\mu_\nu] = \eta_\mu_\nu \cdot \mathbf{T}^\mu_\nu = \mathbf{T} \]
\[\mathbf{V} \cdot \mathbf{V} = V^\mu V_\mu = (V^\mu)^2 - \mathbf{V} \cdot \mathbf{V} = (V^\mu)^2 \]

\[\text{Lorentz Scalar Invariant} \]
SR 4-Vectors & 4-Tensors

More 4-Vector-based Invariants

Phase Space Integration

\[
4\text{-Unit Temporal Differential }dT = (d[\gamma], d[\gamma \beta])
\]

\[
\int F[\text{various Invariants}] d^3p \ d^3x
\]

\[
\int F[\text{various Invariants}] d^3k \ d^3x
\]
SRQM Study: SR 4-Tensor Properties

General $\rightarrow$ Symmetric & Anti-Symmetric

Any SR Tensor $T_{\mu\nu} = (S_{\mu\nu} + A_{\mu\nu})$ can be decomposed into parts:

- **Symmetric** $S_{\mu\nu} = (T_{\mu\nu} + T_{\nu\mu})/2$ with $S_{\mu\nu} = +S_{\nu\mu}$
- **Anti-Symmetric** $A_{\mu\nu} = (T_{\mu\nu} - T_{\nu\mu})/2$ with $A_{\mu\nu} = -A_{\nu\mu}$

$S_{\mu\nu} + A_{\mu\nu} = (T_{\mu\nu} + T_{\nu\mu})/2 + (T_{\mu\nu} - T_{\nu\mu})/2 = T_{\mu\nu}/2 + T_{\nu\mu}/2 + T_{\nu\mu}/2 - T_{\nu\mu}/2 = T_{\mu\nu} + 0 = T_{\mu\nu}$

Independent components: $\{4^2 = 16 = 10 + 6\}$

Max 16 possible

Max 10 possible

Max 6 possible

**General 4-Tensor**

$T_{\mu\nu} = [T_{00}, T_{01}, T_{02}, T_{03}, T_{10}, T_{11}, T_{12}, T_{13}, T_{20}, T_{21}, T_{22}, T_{23}, T_{30}, T_{31}, T_{32}, T_{33}]$

**Symmetric 4-Tensor**

$S_{\mu\nu} = [S_{00}, S_{01}, S_{02}, S_{03}, S_{10}, S_{11}, S_{12}, S_{13}, S_{20}, S_{21}, S_{22}, S_{23}, S_{30}, S_{31}, S_{32}, S_{33}]$

$\text{Tr}[S_{\mu\nu}] = S_{\mu\mu}$

**Anti-Symmetric 4-Tensor**

$A_{\mu\nu} = [A_{00}, A_{01}, A_{02}, A_{03}, A_{10}, A_{11}, A_{12}, A_{13}, A_{20}, A_{21}, A_{22}, A_{23}, A_{30}, A_{31}, A_{32}, A_{33}]$

$\text{Tr}[A_{\mu\nu}] = 0$

**Proof:**

$S_{\mu\nu} A_{\mu\nu} = S_{\mu\nu} (S_{\mu\nu} T_{\nu\mu})/2 = S_{\mu\nu} T_{\nu\mu}/2 + S_{\mu\nu} T_{\nu\mu}/2 = T_{\mu\nu}/2 + T_{\nu\mu}/2 - T_{\nu\mu}/2 = T_{\mu\nu} + 0 = T_{\mu\nu}$

*Note* These don’t have to be composed from a single general tensor.

Physically, the anti-symmetric part contains rotational information and the symmetric part contains information about isotropic scaling and anisotropic shear.
Any Symmetric SR Tensor \( S_{\mu \nu} = (T_{\text{iso}})^{\mu \nu} + T_{\text{aniso}}^{\mu \nu} \) can be decomposed into parts:

**Isotropic** \( T_{\text{iso}}^{\mu \nu} = (1/4)\text{Trace}[S_{\mu \nu}] \eta^{\mu \nu} = (T) \eta^{\mu \nu} \)

**Anisotropic** \( T_{\text{aniso}}^{\mu \nu} = S_{\mu \nu} - T_{\text{iso}}^{\mu \nu} \)

The Anisotropic part is Traceless by construction, and the Isotropic part has the same Trace as the original Symmetric Tensor. The Minkowski Metric is a symmetric, isotropic 4-tensor with \( T=1 \).

**Independent components:**

<table>
<thead>
<tr>
<th>Symmetric 4-Tensor ( S_{\mu \nu} )</th>
<th>Max 10 possible</th>
<th>Symmetric Isotropic 4-Tensor ( T_{\text{iso}}^{\mu \nu} )</th>
<th>Max 1 possible</th>
<th>Symmetric Anisotropic 4-Tensor ( T_{\text{aniso}}^{\mu \nu} )</th>
<th>Max 9 possible</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \begin{bmatrix} S_{00}, S_{01}, S_{02}, S_{03} \ S_{10}, S_{11}, S_{12}, S_{13} \ S_{20}, S_{21}, S_{22}, S_{23} \ S_{30}, S_{31}, S_{32}, S_{33} \end{bmatrix} )</td>
<td></td>
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</tr>
<tr>
<td>( \begin{bmatrix} [T, 0, 0, 0] \ [0, T, 0, 0] \ [0, 0, T, 0] \ [0, 0, 0, T] \end{bmatrix} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \begin{bmatrix} S_{00} - T, S_{01}, S_{02}, S_{03} \ S_{10}, S_{11} + T, S_{12}, S_{13} \ S_{20}, S_{21}, S_{22} + T, S_{23} \ S_{30}, S_{31}, S_{32}, S_{33} + T \end{bmatrix} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

with \( T = (1/4)\text{Trace}[S_{\mu \nu}] \)

Importantly, the Contraction of any Symmetric tensor with any Anti-Symmetric tensor on the same index is always 0.

\( S^{\mu \nu} A_{\mu \nu} = 0 \)

**Proof:**

\( S^{\mu \nu} A_{\mu \nu} = S^{\mu \nu} A_{\nu \mu} \); because we can switch dummy indices

\( = (S^{\mu \nu} A_{\mu \nu}) \); because of symmetry

\( = -S^{\mu \nu} A_{\mu \nu} \); because of anti-symmetry

\( = 0 \); because the only solution of \( \{c = -c\} \) is 0

Physically, the isotropic part represents a direction independent transformation (e.g., a uniform scaling or uniform pressure); the deviatoric part represents the distortion

An Isotropic Tensor has the same components in all possible coordinate-frames.

**Rank 0:** All Scalars are isotropic

**Rank 1:** There are no non-zero isotropic vectors

**Rank 2:** Most general isotropic 2\(^{nd}\) rank tensor must equal to \( \lambda \delta^{\mu \nu} \), for some scalar \( \lambda \)

**Rank 3:** Most general isotropic 3\(^{rd}\) rank tensor must equal to \( \lambda \epsilon^{\mu \nu \lambda} \) for some scalar \( \lambda \)

**Rank 4:** Most general isotropic 4\(^{th}\) rank tensor must equal to \( a \delta^{\mu \nu \lambda \rho} + b \delta^{\mu \nu \rho \lambda} + c \delta^{\mu \rho \lambda \nu} \) for scalars \( (a, b, c) \)

\( \text{Tr}[T_{\mu \nu}] = \eta_{\mu \nu}T_{\mu \nu} = T_{\alpha \beta} = T \)

\( V \cdot V = V^\mu \eta_{\mu \nu} V^\nu = (V^\mu)^2 \)

\( = (V^\nu)^2 \)

\( = \text{Lorentz Scalar Invariant} \)
SRQM Study: SR 4-Tensors

4-Tensor Decomposition

General (rank=2) 4-Tensor $T^{\mu \nu} = T_{\text{symm}}^{\mu \nu} + T_{\text{anti-symm}}^{\mu \nu}$

Symmetric 4-Tensor $T_{\text{symm}}^{\mu \nu}$

- $T_{\text{symm}}^{\mu \nu} = (T^{\mu \nu} + T^{\nu \mu})/2$
- $T_{\text{iso}}^{\mu \nu} + T_{\text{aniso}}^{\mu \nu}$
- Max DoF = 10

Anisotropic Symmetric 4-Tensor $T_{\text{aniso}}^{\mu \nu}$

- $T_{\text{symm}}^{\mu \nu} = T_{\text{aniso}}^{\mu \nu} - T_{\text{iso}}^{\mu \nu}$
- $\text{Tr}[T_{\text{aniso}}^{\mu \nu}] = 0$
- Max DoF = 9

Anti-Symmetric 4-Tensor $T_{\text{anti-symm}}^{\mu \nu}$

- $T_{\text{anti-symm}}^{\mu \nu} = (T^{\mu \nu} - T^{\nu \mu})/2$
- Max DoF = 6

Isotropic Symmetric 4-Tensor $T_{\text{iso}}^{\mu \nu}$

- $T_{\text{iso}}^{\mu \nu} = (\text{Tr}[T_{\text{symm}}^{\mu \nu}]/4) \eta^{\mu \nu}$
- $\text{Tr}[T_{\text{iso}}^{\mu \nu}] = \text{Tr}[T_{\text{symm}}^{\mu \nu}]$
- Max DoF = 1

Maximum Degrees of Freedom (DoF)

- # of possible independent components
- $(\text{Tensor dimension})^\text{(Tensor rank)}$

SR 4-Tensor

- $(2,0)$-Tensor $T^{\mu \nu}$
- $(1,1)$-Tensor $T_\mu^\nu$, or $T_{\nu}^\mu$
- $(0,2)$-Tensor $T_{\mu \nu}$

SR 4-Vector

- $(1,0)$-Tensor $V^\mu = V = (v_0^0, v_1, v_2, v_3)$

SR 4-CoVector: OneForm

- $(0,1)$-Tensor $V_\mu = (v_0, -v_1, -v_2, -v_3)$

SR 4-Scalar

- $(0,0)$-Tensor $S$ or $S_0$
- Lorentz Scalar

Trace[$T^{\mu \nu}$] = $\eta_{\mu \nu}T^{\mu \nu} = T_{\mu}^{\mu}$

$V \cdot V = V^\mu \eta_{\mu \nu}V_\nu = [(v_0^0)^2 - v_1^2 - v_2^2 - v_3^2] = (v_0^0)^2$

= Lorentz Scalar Invariant
SRQM Study: SR 4-Tensors

### SR Tensor Invariants

**A Tensor Study of Physical 4-Vectors**

**SR 4-Tensor Invariants**

(0,0)-Tensor = Lorentz Scalar $S$: Has either (0) or (1) Tensor Invariant, depending on exact meaning (S) itself is Invariant

(1,0)-Tensor = 4-Vector $V^\alpha$: Has (1) Tensor Invariant = The Lorentz Scalar Product

$$V\cdot V = V^\alpha_\mu V^{\mu}_\nu = \eta_{\mu\nu} V^\alpha_\mu V^{\mu}_\nu = \text{Tr} [V^\alpha_\mu V^{\mu}_\nu] = V^\alpha_\mu V^{\mu}_\nu = (V^0 v^0 + v^1_1 v^1_1 + v^2_2 v^2_2 + v^3_3 v^3_3) = (v_0^2 - v \cdot v) = (v_0^2)$$

(2,0)-Tensor $T^{\alpha\beta}$ itself is Invariant

Antisymmetric Triple Product

$$[\alpha\beta\gamma] = \text{Asymm Tri-Product}$$

Inner Product

$$\eta_{\mu\nu}: \text{Has (1) Tensor Invariant} = \text{The Lorentz Scalar Product}$$

The lowered-indices form of a tensor just negativizes the (time-space) and (space-time) sections of the upper-indices tensor

<table>
<thead>
<tr>
<th>$T^{00}$</th>
<th>$T^{01}$</th>
<th>$T^{02}$</th>
<th>$T^{03}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T^{10}$</td>
<td>$T^{11}$</td>
<td>$T^{12}$</td>
<td>$T^{13}$</td>
</tr>
<tr>
<td>$T^{20}$</td>
<td>$T^{21}$</td>
<td>$T^{22}$</td>
<td>$T^{23}$</td>
</tr>
<tr>
<td>$T^{30}$</td>
<td>$T^{31}$</td>
<td>$T^{32}$</td>
<td>$T^{33}$</td>
</tr>
</tbody>
</table>

Invariants sometimes seen as:

- $I_1 = (1/4) \text{Tr} [T^{\mu\nu}]$
- $I_2 = (1/2) \text{Tr} [T^{\mu\nu}]$
- $I_3 = (1/3) \text{Tr} [T^{\mu\nu}]$
- $I_4 = (1/4) \text{Tr} [T^{\mu\nu}]$

The lowered-indices form of a tensor just negativizes the (time-space) and (space-time) sections of the upper-indices tensor

**Set of 4 EigenValues $[T^{\mu\nu}]$**

<table>
<thead>
<tr>
<th>Eigenvalues Tensor Invariants</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T^{\mu\nu}$ Set of 4 Tensors</td>
</tr>
</tbody>
</table>

Determinant Tensor Invariant

<table>
<thead>
<tr>
<th>Determinant Tensor Invariant</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Det}[T^{\mu\nu}]$</td>
</tr>
</tbody>
</table>

Lowered 4-Tensor

$$T_{\mu\nu} = \eta_{\mu\rho} T^{\rho\nu}$$

Traces

<table>
<thead>
<tr>
<th>Trace Tensor Invariant</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Tr} [T^{\mu\nu}] = T_{\nu\mu} = (T^{00} - T^{11} - T^{22} - T^{33})$</td>
</tr>
</tbody>
</table>

### SR 4-Vector: One Form

#### SR 4-Vector

- (1,0)-Tensor $V^\alpha$ or $T^\alpha$
- (0,1)-Tensor $V_\nu$

#### SR 4-Scalar

- (0,0)-Tensor $S$ or $\nu$ Lorentz Scalar
- (0,1)-Tensor $T^{\alpha\beta}$ or $T_{\alpha\beta}$

### SR 4-Vector SRQM Interpretation of QM

- John B. Wilson
- SciReaem.org

http://scirealm.org/4SRQM.pdf

If I got all the math right...
SRQM Study: SR 4-Tensors

SR Tensor Invariants

Tensor Gymnastics

Some Tensor Gymnastics:

Matrix \( A \) = Tensor \( A^c \),
with rows denoted by "r", columns by "c"

Example with dim=4: \( r,c=\{0..3\} \)
Matrix \( A = \begin{bmatrix} A_{00}^{c_{0}} & A_{01}^{c_{1}} & A_{02}^{c_{2}} & A_{03}^{c_{3}} \\ A_{10}^{c_{0}} & A_{11}^{c_{1}} & A_{12}^{c_{2}} & A_{13}^{c_{3}} \\ A_{20}^{c_{0}} & A_{21}^{c_{1}} & A_{22}^{c_{2}} & A_{23}^{c_{3}} \\ A_{30}^{c_{0}} & A_{31}^{c_{1}} & A_{32}^{c_{2}} & A_{33}^{c_{3}} \end{bmatrix} \)

\( M = A \times B = A^c B^r = M^{c_r} \)
, with the rows of \( A \) multiplied by the columns of \( B \)
due to the summation over index "c"

If we have sums over both indices:
\( A^c B^r = M^{c_r} = \text{Trace}[M] \)
The sum over "c" gives the matrix multiplication and then the sum over "d" gives the Trace of the resulting matrix \( M \)

\( A^c B^c = (A \times A)^c = (N)^c = \text{Trace}[N] = \text{Trace}[A^2] = \text{Tr}[A^2] \)

\( A^c B^c = (A \times A^t)^c = (N^t)^c = \text{Trace}[N^t] = \text{Tr}[N^t] = \text{Tr}[A^t] \)

\( A^c B^a = A^c A^d - A^c A^d = (\text{Tr}[A])^2 - \text{Tr}[A^2] \)
, with brackets [ ] around the indices indicating anti-symmetric product

The Trace formula's are independent of tensor dimension.
General Cayley-Hamilton Theorem

\[ A^4 + c_3 A^3 + \ldots + c_0 A^0 = 0 \]

A = square matrix, \( d \) = dimension, \( A^0 = I(d) \)

Characteristic Polynomial: \( p(\lambda) = \text{Det}(A - \lambda I(d)) \)

The following are the Principle Tensor Invariants for dimensions 1..4

**dim = 1:** \( A^1 + c_0 A^0 = 0 \)
\( I_1 = \text{tr}[A] = \text{Det}_{1D}[A] = \lambda_1 \)

**dim = 2:** \( A^2 + c_1 A^1 + c_0 A^0 = 0 \)
\( I_1 = \text{tr}[A] = \Sigma[\text{Eigenvalues}] = \lambda_1 + \lambda_2 \)
\( I_2 = (\text{tr}[A]^2 - \text{tr}[A^2]) / 2 = \text{Det}_{2D}[A] = \Pi[\text{Eigenvalues}] = \lambda_1 \lambda_2 \)

**dim = 3:** \( A^3 + c_2 A^2 + c_1 A^1 + c_0 A^0 = 0 \)
\( I_1 = \text{tr}[A] = \Sigma[\text{Eigenvalues}] = \lambda_1 + \lambda_2 + \lambda_3 \)
\( I_2 = (\text{tr}[A]^2 - \text{tr}[A^2]) / 2 = \lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3 \)
\( I_3 = (\text{tr}[A]^3 - 3 \text{tr}(A^2) \text{tr}[A] + 2 \text{tr}(A^3)) / 6 = \text{Det}_{3D}[A] = \Pi[\text{Eigenvalues}] = \lambda_1 \lambda_2 \lambda_3 \)

**dim = 4:** \( A^4 + c_3 A^3 + c_2 A^2 + c_1 A^1 + c_0 A^0 = 0 \)
\( I_1 = \text{tr}[A] = \Sigma[\text{Eigenvalues}] = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \)
\( I_2 = (\text{tr}[A]^2 - \text{tr}[A^2]) / 2 = \lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3 + \lambda_2 \lambda_4 + \lambda_3 \lambda_4 \)
\( I_3 = (\text{tr}[A]^3 - 3 \text{tr}(A^2) \text{tr}[A] + 3 \text{tr}(A^3)) / 6 = \lambda_1 \lambda_2 \lambda_3 + \lambda_1 \lambda_2 \lambda_4 + \lambda_1 \lambda_3 \lambda_4 + \lambda_2 \lambda_3 \lambda_4 \)
\( I_4 = (\text{tr}[A]^4 - 6 \text{tr}(A^2) \text{tr}[A^2] + 8 \text{tr}(A^3) \text{tr}[A] - 6 \text{tr}(A^4)) / 24 = \text{Det}_{4D}[A] = \Pi[\text{Eigenvalues}] = \lambda_1 \lambda_2 \lambda_3 \lambda_4 \)

Each dimension gives the number of elements from it’s row in Pascal’s Triangle :)

\( l_0 = \Sigma[\text{Unique Eigenvalue Naughts}] = 1 \)
\( l_1 = \Sigma[\text{Unique Eigenvalue Singles}] = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \)
\( l_2 = \Sigma[\text{Unique Eigenvalue Doubles}] = \lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3 + \lambda_2 \lambda_4 + \lambda_3 \lambda_4 \)
\( l_3 = \Sigma[\text{Unique Eigenvalue Triples}] = \lambda_1 \lambda_2 \lambda_3 + \lambda_1 \lambda_2 \lambda_4 + \lambda_1 \lambda_3 \lambda_4 + \lambda_2 \lambda_3 \lambda_4 \)
\( l_4 = \Sigma[\text{Unique Eigenvalue Quadruples}] = \lambda_1 \lambda_2 \lambda_3 \lambda_4 \)
### General Cayley-Hamilton Theorem

\[ A^d + c_{d-1} A^{d-1} + \ldots + c_0 A^0 = 0 \], with \( A \) = square matrix, \( d = \) dimension, \( A^0 = \) identity(d) = \( I_d \), for 4D.

**Characteristic Polynomial:** \( p(\lambda) = \text{Det}[A - \lambda I_d] \)

### Tensor Invariants \( I_n \)

<table>
<thead>
<tr>
<th>( I_n )</th>
<th>Dim = 1</th>
<th>Dim = 2</th>
<th>Dim = 3</th>
<th>Dim = 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_0 ) = 1/0! = 1</td>
<td>( \lambda_1 )</td>
<td>( \lambda_1 + \lambda_2 )</td>
<td>( \lambda_1 + \lambda_2 + \lambda_3 )</td>
<td>( \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 )</td>
</tr>
<tr>
<td>( I_1 ) = tr[( A )]/1!</td>
<td>( \lambda_1 )</td>
<td>( \lambda_1 + \lambda_2 )</td>
<td>( \lambda_1 + \lambda_2 + \lambda_3 )</td>
<td>( \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 )</td>
</tr>
<tr>
<td>( I_2 ) = (tr[( A )^2] - tr[( A^2 )])/2!</td>
<td>( \lambda_1 \lambda_2 )</td>
<td>( \lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3 )</td>
<td>( \lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3 + \lambda_3 \lambda_4 )</td>
<td>( \lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3 + \lambda_3 \lambda_4 + \lambda_4 \lambda_4 )</td>
</tr>
<tr>
<td>( I_3 ) = [3 tr(( A )^3) - 3 tr(( A^2 ))^2 + 2 tr(( A^3 ))]/3!</td>
<td>( \lambda_1 \lambda_2 \lambda_3 )</td>
<td>( \lambda_1 \lambda_2 \lambda_3 + \lambda_1 \lambda_2 \lambda_4 + \lambda_1 \lambda_3 \lambda_4 + \lambda_2 \lambda_3 \lambda_4 )</td>
<td>( \lambda_1 \lambda_2 \lambda_3 + \lambda_1 \lambda_2 \lambda_4 + \lambda_1 \lambda_3 \lambda_4 + \lambda_2 \lambda_3 \lambda_4 + \lambda_3 \lambda_4 \lambda_4 )</td>
<td>( \lambda_1 \lambda_2 \lambda_3 + \lambda_1 \lambda_2 \lambda_4 + \lambda_1 \lambda_3 \lambda_4 + \lambda_2 \lambda_3 \lambda_4 + \lambda_3 \lambda_4 \lambda_4 + \lambda_4 \lambda_5 \lambda_5 )</td>
</tr>
<tr>
<td>( I_4 ) = (tr(( A )^4) - 6 tr(( A^2 )^2) + 3 tr(( A^2 )) tr(( A^2 )))/4!</td>
<td>( \lambda_1 \lambda_2 \lambda_3 \lambda_4 )</td>
<td>( \lambda_1 \lambda_2 \lambda_3 \lambda_4 + \lambda_1 \lambda_2 \lambda_3 \lambda_5 + \lambda_1 \lambda_2 \lambda_4 \lambda_5 + \lambda_1 \lambda_3 \lambda_4 \lambda_5 + \lambda_2 \lambda_3 \lambda_4 \lambda_5 )</td>
<td>( \lambda_1 \lambda_2 \lambda_3 \lambda_4 + \lambda_1 \lambda_2 \lambda_3 \lambda_5 + \lambda_1 \lambda_2 \lambda_4 \lambda_5 + \lambda_1 \lambda_3 \lambda_4 \lambda_5 + \lambda_2 \lambda_3 \lambda_4 \lambda_5 + \lambda_3 \lambda_4 \lambda_5 \lambda_5 )</td>
<td>( \lambda_1 \lambda_2 \lambda_3 \lambda_4 + \lambda_1 \lambda_2 \lambda_3 \lambda_5 + \lambda_1 \lambda_2 \lambda_4 \lambda_5 + \lambda_1 \lambda_3 \lambda_4 \lambda_5 + \lambda_2 \lambda_3 \lambda_4 \lambda_5 + \lambda_3 \lambda_4 \lambda_5 \lambda_5 + \lambda_4 \lambda_5 \lambda_5 \lambda_5 )</td>
</tr>
</tbody>
</table>

**Example:**

For a 2x2 matrix in Euclidean 3-Space:

\[ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \]

- \( I_0 = 1/0! = 1 \)
- \( I_1 = \text{tr}[A] = a + d \)
- \( I_2 = \frac{(a + d)^2 - (a - d)^2}{2} = 2ad \)
- \( I_3 = \frac{3(a + d - a)^3 - 3(a + d)^2 + 2(a + d)}{3} = 0 \)
- \( I_4 = \frac{(a + d)^4 - 6(a + d)^2 + 3(a + d)}{4} = 0 \)

**Euclidean 3-Space:**

\[ A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \]

- \( I_0 = 1/0! = 1 \)
- \( I_1 = \frac{a + e + i}{3} \)
- \( I_2 = \frac{(a + e + i)^2 - (a - e - i)^2}{2} = 2(ae - be + ei - fh) \)
- \( I_3 = 0 \)
- \( I_4 = 0 \)

**Minkowski SpaceTime:**

\[ A = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} \]

- \( I_0 = 1/0! = 1 \)
- \( I_1 = \frac{a + f + k + p}{4} \)
- \( I_2 = 0 \)
- \( I_3 = 0 \)
- \( I_4 = 0 \)
SRQM Study: SR 4-Tensors

SR Tensor Invariants for Faraday EM Tensor

The Faraday EM Tensor $F^{\alpha\beta} \cdot \partial_\alpha A^\beta = 0$ is an anti-symmetric tensor that contains the Electric and Magnetic Fields, defined by the Exterior "Wedge" Product (^). The 3-electric components (e = e_i = e F_i) are in the temporal-sectional planes. The 3-magnetic components (b = b_i = -(1/2) e_j^k F_j^k ) are in the only-sectional plane.

\[ \text{Subtract (2) due to the invariants which provide constraints to get a total of (6) independent components} \]

The 4-Gradient and 4-EMVectorPotential have (4) independent components each, for total of (8).

\[ a) \text{ & c) give 0=0, and do not provide additional constraints} \]

\[ b) \]

\[ (b e / c) \]

\[ d) \text{ & c) give 0=0, and do not provide additional constraints} \]

The 4-Gradient and 4-EMVectorPotential have (4) independent components each, for total of (8). Subtract (2) due to the invariants which provide constraints to get a total of (6) independent components = (6) independent components of a 4x4 anti-symmetric tensor = (3) 3-electric e = (3) 3-magnetic b = (6) independent EM field components

Note: It is possible to have non-zero e and b, yet still have zeroes in the Tensor Invariants. If e is orthogonal to b, then Det[F^[\alpha\beta]] = (b e / c) = 0. If (b (b) = (e c^2), then InnerProd[F^[\alpha\beta]] = 0. These conditions lead to the properties of EM waves = photons = null 4-vectors, which have fields [b] = [e]c and b orthogonal to e, travelling at velocity c.

SR 4-Tensor
(2,0)-Tensor $T_{ij}^\alpha$, (1,1)-Tensor $V_{ij} = V (v_i v_j)$, (0,2)-Tensor $T_{ij}^\alpha$, (1,1)-Tensor $V_{ij}$, Lorentz Scale 4-Vector

SR 4-Vector
(1,0)-Tensor $V = V (v_i)$, (0,1)-Tensor $V_{ij} = V (v_i v_j)$, (0,1)-Tensor $V_{ij}$ or $T_{ij}$, Lorentz Scalar

SR 4-Scalar
(2,0)-Scalar $S_{ij}$, (1,1)-Scalar $S_{ij}$, (0,2)-Scalar $S_{ij}$, Lorentz Scalar

The Faraday EM Tensor $F^{\alpha\beta} = \partial_\alpha A^\beta - \partial_\beta A^\alpha = \partial^\alpha A$ is an anti-symmetric tensor that contains the Electric and Magnetic Fields, defined by the Exterior "Wedge" Product (^). The 3-electric components (e = e_i = e F_i) are in the temporal-sectional planes. The 3-magnetic components (b = b_i = -(1/2) e_j^k F_j^k ) are in the only-sectional plane.

\[ (2,0)-Tensor = (4,0)-Tensor \]

Has (4+) Tensor Invariants (though not all independent)
\[ a) \text{ T}^{\alpha\beta} = \text{Tr} = \text{Sum of EigenValues for (1,1)-Tensors (mixed)} \]
\[ b) \text{ T}^{\alpha\beta} T_{\alpha\beta} = \text{Tr} = \text{Asymmetric Bi-Product} \]
\[ c) T^{\alpha\beta} T_{\alpha\beta} = \text{Asymmetric Tri-Product} \]
\[ d) T^{\alpha\beta} T_{\alpha\beta} T_{\gamma\delta} = \text{Asymmetric Quad-Product} \]

Important: The Faraday EM Tensor has only (2) linearly-independent Lorentz invariants:
\[ 2((b (b) - (e e / c^2)) \]

\[ \text{Asymmetric Tri-Product Tensor Invariant} \]

\[ \text{D} = \frac{1}{\varepsilon_0 \mu_0} \]

\[ 4-(EM)VectorPotential \]
\[ A = A^\alpha = (\phi / c, a) \]

\[ \text{Fundamental EM Invariants:} \]
\[ P = (1/2) F_{\mu\nu} F^{\mu\nu} = - (1/2) F^{\mu\nu} F_{\mu\nu} = \{ (b (b) - (e e / c^2)) \} \]
\[ Q = (1/4) F_{\mu\nu} F^{\mu\nu} = (1/8) e^{\alpha\beta\gamma\delta} F_{\alpha\mu} F_{\beta\nu} \]

\[ \text{Faraday EM Tensor Invariant:} \]
\[ F^{\alpha\beta} = \partial_\alpha A^\beta - \partial_\beta A^\alpha = \partial^\alpha A \]

\[ \text{4-Vector SRQM Interpretation of QM} \]
\[ \text{SciRealm.org} \]
\[ \text{John B. Wilson} \]
\[ \text{SciRealm@aol.com} \]
\[ \text{http://scirealm.org/SRQM.pdf} \]
SRQM Study: SR 4-Tensors

SR Tensor Invariants
for 4-AngularMomentum Tensor

The 4-AngularMomentum Tensor \( M^{\alpha\beta} = X^{\alpha} P^\beta - X^{\beta} P^\alpha \) is an anti-symmetric tensor. The 3-mass-momentum components (\( n = n = M^{i\alpha}/c \)) are in the temporal-spatial sections. The 3-angular-momentum components (\( l = l = + (1/2) \epsilon_{ij}^k M_k^{i\alpha}/c \)) are in the only-spatial section.

(2,0)-Tensor = 4-Tensor \( T^{\alpha\beta} \): Has (4+) Tensor Invariants (though not all independent)

a) \( T_{\alpha\beta} = \text{Trace} = \text{Sum of EigenValues for (1,1)-Tensors (mixed)} \)

b) \( T^{\alpha\beta}_{\alpha\beta} = \text{Asymmetric Bi-Product} \to \text{Inner Product} \)

c) \( T^{\alpha\beta}_{\alpha\beta} \): Asymmetric Tri-Product \to ?Name?

d) \( T^{\alpha\beta}_{\alpha\beta} T^{\gamma\delta}_{\gamma\delta} = \text{Asymmetric Quad-Product} \)

Importantly, the 4-AngularMomentum Tensor has only (2) linearly-independent Lorentz Invariants:

b) \( 2((l)(c^2 n n)) \): see Wikipedia Laplace–Runge–Lenz_vector, sec. Casimir Invariants

c) \( (c(n))^2 \)

a) & \( c \) give 0, and do not provide additional constraints

The 4-Position and 4-Momentum have (4) independent components each, for total of (8).

Subtract (2) due to the invariants which provide constraints to get a total of (6) independent components:

(4) independent components of a 4x4 anti-symmetric tensor

(3) 3-mass-momentum \( n \) + (3) 3-angular-momentum \( l \) = (6) independent 4-AngularMomentum components

4-Momentum \( P = p_{\mu} = (mc, \mathbf{p}) = \frac{E}{c}, \mathbf{p} \)

Trace \( [T^{\alpha\beta}] = \eta_{\alpha\beta} T^{\alpha\beta} = T^{\alpha\beta}_{\alpha\beta} = T \)

V \cdot V = V^{\alpha} V^{\alpha} = (V^{\alpha})^2 = \frac{E}{c} \mathbf{p}^2 = (\mathbf{V} \times \mathbf{p}) \times (\mathbf{V} \times \mathbf{p}) = (\mathbf{V} \times \mathbf{p})^2 = \text{Lorentz Scalar Invariant}

4-AngularMomentum Tensor
\[ M^{\alpha\beta} = X^{\alpha} P^\beta - X^{\beta} P^\alpha = \mathbf{X} \times \mathbf{P} \]

\[ \begin{array}{cccc}
M^{12} & M^{13} & M^{14} & M^{15} \\
M^{21} & M^{23} & M^{24} & M^{25} \\
M^{31} & M^{32} & M^{34} & M^{35} \\
M^{41} & M^{42} & M^{43} & M^{44} \\
\end{array} \]

\[ M_{\mu\nu} = 2((l)(c^2 n n)) \]

\[ M_{\alpha\beta} M^{\alpha\beta} = \text{Inner Product Tensor Invariant} \]

\[ \text{Det}[M^{\alpha\beta}] = (c(n))^2 \]

\( \text{Trace} \) Tensor Invariant

\[ 0 \]

\[ (c(n))^2 \]

\[ (E/c, \mathbf{p}) \times (\mathbf{V} \times \mathbf{p}) \times (\mathbf{V} \times \mathbf{p}) = (\mathbf{V} \times \mathbf{p})^2 \]

\[ (\mathbf{V} \times \mathbf{p})^2 \]

\[ \frac{E}{c} \mathbf{p}^2 \]

\[ \text{Lorentz Scalar Invariant} \]

See Also: Relativistic Angular Momentum.
The Minkowski Metric Tensor \( \eta^{\mu\nu} \) is the tensor all SR 4-Vectors are measured by.

\[
\text{(2,0)-Tensor = 4-Tensor } T^{\mu}_{\nu}; \text{ Has (4+)} \text{ Tensor Invariants (though not all independent)}
\]

- a) \( T^\alpha_\alpha = \text{Trace } = \text{Sum of EigenValues for } (1,1)-\text{Tensors (mixed)} \)
- b) \( T^\alpha_\beta T^{\beta}_\alpha = \text{Asymm Bi-Product } \rightarrow \text{Inner Product} \)
- c) \( T^\alpha_\beta T^{\beta}_\gamma = \text{Asym Tri-Product } \rightarrow \text{Name?} \)
- d) \( T^\alpha_\beta T^{\beta}_\gamma T^{\gamma}_\delta = \text{Asymm Quad-Product } \rightarrow 4D \text{ Determinant } = \text{Product of EigenValues for } (1,1)-\text{Tensors} \)

\[
\begin{align*}
\text{a): Minkowski Trace}[\eta^{\mu\nu}] &= 4 \\
\text{b): Minkowski Inner Product } &\eta_{\mu\nu} &\eta^{\mu\nu} &= 4 \\
\text{c): Minkowski AsymmTr}[\eta^{\mu\nu}] &= 24 = 4! \\
\text{d): Minkowski Def}[\eta^{\mu\nu}] &= -1
\end{align*}
\]

\[
\begin{align*}
a) &\text{ Tr}[\mathbf{A}] &= 4 \\
b) &\text{ Tr}[\mathbf{A}^2] &= -6 + 24 = 18 \\
c) &\text{ Tr}[\mathbf{A}^3] &= 6 - 36 + 24 = 0 \\
d) &\text{ Tr}[\mathbf{A}^4] &= 0 - 0 + 0 = 0
\end{align*}
\]

\[
\begin{align*}
\Lambda_{\mu\nu} &= \text{Diag}[\eta^{\mu\nu}] \\
\det(\text{Exp}[\mathbf{A}]) &= \text{Exp}[\text{Tr}[\mathbf{A}]] \\
\det(\text{det}(\mathbf{A})) &= (\text{tr}(\mathbf{A}))^4 - 6(\text{tr}(\mathbf{A}^2)) + 3(\text{tr}(\mathbf{A}^3)) + 8(\text{tr}(\mathbf{A}^4)) = 4^4 - 6(4^2) + 3(4) + 8 = 16 + 24 = 40
\end{align*}
\]

\[
\begin{align*}
\text{SRQM Study: SR 4-Tensors} & \quad \text{for Minkowski Metric Tensor} \\
\text{SR Tensor Invariants} & \quad \text{EigenValues not defined for the standard Minkowski Metric Tensor since it is a type } (2,0)-\text{Tensor, all upper indices. However, they are defined for the mixed form } (1,1)-\text{Tensors, mixed indices}
\end{align*}
\]

\[
\begin{align*}
\text{SR 4-Tensor} & \quad \text{SR 4-Vector} \\
\text{(2,0)-Tensor } T^{\mu\nu} & \quad \text{(1,0)-Tensor V} \text{ or V} \text{ or T} \text{ or T} \text{ or T} \text{ or T} \\
\text{SR 4-Vector} & \quad \text{SR 4-CoVector:OneForm} \\
\text{(1,0)-Tensor V} & \text{ or V} \text{ or T} \text{ or T} \text{ or T} \text{ or T} \\
\text{SR 4-Scalar} & \quad \text{SR 4-Scalar} \text{ or S or S} \\
\text{(0,0)-Tensor S or S} & \text{ Lorentz Scalar}
\end{align*}
\]

\[
\begin{align*}
\text{Det}[T^\alpha_\nu] &= \Pi_\lambda[\lambda]; \text{ with } \{\lambda_\lambda\} = \text{EigenValues} \\
\text{Characteristic Eqns: } &\text{Det}[T^\alpha_\nu - \lambda_k I_{4\times 4}] = 0
\end{align*}
\]

\[
\begin{align*}
\text{5D Determinant} &= \text{Product of EigenValues for } (1,1)-\text{Tensors} \\
\text{4D SpaceTime} &\text{In GR} \\
\text{GR Trace Tensor Invariant} &\text{4D SpaceTime} \\
\text{4-Gradient} &\text{In GR} \\
\text{Diag}[1,1,1,1] &\text{Diag}[1,-1,1] \\
\text{Det}[\eta^{\mu\nu}] &= 24 = 4! \\
\text{SR: Minkowski Metric} &\text{ "Particle Physics" Convention} \\
\text{Determinant Tensor Invariant} &\text{Asym Tri-Product Tensor Invariant} \\
\text{Asym Tri-Product} &\text{Invariants}
\end{align*}
\]

\[
\begin{align*}
\text{SR 4-Vector} & \quad \text{SR 4-Tensor} \\
\text{SR 4-Vector} & \quad \text{SR 4-Vector} \\
\text{SR 4-Vector} & \quad \text{SR 4-Vector} \\
\text{SR 4-Scalar} & \quad \text{SR 4-Scalar} \\
\text{(0,0)-Tensor S or S} & \text{Lorentz Scalar}
\end{align*}
\]
SRQM Study: SR 4-Tensors

SR Tensor Invariants

for Perfect Fluid Stress-Energy Tensor

The Perfect Fluid Stress-Energy Tensor $T^{\mu\nu}$ is the tensor of a relativistic fluid.

$(2,0)$-Tensor = 4-Tensor $T^{\mu\nu}$: Has $(4+)$ Tensor Invariants.

- Perfect Fluid Invariants:
  - $\rho_{eo}$
  - $p_{o}$
  - $p_{o}$
  - $p_{o}$

The Perfect Fluid Stress-Energy Tensor is defined as:

$$T^{\mu\nu} = T^{\rho\sigma} T_{\rho\sigma}^{\mu\nu}$$

where $T^{\rho\sigma}$ is the $(2,0)$-Tensor.

4-Force Density $F^{\mu\nu} = \rho_{eo} - 3p_{o}$

SR Conservation of Stress-Energy $T^{\mu\nu}$

If $T^{\mu\nu} = 0$, then $\rho_{eo} - 3p_{o} = 0$.

4-Vector SRQM Interpretation of QM

http://scirealm.org/SRQM.pdf
**SRQM Study: SR 4-Tensors**

**SR Tensor Invariants**

for Maxwell 4D EM Stress-Energy Tensor

---

### SR Perfect Fluid 4-Tensor

\[ T_{\text{perfect,fluid}}^{\mu \nu} = (\rho_{eo}) V_{\nu} \mathbf{V}^\mu + (-p_{eo}) \mathbf{V}^{\mu \nu} \rightarrow \]

\[
\begin{bmatrix}
\rho_{eo} & 0 & 0 & 0 \\
0 & p_{eo} & 0 & 0 \\
0 & 0 & p_{eo} & 0 \\
0 & 0 & 0 & p_{eo}
\end{bmatrix}
\]

---

### 4-Force Density \( F_{\text{density}}^{\mu} \)

\[ -\partial T^{\mu \nu} = F_{\text{density}}^{\mu} \]

---

**SR Conservation of Stress-Energy**

\[ T^{\mu \nu} = \frac{1}{2}(\mathbf{e} \cdot \mathbf{b} / \mu_0) \]

Note this is positive-definite

---

**Trace Tensor Invariant**

\[ \text{Tr}[T^{\mu \nu}] = 0 \]

\[ = T^{00} - T^{11} - T^{22} - T^{33} \]

\[ = T^{00} - 3T^{00}(1 \text{ from } \delta^{ij}) + (\frac{2}{3}T^{00} \text{ from } T^{xx} + T^{yy} + T^{zz}) \]

\[ = T^{00} - 3T^{00} + 2\frac{1}{3}T^{00} = 0 \]

---

**Maxwell 4D EM Stress-Energy Tensor**

\[ T_{\text{EM}}^{\mu \nu} = -(1/\mu_0)[F_{\mu \nu}F_\alpha^{\mu \nu} - (1/4)\eta^{\mu \nu}F_{\alpha \beta}F_{\alpha \beta}] \quad \rightarrow \text{No RestFrame, Light-Like, Null} \]

---

**Maxwell 4D EM Stress-Energy Tensor**

\[ T_{\text{EM}}^{\mu \nu} = -T^{00}, T^{\nu \alpha} = T^{\nu \alpha} = T^{\nu \alpha} = T^{\nu \alpha} \]

\[ \text{EoS}[T^{\mu \nu}] = w = p_{eo}/\rho_{eo} \]

---

**Eigenvalues Tensor Invariants**

\[ \frac{1}{2} \left( \mathbf{e} \cdot \mathbf{b} + b^2/\mu_0 \right) s^{0/c} - \sigma^{ij} \]

\[ w/ 3D Maxwell Stress Tensor \]

\[ \sigma^{ij} = \frac{\varepsilon_0 e R_t+b^2}{e^2 \mu_0} - \frac{1}{2} \left( \mathbf{e} \cdot \mathbf{b} + b^2/\mu_0 \right) \delta^{ij} \]

\[ = \varepsilon_0 e R_t+b^2/\mu_0 - T^{00} \delta^{ij} \]

---

**SR 4-Tensor**

(2,0)-Tensor \( T^{\mu \nu}_0 \)

---

**SR 4-Vector**

(1,0)-Tensor \( V^\nu = (\mathbf{v}^\nu) \)

---

**SR 4-Vector**

(0,1)-Tensor \( V_\nu = (\mathbf{v}^\nu) \)

---

**SR 4-Scalar**

(0,0)-Tensor \( S = s^{0/c} \)

---

**SR 4-Scalar**

Lorentz Scalar

\[ \text{Det}[T^{\mu \nu}_0] = \Pi_4[\lambda_4]; \text{ with } \{\lambda_4\} = \text{EigenValues} \]

---

**Characteristic Eqns:**

\[ \text{Det} [T_0^{\mu \nu} - \lambda_1 I_{4x4}] = 0 \]

---

**Tr}\[T^{\mu \nu}] = 0 \]

---

**Trace**

\[ \text{Tr} [V^{\mu \nu}V] = (V^\mu V^\nu) - (V^\nu V^\mu) = (V^\nu V^\mu) = (V^\nu V^\mu) \]

---

**= Lorentz Scalar Invariant**
SRQM Study: SR 4-Tensors
SR Tensor Invariants for Maxwell 4D EM Stress-Energy Tensor

Maxwell 4D EM Stress-Energy Tensor

\[ T_{\mu \nu}^{EM} = -(1/\mu_0)\{F_\mu F_\nu + \eta_{\mu \nu} F_\alpha F_\sigma \eta^{\alpha \sigma}\} \rightarrow \text{(No RestFrame,Light-Like,Null)} \]

\[ =(-1/\mu_0)\{F_\mu F_\nu + \eta_{\mu \nu} F_\alpha F_\sigma \eta^{\alpha \sigma}\} \]

\[ =((-1/\mu_0)\{F_\mu F_\nu + \eta_{\mu \nu} F_\alpha F_\sigma \eta^{\alpha \sigma}\} \]

00th component:\n\[ =\{\{e_\mu e_\nu\}\} + (1/2)\{(b_\mu/b)_\nu e_\sigma(e_\sigma e_\nu)\} \]

11th component:\n\[ =\{(e_\mu e_\nu)\} + (1/2)\{(b_\mu/b)_\nu e_\sigma(e_\sigma e_\nu)\} \]

\[ =+(1/2)\{(e_\mu e_\nu)+(b_\mu/b)_\nu e_\sigma(e_\sigma e_\nu)\} \]

4-Tensor in \((\mp,\mp,\mp,\pm)\), neg of formula in \((\pm,\pm,\pm,\mp)\) Symmetric

4-Vector in \((\mp,\mp,\mp,\pm)\)

\[ \text{Tr}[T_{\mu \nu}] = 0 \]

Trace Tensor Invariant

\[ \text{Tr}[T_{\mu \nu}] = 0 \]

\[ = T^{00} - T^{11} - T^{22} - T^{33} \]

\[ = T^{00} - 3T^{00}(1 - \delta^{ii}) + (2T^{00} + T^{ii}) \]

\[ = 0 \]

= Sum of EigenValues

w/ 3D Maxwell Stress Tensor

\[ T_{\mu \nu}^{MaxwellEM} \rightarrow \]

\( s_i^j = \)

\[ = e_\mu e_\nu + b_\mu/b_\nu /\mu_0 \]

\[ -\frac{1}{2}(e_\mu e_\nu + b_\mu/b_\nu) \delta_i^j \]

\[ = e_\mu e_\nu + b_\mu/b_\nu \]

\[ -T^{00}\delta_i^j \]
The Lorentz Transform Tensor \( \Lambda^\mu_\nu = \partial x^\mu / \partial x^\nu = \delta_\nu^\mu \) is the tensor all SR 4-Vectors must transform by.

### SR: Lorentz Transform
\[
\Lambda^\mu_\nu = \partial x^\mu / \partial x^\nu = \delta_\nu^\mu
\]

### 4D Kronecker Delta
\[
\delta_\nu^\mu = \delta^{\mu}_\nu
\]

The properties of the Lorentz Transforms give interesting relations to the 4D Kronecker Delta. When taking an inner product of a Lorentz Transform, each index combo sums to either 0 or 1. Different index combos give 0, same index combos give 1, the usual Delta rule \( = 1 \) for \( \mu = \nu \), else \( = 0 \).

#### For Rotations:
- 0\(^{th}\) with 0\(^{th}\) : \((1)(1)+(0)(0)+(0)(0)+(0)(0) = 1
- 0\(^{th}\) with 1\(^{st}\) : \((1)(0)+(0)(\cos\theta)+(0)(-\sin\theta)+(1)(0) = 0
- 0\(^{th}\) with 3\(^{rd}\) : \((1)(0)+(0)(0)+(0)(0)+(0)(1) = 0

#### For Boosts:
- 0\(^{th}\) with 0\(^{th}\) : \((\gamma)(\gamma)-(-\beta)(\gamma\beta)+(0)(0)+(0)(0)+(0)(0) = \gamma^2[1-\beta^2] = 1
- 0\(^{th}\) with 1\(^{st}\) : \((\gamma)(\gamma)+(\gamma)(-\beta)+(0)(0)+(0)(0) = \gamma^2[\beta-\beta] = 0
- 0\(^{th}\) with 3\(^{rd}\) : \((\gamma)(0)+(-\beta)(0)+(0)+0(1) = 0

### Inner Product Tensor Invariant
\[
\Lambda^\alpha_\nu \Lambda^\nu_\alpha = 4 = \delta^{\alpha}_\nu \delta^\nu_\alpha
\]

The fact that each row gives a single (1) leads to the overall inner product \( \Lambda^\mu_\nu \Lambda^\nu_\mu = 4 = \delta^{\mu}_\nu \delta^\nu_\mu \) for 4D Lorentz Transforms \((1+1+1+1 = 4)\).
SRQM Study: SR 4-Tensors

SR Tensor Invariants for Continuous Lorentz Transform Tensors

The Lorentz Transform Tensor \( \Lambda_{\mu\nu} = \partial x^\mu / \partial x'^\nu = \delta_{\mu\nu} \) is the tensor all SR 4-Vectors must transform by.

\[ (2,0)\text{-Tensor} = 4\text{-Tensor} T^{\mu\nu}; \text{Has (4+) Tensor Invariants (though not all independent)} \]

- a) Lorentz Trace[\(\Lambda^\mu\nu\)] = \{0..4..\Infinity\} Lorentz Boost meets Rotation at Identity of 4
- b) Lorentz Inner Product \(\Lambda_{\mu\nu}\Lambda^{\mu\nu} = \) 4 from \(\{\eta_{\mu\nu}\Lambda_{\mu\nu} = \eta_{\delta\sigma}\) and \(\{\eta_{\nu\nu}\} = 4\)
- c) Lorentz AsymmTri[\(\Lambda^\mu\nu\)]
- d) Lorentz Det[\(\Lambda^\mu\nu\)] = +1 for Proper Transforms, Continuous Transforms Proper

An even more general version would be with a & b as arbitrary complex values:

\[ \text{EigenValues}[\Lambda^\mu\nu] = \{e^a, e^a, e^b, e^b\} \]

\[ \text{Sum of EigenValues}[\Lambda^\mu\nu] = \text{Tr}[\Lambda^\mu\nu] = \Lambda_{\mu\nu} = \{e^a + e^b + a + b\} \]
\[ \text{Determinant Tensor Invariant} \]

\[ \text{Det}[\text{Proper} \Lambda^\mu\nu] = +1 \text{ Proper Transform always +1} \]

\[ \text{SR: Lorentz Transform} \]
\[ \delta R^\mu / \partial x = \Lambda^\mu_{\nu} = (\Lambda^{-1})^\mu_{\nu} : \Lambda^\mu_{\mu} = \eta_{\mu\nu} = \delta^\mu_{\nu} \]
\[ \text{Product of EigenValues}[\Lambda^\mu_{\nu}] = \text{Det}[\Lambda^\mu_{\nu}] = \{e^a e^a, e^a e^b, e^b e^b, e^b e^a\} \]
\[ \text{Det}[\Lambda^\mu_{\nu}] = \pm 1 \]
\[ \Lambda_{\mu\nu} \Lambda^{\mu\nu} = 4 \]

\[ \text{SR 4-Tensor} \]
\[ (2,0)\text{-Tensor} T^{\mu\nu} \]
\[ (1,1)\text{-Tensor} T^\mu_{\nu} \text{ or } T^\nu_{\mu} \]
\[ (0,2)\text{-Tensor} T_{\mu\nu} \]

\[ \text{SR 4-Vector} \]
\[ (1,0)\text{-Tensor} V^\mu = (v^\mu, v^\nu) \]
\[ (0,1)\text{-Tensor} V_\nu = (v_\nu, v_\mu) \]

\[ \text{SR 4-Scalar} \]
\[ (0,0)\text{-Tensor} S \text{ or } S_0 \]

\[ \text{Lorentz Scalar} \]

\[ \text{Det}[T^\mu_{\nu}] = \Pi_{\nu}[\lambda_{\nu}]; \text{with } \{\lambda_{\nu}\} = \text{EigenValues} \]

\[ \text{Characteristic Eqns: Det}[T^\mu_{\nu} - \lambda_0 I_{\nu\nu}] = 0 \]

\[ \text{Trace}[T^\mu_{\nu}] = \eta_{\nu\nu} T^\mu = T^\mu_{\nu} = T_{\nu}^\mu = T \]
\[ V \cdot V = V^\mu V^\nu = (v^\mu v^\nu) = (v^0)^2 \]

\[ \text{Lorentz Scalar Invariant} \]
### SRQM Study: SR 4-Tensors

#### SR Tensor Invariants for Discrete Lorentz Transform Tensors

<table>
<thead>
<tr>
<th>SR: Lorentz Transform</th>
<th>Inner Product Tensor Invariant</th>
<th>Trace Tensor Invariant</th>
<th>Determinant Tensor Invariant</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \partial [R^\mu] = \partial R^\mu / \partial R^\nu = \Lambda^\mu_\nu )</td>
<td>( \Lambda^\mu_\nu = (\Lambda^\nu_\mu)^\ast ) ( \Lambda^\nu_\mu \Lambda^\alpha_\nu = \eta^\nu_\mu = \delta^\nu_\mu )</td>
<td>( \det[\Lambda^\mu_\nu] = \pm 1 )</td>
<td>( \det[\Lambda^\mu_\nu] = \pm 1 ) Proper Transform = +1 Improper Transform = -1</td>
</tr>
<tr>
<td>Lorentz SR TPCombi Tensor ( \Lambda^\mu_\nu, \rightarrow TP^\nu_\mu ) ( -\nu^\mu = -\delta^\nu_\mu ) ( \Lambda^\nu_\mu \Lambda^\nu_\mu = 4 = \delta^\nu_\mu )</td>
<td>Det[TP\nu_\mu] = -1 -1 -1 -1</td>
<td>Sum of ( \text{EigenValues}[TP^\nu_\mu] ) = Set{-1,-1,-1,-1}</td>
<td>Product of ( \text{EigenValues}[TP^\nu_\mu] ) = Det[TP\nu_\mu] = -1 -1 -1 -1</td>
</tr>
<tr>
<td>Lorentz SR Parity-Inversion Tensor ( \Lambda^\nu_\mu, \rightarrow TP^\nu_\mu ) ( -\nu^\mu = -\delta^\nu_\mu ) ( \Lambda^\nu_\mu \Lambda^\nu_\mu = 4 = \delta^\nu_\mu )</td>
<td>Lorentz SR Flip-xy-Combi Tensor ( \Lambda^\nu_\mu, \rightarrow TP^\nu_\mu ) ( -\nu^\mu = -\delta^\nu_\mu ) ( \Lambda^\nu_\mu \Lambda^\nu_\mu = 4 = \delta^\nu_\mu )</td>
<td>Lorentz SR Time-Reversal Tensor ( \Lambda^\nu_\mu, \rightarrow TP^\nu_\mu ) ( -\nu^\mu = -\delta^\nu_\mu ) ( \Lambda^\nu_\mu \Lambda^\nu_\mu = 4 = \delta^\nu_\mu )</td>
<td>Lorentz SR Identity Tensor ( \Lambda^\nu_\mu, \rightarrow TP^\nu_\mu ) ( -\nu^\mu = -\delta^\nu_\mu ) ( \Lambda^\nu_\mu \Lambda^\nu_\mu = 4 = \delta^\nu_\mu )</td>
</tr>
</tbody>
</table>

The Trace of various discrete Lorentz transforms varies in steps from \{-4,-2,0,2,4\}

This includes Mirror Flips, Time Reversal, and Parity Inversion – essentially taking all combinations of ±1 on the diagonal of the transform.
The Flip-xy-Combo is the equivalent of a π-Rotation-z. I suspect that this may be related to exchange symmetry and the Spin-Statistics idea that a particle-exchange is the equivalent of a spin-rotation.

A single Flip would not be an exchange because it leaves a mirror-inversion of \(\text{right-\leftarrow\text{-left}}\).

But the extra Flip along an orthogonal axis corrects the mirror-inversion, and would be an overall exchange because the particle is in a different location.

\[
\text{SR: Lorentz Transform} \quad \frac{\partial [R^\mu]}{\partial R^\nu} = \Lambda^\mu_v \quad \text{where} \quad \Lambda^\mu_v = (\Lambda^\mu_v)_{\nu}^\gamma \quad \text{and} \quad \eta^\mu_v = \delta^\mu_v \quad \text{det} \Lambda = \pm 1 \quad \Delta_{\mu\nu}\Lambda^{\mu\nu} = 4
\]

\[
\text{Lorentz SR} \quad 0\text{-Rotation-z Tensor} \quad \Lambda^\mu_\nu \rightarrow R^\mu_\nu \quad \quad \text{Identity Tensor} \quad \Lambda^\mu_\nu \rightarrow \eta^\mu_\nu \quad = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{is} \quad \text{Minkowski Delta}
\]

\[
\\text{Lorentz SR} \quad \text{Flip-x Tensor} \quad \Lambda^\mu_\nu \rightarrow F^\mu_\nu \quad \quad \text{Lorentz SR} \quad \text{Flip-y Tensor} \quad \Lambda^\mu_\nu \rightarrow F^\mu_\nu \quad \quad \text{Lorentz SR} \quad \text{π-Rotation-z Tensor} \quad \Lambda^\mu_\nu \rightarrow R^\mu_\nu
\]

\[
\text{SRQM Study: SR 4-Tensors}
\]

\[
\text{More SR Tensor Invariants for Discrete Lorentz Transform Tensors}
\]

\[
\text{EigenValues}[R^\mu_\nu] = \{1, e^i\pi, e^{i\pi}, 1\}
\]

\[
\text{EigenValues}[\eta^\mu_\nu] = \{-1, -1, 1, 1\}
\]

\[
\text{EigenValues}[F^\mu_\nu] = \{-1, 1, 1, 1\}
\]

\[
\text{EigenValues}[F^\mu_\nu] = \{-1, -1, 1, 1\}
\]

\[
\text{EigenValues}[R^{\mu}_\nu] = \{-1, 1, 1, 1\}
\]

\[
\text{EigenValues}[R^{\mu}_\nu] = \{1, e^i\pi, e^{i\pi}, 1\}
\]

\[
\text{SR 4-Tensor} \quad (2,0) \text{-Tensor} T^{\mu\nu}_{\nu\mu} \quad \text{SR 4-CoVector: OneForm} \quad (0,1) \text{-Tensor} V_\nu = (v_\nu)\nu
\]

\[
\text{SR 4-Scalar} \quad (0,0) \text{-Tensor} S \text{ or } S_{\alpha\beta} \quad \text{Lorentz Scalar}
\]

\[
\text{Det}[T^\mu_\nu] = \Pi_{\nu}[\lambda_{\gamma}] \quad \text{with} \quad \{\lambda_{\gamma}\} = \text{EigenValues}
\]

\[
\text{Characteristic Eqns:} \quad \text{Det}[T^\mu_\nu - \lambda_{\gamma}I_{\gamma\delta}] = 0
\]

\[
\text{Trace}[T^\mu_\nu] = \eta^\nu_\nu \tau^\mu_\nu = T^\mu_\nu = T
\]

\[
V \cdot V = V^\mu_\nu V^\nu_\mu \quad = (v^\mu_\nu)^2 - v^\mu_\nu v^\mu_\nu = (v^\mu_\nu)^2
\]

\[
\text{Lorentz Scalar Invariant}
\]
SR 4-Scalars, 4-Vectors, 4-Tensors beautifully and elegantly display the relations between lots of different physical properties and relations. Their notation makes navigation through the physics very simple.

They also devolve very nicely into the limiting/approximate Newtonian cases of \( |v| << c \) by letting \( \gamma \rightarrow 1 \) and \( \gamma' = \frac{1}{\sqrt{1-v^2/c^2}} \).

SR tells us that several different physical properties are actually dual aspects of the same thing, with the only real difference being one’s point of view, or reference frame.

SR 4-Scalars, 4-Vectors, and 4-Tensors elegantly join many dual physical properties and relations. Their notation makes navigation through the physics very simple.

Examples of 4-Vectors = (1,0)-Tensors include:
- (Time, Space), (Energy, Momentum), (Power, Force), (Frequency, WaveNumber),
- (Time Differential, Spatial Gradient), (Number Density, Number Flux),
- (Charge Density, Current Density), (EM-Scalar Potential, EM-Vector Potential), etc.

One can also examine 4-Tensors, which are type (2,0)-Tensors.

The Faraday EM Tensor similarly combines EM fields:
Electric \( e = e' = (e^x, e^y, e^z) \) and Magnetic \( b = b' = (b^x, b^y, b^z) \)

\[
F^{\alpha \beta} = \begin{pmatrix}
0 & -e^y/c & +e^z/c \\
-e^y/c & 0 & -e^x/c \\
+e^z/c & e^x/c & 0
\end{pmatrix}
\]

Also, things are even more related than that. The 4-Momentum is just a constant times 4-Velocity. The 4-WaveVector is just a constant times 4-Velocity.

In addition, the very important conservation/continuity equations seem to just fall out of the notation. The universe apparently has some simple laws which can be easy to write down by using a little math and a super notation.

The 4-WaveVector is just a constant times 4-Velocity.

The 4-Momentum is just a constant times 4-Velocity.

Also, things are even more related than that.

Faraday EM Tensor \( F^{\alpha \beta} \) is invariant:

\[
F^{\alpha \beta} = \begin{pmatrix}
0 & -e^x/c & +e^z/c & -e^y/c \\
e^x/c & 0 & -b^z & +b^y \\
e^z/c & -b^y & 0 & +b^x \\
e^y/c & +b^z & -b^x & 0
\end{pmatrix}
\]

\[
\text{Trace}[T^{\alpha \beta}] = \eta_{\mu \nu}T^{\mu \nu} = T \rightarrow T_{\nu} = T
\]

\[
V \cdot V = V^\nu \eta_{\nu \nu} V^\nu = (V^\nu)^2 - (V^\nu V^\nu) = (V^\nu)^2
\]

\[
\text{Lorentz Scalar}
\]

John B. Wilson
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http://scirealm.org/SRQM.pdf
**SRQM Study:**

**SR Gradient 4-Vectors = (1,0)-Tensors**

**SR Gradient One-Forms = (0,1)-Tensors**

<table>
<thead>
<tr>
<th>4-Vector = Type (1,0)-Tensor</th>
<th>[Temporal : Spatial] components</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-Position ( R = R^\mu = (ct,r) )</td>
<td>Time (t) : Space (r)</td>
</tr>
<tr>
<td>4-Gradient ( \partial_R = \partial = \partial^\mu = \partial/\partial R_\mu = (\partial/\partial c, \nabla) )</td>
<td>Time Differential (( \partial_t )) : Spatial Gradient(( \nabla ))</td>
</tr>
</tbody>
</table>

**Standard 4-Vector**

<table>
<thead>
<tr>
<th>4-Position ( R = R^\mu = (ct,r) )</th>
<th>Related Gradient 4-Vector (from index-raised Gradient One-Form)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-Velocity ( U = U^\mu = \gamma(c,u) )</td>
<td>4-PositionGradient ( \partial_R = \partial_R^\mu = \partial/\partial R_\mu = (\partial/\partial c, \nabla_R) = \partial = \partial^\mu = 4\text{-Gradient} )</td>
</tr>
<tr>
<td>4-Momentum ( P = P^\mu = (E/c,p) )</td>
<td>4-VelocityGradient ( \partial_U = \partial_U^\mu = \partial/\partial U_\mu = (\partial/\partial c, \nabla_U) )</td>
</tr>
<tr>
<td>4-WaveVector ( K = K^\mu = (\omega/c,k) )</td>
<td>4-MomentumGradient ( \partial_P = \partial_P^\mu = \partial/\partial P_\mu = (\partial/\partial c, \nabla_P) )</td>
</tr>
</tbody>
</table>

| 4-WaveGradient \( \partial_K = \partial_K^\mu = \partial/\partial K_\mu = (\partial/\partial c, \nabla_K) \) | |

In each case, the (Whichever)Gradient 4-Vector is derived from an SR One-Form or 4-CoVector, which is a type (0,1)-Tensor

ex. One-Form PositionGradient \( \partial_R^\nu = \partial/\partial R^\nu = (\partial/\partial c, \nabla_R) \)

The (Whichever)Gradient 4-Vector is the index-raised version of the SR One-Form (Whichever)Gradient

ex. 4-PositionGradient \( \partial_R^\mu = \partial/\partial R_\mu = (\partial/\partial c, \nabla_R) = \eta^{\nu\mu}\partial_R^\nu = \eta^{\nu\mu}\partial/\partial R^\nu = \eta^{\nu\mu}(\partial_R/c, \nabla_R) = \eta^{\nu\mu}(\text{One-Form PositionGradient}) \),

This is why the 4-Gradient is commonly seen with a minus sign in the spatial component, unlike the other regular 4-Vectors, which have all positive components.

4-Tensors can be constructed from the Tensor Outer Product of 4-Vectors
Some Basic 4-Vectors

Minkowski SpaceTime Diagram

Events & Dimensions

past       future
now       here

elsewhere
now       here

"Stack of Motion Picture Photos"

Note the matching dimensional units: (4D SpaceTime)
\( \Delta t \) is [time], \( |\Delta r| \) is [length]

\( \Delta t \) time-like interval
\( \Delta r \) space-like interval

Classical Mechanics

1/c

4-Displacement
\[ \Delta R_{\text{cm}} = (c\Delta t, \Delta r) \]

1/c

\( \Delta t \)

3-displacement
\( \Delta r = \Delta r \rightarrow (\Delta x, \Delta y, \Delta z) \)

Special Relativity

\( c \) light-like interval (0) = null
\( \Delta r \) space-like interval (-)

Note the matching dimensional units: (4D SpaceTime)
\( \Delta t \) is [length/time]*[time] = [length], \( |\Delta r| \) is [length], \( |\Delta R| \) is [length]

\( \tau \) is the Proper Time = "rest-time", time as measured by something not moving spatially
The Minkowski Diagram provides a great visual representation of SpaceTime

SR 4-Scalar
\( \eta^\mu = \eta_{\nu} \eta^{\mu} = T^{\mu}_\nu = T_{\nu}^\mu \)
\( V \cdot V = \eta^{\mu\nu} V^\mu V_\nu = (v_0^2 - v_x^2 - v_y^2 - v_z^2) = \text{Lorentz Scalar Invariant} \)

Trace\[T^{\mu\nu} = \eta_{\mu\nu} T^{\mu\nu} = 0 \]

SR 4-Vector
\( \eta^{\mu\nu} = (1,0,0,0) \)
\( V_\nu = (v_0, v_x, v_y, v_z) \)

SR 4-Vector
\( \eta^{\mu\nu} = (0,1,0,0) \)
\( V_\nu = (0, v_x, v_y, v_z) \)

SR 4-Vector
\( \eta^{\mu\nu} = (0,0,1,0) \)
\( V_\nu = (0, 0, v_y, v_z) \)

SR 4-Vector
\( \eta^{\mu\nu} = (0,0,0,1) \)
\( V_\nu = (0, 0, 0, v_z) \)

SR 4-Vector
\( \eta^{\mu\nu} = (0,0,0,0) \)
\( V_\nu = (0, 0, 0, 0) \)

SR 4-Vector
\( \eta^{\mu\nu} = (0,0,0,0) \)
\( V_\nu = (0, v_x, v_y, v_z) \)

SR 4-Vector
\( \eta^{\mu\nu} = (0,0,0,0) \)
\( V_\nu = (v_0, v_x, v_y, v_z) \)

Classical (scalar)

- Lorentz Invariant

3-vector

Galilean Invariant

Not Lorentz Invariant

Galilean

Lorentz

Invariant

Invariant
Some Basic 4-Vectors

Minkowski SpaceTime Diagram, WorldLines, LightSpeed to the Future!

- $\Delta t$ | time-like interval (+)
- $\Delta r$ | light-like interval (0) = null
- at-rest WorldLine ($u=0$)
- inertial motion WorldLine ($0<u<c$)
- future
- elsewhere
- now
- here
- past
- -c

An Event (*) is a point in SpaceTime. The 4-Position points to an Event.

A WorldLine is a series of connected Events which trace out a path in SpaceTime, such as the track of a moving particle.

- 4-Displacement $\Delta R = (c\Delta t, \Delta r)$
- 4-Position $R = (ct, r) = \langle \text{Event} \rangle$

4-Position is Lorentz Invariant, but not Poincaré Invariant. A standard 4-Displacement is both.

$\Delta R \cdot \Delta R = [(c\Delta t)^2 - \Delta r \cdot \Delta r] = 0$ for time-like (+)

$-(\Delta r_o)^2$ for space-like (−)

Massive particles move temporally into the future at the speed-of-light (c) in their own rest-frame.

Massless particles (photonic) move nullly into the future at the speed-of-light (c), and have no rest-frame.
Since the SpaceTime magnitude of $U$ is a constant ($c$), changes in the components of $U$ are like rotating the 4-Vector without changing its length. It keeps the same magnitude.

Rotations, purely spatial changes, (eg. along x,y) result in circular displacements.

Boosts, or temporal-spatial changes, (eg. along x,t) result in hyperbolic displacements.

The interval between the origin and a given topograph-line is a Lorentz Invariant Constant.

$U \cdot U = \gamma(c,u) \cdot \gamma(c,u) = \gamma^2(c^2 - u \cdot u) = (c^2)$

**Rotation (x,y): Purely Spatial**

**Boost (x,t): Spatial-Temporal**
Since the SpaceTime magnitude of $U$ is a constant $(c)$, changes in the components of $U$ are like rotating the 4-Vector without changing its length. It keeps the same magnitude $(c)$. Rotations, purely spatial changes, {eg. along x,y} result in circular displacements. Boosts, or temporal-spatial changes, {eg. along x,t} result in hyperbolic displacements. The interval between the origin and a given topograph-line is a Lorentz Invariant Constant.

The Minkowski Diagram provides a great visual representation of SpaceTime.
SRQM: Some Basic 4-Vectors

4-Position, 4-Velocity, 4-Acceleration

SpaceTime Kinematics

A Tensor Study

\[ R \cdot U / U \cdot U = (ct, r) \cdot \gamma(c, u) / c^2 = \gamma(c^2 t - r \cdot u) / c^2 = (c^2 t_c) / c^2 \]

\[ \delta = (\partial_t / c, -\nabla) \rightarrow (\partial_t / c, -\partial_r - \partial_y - \partial_z) \]

\[ U \cdot \partial = \gamma(c, u) \cdot (\partial_t / c, -\nabla) = \gamma d/dt \]

\[ = \gamma^2 (c \beta' , \gamma' u + \gamma a) \]

\[ = \gamma^2 (u \cdot a) / c, \gamma^2 (u \cdot a) / c^2 + a \]

\[ 4-Vectors: \]

\[ R = \langle Event \rangle \]

\[ U = = dR / dt \]

\[ = A = U / \tau \]

\[ \tau = d/dt \]

\[ = \gamma \]

\[ = \gamma \]

\[ \gamma = 1 / \sqrt{1 - (v/c)^2} \]

Since time:space don't mix in CM, typically use time \( t \) & 3-position \( r \) separately.

Since temporal velocity (c) always constant in CM, typically use just 3-velocity \( u \).

Since temporal acceleration (0) always constant in CM, typically use just 3-acceleration \( a \).

The relativistic Gamma factor \( \gamma = 1 / \sqrt{1 - (v/c)^2} \).

The 1st order Newtonian Limit gives \( \gamma \sim 1 + O((v/c)^2) \).

The 2nd order Newtonian Limit gives \( \gamma \sim 1 + (v/c)^2 / 2 + O((v/c)^4) \).

For historical reasons, velocity can be represented by either (v) or (u).

\[ r \rightarrow (x, y, z) \]

\[ u \rightarrow (u', u', u') \]

\[ \gamma' = d/dt = \gamma^2 (u \cdot a) / c^2 \]

\[ T^\mu_{\nu} = \eta_{\mu \nu} T^\mu_{\nu} = T_{\nu}^\mu = T_{\mu \nu} \]

\[ V \cdot V = V^\nu \eta_{\nu \nu} V^\mu = [(v)^2] \cdot V \cdot V = [(v)^2] \] is Lorentz Scalar Invariant
SRQM: Some Basic 4-Vectors

4-Position, 4-Velocity, 4-Acceleration, (RestMass), 4-Momentum, 4-Force

SpaceTime Dynamics

4-Vectors:
- \( R = (ct, r) \) : 4-Position
- \( U = \gamma(c, u) = dR/d\tau \) : 4-Velocity
- \( A = \gamma(c, u) \) : 4-Acceleration
- \( P = m_o U \) : 4-Momentum
- \( F = \gamma \) : 4-Force

E/c^2 = m_o

This group of 4-Vectors are the main ones that are connected by the ProperTime Derivative.

\[
U \cdot \partial = d/d\tau = \gamma d/dt = \gamma (\partial_t + u \cdot \nabla)
\]

The classical part of it, the convective derivative, \( (\partial_t + u \cdot \nabla) \), is known by many different names:
- The convective derivative is a derivative taken with respect to a moving coordinate system. It is also called the advective derivative, derivative following the motion, hydrodynamic derivative, Lagrangian derivative, material derivative, particle derivative, substantial derivative, substantive derivative, Stokes derivative, or total derivative.

SR 4-Tensor
- \( (2,0) \)-Tensor \( T^{uv} \)
- \( (1,1) \)-Tensor \( V^u = (v^0, v) \)

SR 4-Vector
- \( (1,0) \)-Tensor \( V^u = (v^0, v) \)
- \( (0,2) \)-Tensor \( T_{uv} \)

SR 4-Scalar
- \( (0,0) \)-Tensor \( S \) or \( S_o \)
- Lorentz Scalar

SR 4-CoVector: OneForm
- \( V_u = (v_0, v) \)

Trace[\( T^{uv} \)] = \( \eta_{uv} T^{uv} = T^{uv} = T \)

\( V \cdot V = V^u \eta_{uv} V^v = (v^0)^2 - v \cdot v = (v^0)^2 \)

= Lorentz Scalar Invariant
SRQM: Some Basic 4-Vectors

4-Velocity, 4-Momentum, \( E = mc^2 \)

**4-Velocity**

\[ U = \gamma(c, u) \]

\[ U \cdot U = (c)^2 \]

**4-Momentum**

\[ P = (E/c, p) = (mc, p) \]

\[ P \cdot P = (E/c)^2 = (mc)^2 \]

**Temporal part:**

\[ E = mc^2 = \gamma m_o c^2 = \gamma E_o \]

\[ E = m_o c^2 + (\gamma - 1)m_o c^2 \]

\[ E = E_o + (\gamma - 1)E_o \]

\( (\text{rest}) + (\text{kinetic}) \)

**Spatial part:**

\[ p = \gamma m_o u = m u \]

Since time:space don't mix in CM, typically use energy \( E \) & 3-momentum \( p \) separately.

The relativistic Gamma factor \( \gamma = 1/\sqrt{1-(v/c)^2} \)

The 1st order Newtonian Limit gives \( \gamma \sim 1 + O[(v/c)^2] \)

The 2nd order Newtonian Limit gives \( \gamma \sim 1 + (v/c)^2/2 + O[(v/c)^4] \)

For historical reasons, velocity can be represented by either \( v \) or \( u \).

**Temporal part:**

\[ E \sim (1+(v/c)^2/2)m_o c^2 = m_o c^2 + m_o v^2/2 \]

\[ E_o + |p|^2/2m_o \]

\( (\text{rest}) + (\text{kinetic}) \)

**Spatial part:**

\[ p \sim (1)m_o u = m_o u \rightarrow m u \]
SRQM: Some Basic 4-Vectors

4-Velocity, 4-Acceleration, SpaceTime Orthogonality

4-Velocity
\[ \mathbf{U} = \gamma (c, \mathbf{u}) \]

4-Acceleration
\[ \mathbf{A} = \gamma (c \gamma', c \mathbf{u} + \gamma' \mathbf{a}) \]

The Lorentz Scalar Product can be used to show SpaceTime orthogonality when the result is zero.

\[ \mathbf{U} \cdot \mathbf{A} = \mathbf{U} \cdot \mathbf{U}' = 0 \]

\[ \mathbf{U} \perp \mathbf{A} \]

4-Velocity is the direction along a WorldLine.
4-Acceleration is the thing which causes a WorldLine to bend/curve.
SRQM: Some Basic 4-Vectors
4-Displacement, 4-Velocity, Relativity of Simultaneity

4-Velocity
\[ U = \gamma (c, u) \]

4-Gradient
\[ \partial \equiv \left( \frac{\partial}{\gamma c}, -\nabla \right) \]

4-Acceleration
\[ A = \gamma (c \gamma', \gamma' u + \gamma a) \]

4-Displacement
\[ \Delta X = (c \Delta t, \Delta x) \]

4-Position
\[ X = (ct, x) \]

\[ U \cdot \Delta X = \gamma (c, u) \cdot (c \Delta t, \Delta x) = c^2 \Delta t - u \cdot \Delta x \]

If Lorentz Scalar \( U \cdot \Delta X = 0 = c^2 \Delta t \), then the ProperTime displacement \( (\Delta t) \) is zero, and the event separation \( (\Delta X = X_2 - X_1) \) is orthogonal to the worldline \( U \).

\( X_1 \) and \( X_2 \) are therefore simultaneous for the observer on this worldline \( U \).

Examine the equation we get \( \gamma (c^2 \Delta t - u \cdot \Delta x) = 0 \). The coordinate time difference between the events is \( (\Delta t = u \cdot \Delta x / c^2) \). The condition for simultaneity in an alternate frame (moving at 3-velocity \( u \) wrt. the worldline \( U \)) is \( \Delta t = 0 \), which implies \( (u \cdot \Delta x) = 0 \).

This can be met by:
- \( (|u| = 0) \), the alternate observer is not moving wrt. the events, i.e. is on worldline \( U \) or on a worldline parallel to \( U \).
- \( (|\Delta x| = 0) \), the events are at the same spatial location (co-local).
- \( (u \cdot \Delta x = 0) \), the alternate observer’s motion is perpendicular (orthogonal) to the spatial separation \( \Delta x \) of the events in that frame.

If none of these conditions is met, then the events will not be simultaneous in the alternate reference frame. This is the mathematics behind the concept of Relativity of Simultaneity.
SR Diagram:
SR Motion * Lorentz Scalar = Interesting Physical 4-Vector

- **4-Discplacement**: \(\Delta R = (c\Delta t, \Delta r)\)
- **4-Position**: \(R = (ct, r)\)
- **4-Motion**: Lorentz Scalar
- **4-Velocity**: \(U = \gamma (c, u)\)
- **4-Momentum**: \(P = m (c, u) = (mc, p) = (E/c, p)\)
- **4-NumberFlux**: \(N = (n, c)n = n (c, u)\)
- **4-ChargeFlux**: \(J = (\rho, c)\)
- **4-CurrentDensity**: \(\mathbf{J} = (\rho, c)\)
- **4-EMVectorPotential**: \(\mathbf{A} = (\phi/c^2)\)
- **4-Momentum**: \(P = m (c, u) = (mc, p) = (E/c, p)\)
- **4-WaveVector**: \(K = (\omega/c, k)\)

Interested note:
Most 4-Vectors have 4 independent components. (1 temporal, 3 spatial)

The 4-Velocity has only the 3 spatial however, due to its invariant magnitude\(\gamma U \cdot U = c^2\).

This fact allows one to multiply it by a Lorentz Scalar to make a new 4-Vector with 4 independent components, as shown in the diagram.

Proof of non-varying (c).
SRQM Diagram:
ProperTime Derivative
Very Fundamental Results

4-Displacement
\[ \Delta \mathbf{R} = (c \Delta \tau, \Delta \mathbf{r}) \]
d\( \mathbf{R} = (c d \tau, d \mathbf{r}) \)
4-Momentum
\[ \mathbf{P} = (E/c, \mathbf{p}) = (mc, \mathbf{p}) \]

\[ U \cdot U = c^2 \]
\[ d/d\tau \text{ of } \mathbf{R} = d/d\tau (4) = 0 \]
\[ (U \cdot U)(d/d\tau) = (U \cdot d)[c^2] = 0 \]
\[ d/d\tau (U \cdot U) = d/d\tau (U \cdot U) = 0 \]
\[ U \cdot \mathbf{A} = U \cdot \mathbf{U}' = 0 \]
\[ \mathbf{A} = \mathbf{U} \times \mathbf{U} + \mathbf{U} \times \mathbf{U} = 0 \]

4-Gradient
\[ \partial = (\partial/c, -\nabla) \]
\[ \text{Continuity of 4-Velocity Flow: } \partial \mathbf{U} = 0 \]
\[ \text{ProperTime Derivative} \]
\[ \mathbf{A} = \gamma (c \partial, \mathbf{U}) \]
\[ \mathbf{F} = \gamma \mathbf{E} \]

4-Acceleration
\[ \mathbf{A} = \gamma (c \partial, \mathbf{U}) - \gamma (\partial/c, -\nabla) \mathbf{U} \times \mathbf{U} = 0 \]

ProperTime Derivative
\[ \partial \cdot \mathbf{R} = 4 \]

\[ \text{SpaceTime Dimension is 4} \]

SR 4-Vector
\[ (2,0) \text{-Tensor } \mathbf{T}_{\mu \nu} \]
\[ (1,1) \text{-Tensor } \mathbf{V}^\nu = (v^\nu, \mathbf{V}) \]
\[ (0,2) \text{-Tensor } \mathbf{T}_{\mu \nu} \]

SR 4-Scalar
\[ \text{Lorentz Scalar} \]

SR 4-Vector
\[ (1,0) \text{-Tensor } \mathbf{V}^\nu = (v^\nu, \mathbf{V}) \]
\[ (0,1) \text{-Tensor } \mathbf{V}_\mu = (v_\mu, -\mathbf{V}) \]

SR 4-Scalar
\[ \text{Lorentz Scalar} \]

Trace[T_{\mu \nu}] = \eta_{\mu \nu} T_{\mu \nu} = T_{\nu \nu} = T

\[ \mathbf{V} \cdot \mathbf{V} = (v^\mu, \mathbf{V}) \]

\[ \text{Lorentz Scalar Invariant} \]
SRQM Diagram: Local Continuity of 4-Velocity leads to all of the Conservation Laws

Conservation Laws:

All of the Physical Conservation Laws are in the form of a 4-Divergence, which is a Lorentz Invariant Scalar equation.

These are local continuity equations which basically say that the temporal change in a quantity is balanced by the flow of that quantity into or out of a local spatial region.

Conservation of Charge:
\[ \partial \cdot J = 0 \]

Example:
\[ \partial (\rho c) = 0 \]
\[ \partial J = 0 \]
\[ (\partial / c \rho c + \nabla j) = 0 \]
\[ (\partial p + \nabla j) = 0 \]
= Conservation of Charge
= A Continuity Equation

SR 4-Tensor
(2,0)-Tensor \( T^{\mu \nu} \)
(1,0)-Tensor \( V^\mu = V \) \( (v^0, v) \)
SR 4-CoVector: OneForm
(0,1)-Tensor \( V_\mu = (v_0, -v) \)
SR 4-Scalar
(0,0)-Tensor \( S \) or \( S_0 \) Lorentz Scalar

4-Vector SRQM Interpretation of QM
SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

\[ \text{Trace}[T^{\mu \nu}] = \eta^{\mu \nu} T^{\mu \nu} = T^0_\mu = T \]
\[ V \cdot V = V^\mu \eta_{\mu \nu} V^\nu = (v^0)^2 - v \cdot v = (v_0)^2 \]
= Lorentz Invariant
SRQM Diagram:

SRQM Motion * Lorentz Scalar Conservation Laws, Continuity Eqns

SR 4-Tensor
(2,0)-Tensor $T^{iv}$
(1,1)-Tensor $T^{iv}$, or $T^{uv}$
(0,2)-Tensor $T_{iv}$

SR 4-Vector
(1,0)-Tensor $V^{i} = (\gamma v, \mathbf{V})$
SR 4-CoVector: OneForm
(0,1)-Tensor $V_{i} = (\mathbf{v}, c)$
SR 4-Scalar
(0,0)-Tensor $S$ or $\rho$ Lorentz Scalar

Conservation Laws:
All of the Physical Conservation Laws are in the form of a 4-Divergence, which is a Lorentz Invariant Scalar equation.

Conservation of Charge: $\partial J = (\partial / c + \nabla) j = 0$

Conservation of Momentum: $\partial P = 0$

Conservation of Mass: $\partial \rho = 0$

Conservation of Energy: $\partial E = 0$

Conservation of Charge: $\partial A = 0$

These are local continuity equations which basically say that the temporal change in a quantity is balanced by the flow of that quantity into or out of a local spatial region.

SRQM Motion: $\mathbf{v} = \mathbf{V} / c$

Einstein de Broglie:
$K = (\omega / c, k) = (\omega / c_n) \mathbf{n} / \sqrt{\gamma}$

4-WaveVector:
$\mathbf{K} = (\omega / c_n, k)$

4-Momentum:
$P = m(c, u) = (m c, \mathbf{p}) = (E / c, \mathbf{p})$

4-MassFlux
$N = (n, c, u) = n(c, u)$

4-NumberFlux
$J = (pc, j) = p(c, u)$

4-ProbCurrDensity
$J_{prob} = (\rho_{prob}, c, j_{prob})$

4-ProbDensityFlux
$J_{prob} = (\rho_{prob}, c, j_{prob})$

4-ProbabilityFlux
$\rho_{prob} = \chi^{\gamma} = |\langle \psi | \gamma \psi \rangle|^2$

4-MassDensity
$\rho = m(c, u)$

4-MomentumDensity
$G = \rho m(c, u)$

4-CurrentDensity
$J = (pc, j) = p(c, u)$

4-Vector SRQM Interpretation of QM
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SRQM: Some Basic 4-Vectors

4-Velocity, 4-Gradient, Time Dilation

The Minkowski Diagram provides a great visual representation of SpaceTime.

at-rest worldline \( U_0 \) (\( u=0 \))

const inertial motion worldline \( U \) (\( 0<u<c \)) trades some time for space

\[
\text{4-Velocity: } U = \gamma(c, u) = \frac{c}{\sqrt{1-(u/c)^2}} = \frac{1}{\sqrt{1-\beta^2}}
\]

\[
\text{4-Gradient: } \partial = (\partial_t/c, -\nabla)
\]

Everything moves into future (+t) at the speed-of-light (c) in its own spatial rest-frame.

Since the SpaceTime magnitude of \( U \) is a constant, changes in the components of \( U \) are like "rotating" the 4-Vector without changing its length. However, as \( U \) gains some spatial velocity, it loses some "relative" temporal velocity. Objects that move in some reference frame "age" more slowly relative to those at rest in the same reference frame.

Time Dilation:

\[
\Delta t = \gamma \Delta \tau = \gamma \Delta t_c
\]

\[
dt = \gamma d\tau
\]

\[
d/d\tau = \gamma d/dt
\]

Each observer will see the other as aging more slowly; similarly to two people moving oppositely along a train track, seeing the other as appearing smaller in the distance.

Trace\[T_{\mu\nu}\] = \( \eta_{\mu\nu} T^{\mu\nu} = T^\nu_\nu = T \]

\[
V \cdot V = V^\mu V_\mu = (v^i)^2 - v \cdot v = (v^i)^2
\]

= Lorentz Scalar Invariant

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SRQM: Some Basic 4-Vectors

Lorentz Invariant d’Alembertian \((\partial \cdot \partial)\)

The d’Alembertian \(\{ \partial \cdot \partial = (\partial_t/c)^2 - \nabla \cdot \nabla \}\) is a 4D Lorentz Scalar Invariant

It is used as the basis of many “wave-type” equations.

\((\partial \cdot \partial)\phi[X] = (\partial \cdot \partial)\phi[t,x] = 0\) is the standard relativistic wave equation

\((\partial \cdot \partial)\mathbf{A}[X] = (\partial \cdot \partial)\mathbf{A}[t,x] = 0\) is the Maxwell EM Wave equation in Lorenz Gauge \((\partial \cdot \mathbf{A})=0\)

\((\partial \cdot \partial)\psi[X] = -(m_0 c/\hbar)^2 \psi[X]\) is the standard relativistic quantum Klein-Gordon equation

\((\partial \cdot \partial)\mathbf{G}[X-X'] = \delta^4[X-X'] : \mathbf{G}[X-X']\) is a 4D Green’s Function, \(\delta^4[X-X']\) is a 4D Dirac Delta function

\(\delta^4[X-X'] = 1/(2\pi)^4 \int d^4K e^{-iK\cdot(X-X')} = \delta[ct-ct'] \delta^{(3)}[x-x'] \delta[y-y'] \delta[z-z']\)

\(\delta^4[X-X'] = 1/\text{Vol} \delta^4[X-X'] d^4X = \{1 \text{ if } \mathbf{X}' \text{ is in the 4D Volume, 0 otherwise}\}\)

The Covariant 4D versions of the Green’s Function and the Dirac Delta Function.

Given a linear ordinary differential equation (ODE), \(L(\text{solution}) = \text{source}\), one can first solve \(L(\text{green}) = \delta[s]\), for each \(s\), and realizing that, since the source is a sum of delta functions, the solution is a sum of Green’s functions as well, by linearity of \(L\).
SRQM: Some Basic 4-Vectors

SR 4-WaveVector K

Solution to d’Alembertian (∂⋅∂)

\[ K = (ω/c)^2 U = (ω/c, k) = (ω/c, ωn/v_{\text{phase}}) = (ω/c, ωu/c') = (ω/c^2)(c, u) = (ω/c)(1, β) = (1/cτ, n/λ) \]

There are multiple ways of writing out the components of the 4-WaveVector, with each one giving an interesting take on what the 4-WaveVector means.

An SR wave \( \Psi \) is actually composed of two tensors:
1. 4-Vector propagation part = \( K^c \) (the engine), in \( e^{i(-ik\cdot x)} \)
2. Variable amplitude part = \( A \) (the load), depends on what is waving...

One comparison I find very interesting is:
\[ R \cdot R = (ct)^2 \Rightarrow K \cdot K = (1/ct^2)^2 \]
\[ \partial \partial \Rightarrow (\partial/c\partial t)^2 \Rightarrow (\partial/c\partial t)^2 \]

I believe the last one is correct: \( (\partial/\partial t)[R] = 0 = (\partial/c\partial t^2)[R] = A_0/c^2 = 0 \) The 4-Acceleration seen in the ProperTime Frame = RestFrame = 0 Normally \( (\partial/\partial t^2)[R] = A \), which could be non-zero. But that is for the total derivative, not the partial derivative.
SRQM: Some Basic 4-Vectors

4-Velocity, 4-WaveVector

Wave Properties, Relativistic Doppler Effect

The Phase Velocity of a Photon \( v_{\text{phase}} = c \) equals the Particle Velocity of a Photon \( v = c \).

The Phase Velocity of a Massive Particle \( v_{\text{phase}} > c \) is greater than the Velocity of a Massive Particle \( v < c \).

SR 4-Tensor

- \( (2,0) \)-Tensor \( T_{\mu}^{\nu} \)
- \( (1,1) \)-Tensor \( V_{\nu} = V = (v^\nu, v) \)
- \( (0,2) \)-Tensor \( T_{\mu}^{\nu} \)

SR 4-Vector

- \( (1,0) \)-Tensor \( V^{\nu} = V = (\omega, \gamma) \)

SR 4-Scalar

- \( (0,0) \)-Tensor \( S \) or \( S_0 \)
- Lorentz Scalar

SR 4-CoVector: One-Form

- \( (0,1) \)-Tensor \( V_{\nu} = V = (\omega, \gamma) \)

Relativistic SR Doppler Effect

Choose an observer frame for which:

\[ K = (\omega/c, k) \]

with \( k, \hat{n} \) pointing toward observer.

\[ U_{\text{obs}} = (c, 0) \quad K \cdot U_{\text{obs}} = (\omega/c, k) \cdot (c, 0) = \omega = \omega_{\text{obs}}^o \]

\[ U_{\text{emit}} = (c, 0) \quad K \cdot U_{\text{emit}} = (\omega/c, k) \gamma(c, 0) = \gamma(\omega - k \cdot u) = \omega_{\text{emit}}^o \]

\[ K \cdot U_{\text{obs}} / K \cdot U_{\text{emit}} = \omega_{\text{obs}}^o / \omega_{\text{emit}}^o = \omega / [\gamma(\omega - k \cdot u)] \]

For photons, \( K = 0 \rightarrow k = (\omega/c) \hat{n} \)

\[ \omega_{\text{obs}} / \omega_{\text{emit}} = \omega / [\gamma(\omega - (\omega/c) \hat{n} \cdot u)] = 1 / [\gamma(1 - \hat{n} \cdot \beta)] \]

\[ \omega_{\text{obs}} / \omega_{\text{emit}} = \gamma \omega_{\text{obs}}^o / (\gamma \omega_{\text{emit}}^o) = \omega_{\text{obs}}^o / \omega_{\text{emit}}^o \]

\[ \omega_{\text{obs}} = \omega_{\text{emit}} / [\gamma(1 - \hat{n} \cdot \beta)] = \omega_{\text{emit}}^o \sqrt{1 + |\beta|} / \sqrt{1 - |\beta|} \]

with \( \gamma = 1 / \sqrt{1 - |\beta|^2} = 1 / [\sqrt{1 + |\beta|} \sqrt{1 - |\beta|}] \)

For motion of emitter \( \beta \): (in observer frame of reference)

Away from obs, \( (\hat{n} \cdot \beta) = -\beta \), \( \omega_{\text{obs}} = \omega_{\text{emit}}^o \sqrt{1 + |\beta|} / \sqrt{1 - |\beta|} \) = \text{Red Shift}

Toward obs, \( (\hat{n} \cdot \beta) = +\beta \), \( \omega_{\text{obs}} = \omega_{\text{emit}}^o \sqrt{1 + |\beta|} / \sqrt{1 - |\beta|} \) = \text{Blue Shift}

Transverse, \( (\hat{n} \cdot \beta) = 0 \), \( \omega_{\text{obs}} = \omega_{\text{emit}}^o \gamma = \text{Transverse Doppler Shift} \)
SRQM: Some Basic 4-Vectors

4-Velocity, 4-WaveVector

Wave Properties, Relativistic Aberration

The Phase Velocity of a Photon \( \{ v_{\text{phase}} = c \} \) equals the Particle Velocity of a Photon \( \{ u = c \} \)
The Phase Velocity of a Massive Particle \( \{ v_{\text{phase}} > c \} \) is greater than the Velocity of a Massive Particle \( \{ u < c \} \)
SRQM: Some Basic 4-Vectors

4-Momentum, 4-WaveVector, 4-Position, 4-Velocity, 4-Gradient, Wave-Particle

See Hamilton-Jacobi Formulation of Mechanics
for info on the Lorentz Scalar Invariant SR Action. 
{ P = (E/c, p) = -∂[S] = (-∂[S], V[S]) }
{ temporal component } E = -∂t[S] = -∂[S] 
{ spatial component } p = V[S]

**Note** This is the Action (S) for a free particle. Generally Action is for the 4-TotalMomentum P of a system.

See SR Wave Definition
for info on the Lorentz Scalar Invariant SR WavePhase.
{ K = (ω/c, k) = -∂[Φ] = (-∂[Φ], V[Φ]) }
{ temporal component } ω = -∂t[Φ] = -∂[Φ] 
{ spatial component } k = V[Φ]

**Note** This is the Phase (Φ) for a single free plane-wave. Generally WavePhase is for the 4-TotalWaveVector of a system.

SR 4-Tensor
(2,0)-Tensor $T^{μν}$
(1,1)-Tensor $T^μν$, or $T_{μν}$
(0,2)-Tensor $T_{μν}$

SR 4-Vector
(1,0)-Tensor $V^μ = V = (v^μ, v^ν)$
SR 4-CoVector: OneForm
(0,1)-Tensor $V_ν = (v_ν, v_μ)$

SR 4-Scalar
(0,0)-Tensor $S$ or $S_ρ$ = Lorentz Scalar

Existing SR Rules
Quantum Principles

4-Vector SRQM Interpretation of QM

Hamilton-Jacobi
$P = hK$

Euler-Lagrange
$\frac{∂}{∂t}P \cdot U = E_μ$

Phase, Plane
$K \cdot U = ω_o$

Wave-Particle
$K = (ω/c, k)$

Rest Mass: Energy
E = $mc^2$

4-WaveVector
$K = (ω/c, k)$

Rest Angle Frequency
$ω_o = c^2/k$

4-Momentum
$P = (mc, p) = (E/c, p)$

4-Gradient
$d[Φ] = 4$-Gradient of Phase

δ[R] = $η^{μν}$ Diag[1,-1,-1,-1]

Minkowski Metric

Proper Time
$U \cdot ∂/∂t = γd/dt$

Derivative

SpaceTime
$∂ = δ[R] = 4$-Dimension

Diag

Diagonalization

Diag[1,-1,-1,-1]
Some Cool Minkowski Metric Tensor Tricks

4-Gradient, 4-Position, 4-Velocity

SpaceTime is 4D

\( \gamma \)-d/\( d\tau \) = \gamma d/dt

4-Gradient
\( \partial = \gamma (\mathbf{c}, \mathbf{u}) \)
\( \partial/\gamma (\mathbf{c}, \mathbf{u}) = (\partial/\gamma \mathbf{c}, \mathbf{u}) \)
\( \partial \mathbf{v} = \gamma \partial \mathbf{v} \)

4-Position
\( \mathbf{R} = (ct, r) \)
\( \mathbf{v} \cdot \mathbf{v} = \gamma v^2 \)

\( \eta_{\alpha\beta} \) (Minkowski Metric)
\( Tr[\eta] = 4 \)

4-Vector SRQM Interpretation of QM

A Tensor Study of Physical 4-Vectors

SR 4-Tensor
(2,0)-Tensor \( T^{\mu\nu} \)
(1,1)-Tensor \( T^{\mu}_\nu \), or \( T^\nu_\nu \)
(0,2)-Tensor \( T^\nu_\mu \)

SR 4-Vector
(1,0)-Tensor \( V^\nu \), or \( V^\nu \)
(0,1)-Tensor \( V_\nu \), or \( V_\nu \)

SR 4-Scalar
(0,0)-Tensor \( S \) or \( S \)

Lorentz Scalar

Interpretation

\( \gamma v^2 = \gamma \partial^2 \mathbf{v}/\gamma c^2 \)

\( \gamma \partial/\gamma d\tau = \gamma d/dt \)

\( \gamma = \sqrt{1 - v^2/c^2} \)

\( \eta_{\alpha\beta} \) (Minkowski Metric)

\( Tr[\eta] = 4 \)

\( \gamma \partial^2 \mathbf{v}/\gamma c^2 \)

\( \gamma \partial/\gamma d\tau \)

\( \gamma v^2 \)

\( \gamma = \sqrt{1 - v^2/c^2} \)

\( \gamma d/dt \)

Relativistic Factor

\( \gamma = \sqrt{1 - v^2/c^2} \)

\( \gamma d/dt \)

Lorentz Scalar

\( \gamma v^2 = \gamma \partial^2 \mathbf{v}/\gamma c^2 \)

\( \gamma \partial/\gamma d\tau = \gamma d/dt \)

\( \gamma v^2 \)

\( \gamma = \sqrt{1 - v^2/c^2} \)

\( \gamma d/dt \)

Relativistic Factor

\( \gamma = \sqrt{1 - v^2/c^2} \)

\( \gamma d/dt \)

Lorentz Scalar

\( \gamma v^2 = \gamma \partial^2 \mathbf{v}/\gamma c^2 \)

\( \gamma \partial/\gamma d\tau = \gamma d/dt \)

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Relativistic Factor

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Relativistic Factor

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\( \gamma d/dt \)

Lorentz Scalar

\( \gamma v^2 = \gamma \partial^2 \mathbf{v}/\gamma c^2 \)

\( \gamma \partial/\gamma d\tau = \gamma d/dt \)
SRQM+EM Diagram:

4-Vectors

- 4-Displacement: $\Delta R = (c\Delta t, \Delta r)$
- 4-Position: $R = (ct, r)$
- 4-Unit Temporal: $T = (\gamma, 1)$
- 4-Number Flux: $N = (nc, n) = n(c, u)$
- 4-Charge Flux: $J = (pc, j) = p(c, u)$
- 4-Prob Current Density: $J_{prob} = (\rho_c, j)$
- 4-Mass Flux: $G = (\rho_m, c) = (\rho, c)$
- 4-EM Vector Potential: $A = (\phi/c, a)$
- 4-Force Density: $F_{den} = (\gamma(E_{den}/c, f_{den}))$
- 4-Gradient: $\nabla = (\partial_t/c, -\nabla)$
- 4-Acceleration: $A = (\gamma(c\gamma', \gamma' u + \gamma a))$
- 4-Polarization: $E = (\epsilon', 1) = (\epsilon\beta, 1)$
- 4-Wave Vector: $K = (\omega/c, k)$
- 4-Total Wave Vector: $K_T = (\omega/c, k)$
- 4-Total Momentum: $P_T = (E/c, p_T) = (H/c, p_T)$
- 4-Force: $F = (\gamma(E/c, f = \dot{p})$
- 4-Total Momentum Field: $P_{f} = (E/c, p_f)$

4-Unit Spatial: $S = (\gamma n, n) = (n, n)$

4-Position: $R = (ct, r)$

4-Gradient: $\nabla = (\partial_t/c, -\nabla)$

4-Momentum: $P = (mc, p) = (E/c, p)$

SR 4-Tensor:
- (2,0)-Tensor $T^{uv}$
- (1,1)-Tensor $V = V = (v^0, v)$
- (0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector:
- (1,0)-Tensor $V^\mu = V = (v^0, v)$
- (0,1)-Tensor $V_\mu = (v_0, v)$

SR 4-Scalar:
- (0,0)-Tensor $S$ or $S_\mu$ Lorentz Scalar

SR → QM

Physics

A Tensor Study of Physical 4-Vectors

SRQM Interpretation

John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

(\epsilon, \mu, \nu, \rho, \sigma) ∈ T^{uv} = T^{\mu\nu} = T^{\rho\sigma} = T$

$V \cdot V = V^\mu V_\mu = (v_0)^2 - v \cdot v = (v_0)^2$

Lorentz Scalar Invariant

SRQM → EM

4-Vector QM Interpretation of SR

SciRealm.org

John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf
SRQM+EM Diagram:
4-Vectors, 4-Tensors

SR Perfect Fluid
\[ T^{\mu\nu} = \eta^{\mu\nu} T + \eta^{\mu\nu} - \eta^{\mu\nu} \]

StressEnergy 4-Tensor

4-Force
\[ F = \gamma(E/c, f = \dot{p}) \]

4-TotalMomentum
\[ P = (E/c, p) = (H/c, p) \]

4-MomentumField
\[ P_f = (E/c, p) = (P + Q)/c + qA \]

SR Perfect Fluid
\[ T^{\mu\nu} = \eta^{\mu\nu} T + \eta^{\mu\nu} - \eta^{\mu\nu} \]

StressEnergy 4-Tensor

4-Force
\[ F = \gamma(E/c, f = \dot{p}) \]

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SR Perfect Fluid
\[ T^{\mu\nu} = \eta^{\mu\nu} T + \eta^{\mu\nu} - \eta^{\mu\nu} \]

StressEnergy 4-Tensor

4-Force
\[ F = \gamma(E/c, f = \dot{p}) \]

4-TotalMomentum
\[ P = (E/c, p) = (H/c, p) \]

4-MomentumField
\[ P_f = (E/c, p) = (P + Q)/c + qA \]
SRQM Diagram: Physical Constants Emphasized

\[ K \cdot R = -\Phi_{\text{phase}} \]

\[ P_T \cdot R = -S_{\text{action}} \]

\[ \text{SR Conservation of StressEnergy if } F_{\text{den}} = 0 \]

\[ \partial \cdot G_{\mu\nu} = 0 \]

8\pi G/c^4

\[ E_0/\omega_0 = \eta_{\mu\nu} \]

\[ \text{SR Conservation of Einstein Tensor} \]

\[ \text{SR Conservation of EM Field} \]

\[ \partial \cdot J = 0 \]

\[ \text{Conservation of Particle# : Probability} \]

\[ \partial \cdot A = 0 \]

\[ \text{Conservation of EM Field} = \text{Lorentz Gauge} \]

Notice that all the main “Universal” or “Fundamental” Physical Constants are here: G, c, \( h \), \( \varepsilon \), \( \mu \).

Some depend on the actual particle type: q, m, \( \omega_0 \)
Some depend on regional conditions: \( \tau, \rho \), \( \rho_0, \psi \), \( \psi^* \psi \)
Some depend on interaction: \( \Phi_{\text{phase}}, S_{\text{action}} \)

Some are mathematical: 0, 4, \( \pi, [\text{Diag}[1, -1, -1, -1]], d/d\tau \)

Conservation Laws are also a type of “zero” constant in this regard.

The majority of the constants are Lorentz Scalars, but some are 4-Vector or 4-Tensor, and all are valid for all inertial observers.

Fundamental Physical Constants are SR Lorentz Scalars

The fact that these “tie together” a network of 4-Vectors is a good argument for why their values are constant. Changing even one would change the relationship properties among all of the 4-Vectors.
SRQM Diagram: Projection Tensors
Temporal, Spatial, Null, SpaceTime

Projection Tensors act as follows:
Generic 4-Vector: \( A^\mu = (a^0, a^1, a^2, a^3) \)

Temporal Projection: \( V^\mu_\nu = \eta^\mu_\nu V^\nu_\sigma \rightarrow \text{Diag}[1,0,0,0] \)
\( V^\mu_\nu A^\nu = (a^0,0,0,0) = (a^0,0) \)

Spatial Projection: \( H^\mu_\nu = \eta^\mu_\nu H^\nu_\sigma \rightarrow \text{Diag}[0,1,1,1] \)
\( H^\mu_\nu A^\nu = (0,a^1,a^2,a^3) = (0,a) \)

SpaceTime Projection: \( V^\mu_\nu + H^\mu_\nu A^\nu = \eta^\mu_\nu A^\nu = (a^0,a) \)

\( V^\mu_\nu + H^\mu_\nu = \eta^\mu_\nu = \delta^\mu_\nu \)
\( V^\mu_\nu + H^\mu_\nu = \eta^\mu_\nu \)
The Minkowski Metric Tensor is the Sum of Temporal & Spatial Projection Tensors, all of which are dimensionless.
SRQM Diagram: Projection Tensors & Perfect-Fluid Stress-Energy Tensor

Projection Tensors act as follows:
- \( \Delta t \):
  - Time-like Interval (\( t_2 - t_1 \))
  - "Vertical" Temporal Projection
- \( \Delta r \):
  - Space-like Interval (\( r_2 - r_1 \))
  - "Horizontal" Spatial Projection
- \( \Delta s \):
  - Null Interval (\( s_2 - s_1 = 0 \))
  - "Null" Projection

Perfect-Fluid Stress-Energy 4-Tensor:

\[ T^{\mu\nu} = ((\rho p_o + p_c/c^2)u^\mu u^\nu - (p_o)\eta^{\mu\nu}) \]

\( T^\mu = (\rho p_o)\eta^\mu - (p_o)H^\mu \)

\( T^\mu = (\rho p_o)V^\mu + (-p_o)H^\mu \)

\( \Delta t = (V^\mu)^+ \)

\( \Delta r = (V^\mu)^- \)

\( \Delta s = (V^\mu)^0 \)

Conservation of rest-energy-density:

\( \partial \cdot T^\mu = 0 \)

Perfect-Fluid Stress-Energy 4-Tensor:

\( T^\mu_{\text{rest}} \rightarrow \text{Diag}[\rho p_o, \rho p_o, \rho p_o] \)

\( T^\mu = (\rho p_o)\eta^\mu + (-p_o)H^\mu \)

Perfect-Fluid Stress-Energy 4-Tensor:

\( T^\mu = (\rho p_o)V^\mu + (-p_o)H^\mu \)

SR 4-Tensor
- (2,0)-Tensor \( T^\mu_{\text{SR}} \)
- (1,1)-Tensor \( T^\mu_{\text{SR}} \)
- (0,2)-Tensor \( T^\mu_{\text{SR}} \)

SR 4-Vector:
- (1,0)-Tensor \( V^\mu = (v^\nu, 0) \)

SR 4-Scalar:
- (0,0)-Tensor \( S_{\text{SR}} \)

SR 4-CoVector:
- (OneForm) \( V_\nu = (v_\mu, 0, 0) \)

Conservation of stress-energy:

\( \partial \cdot T^\mu = 0 \)

\( \Delta t = (V^\mu)^+ \)

\( \Delta r = (V^\mu)^- \)

\( \Delta s = (V^\mu)^0 \)

Lorentz Scalar Invariant:

\( [V \cdot V] = (V^\mu)^+ (V^\nu)^- \cdot (V^\mu)^+ (V^\nu)^- = (V^\mu V^\nu)^2 \)
Matter-Dust is a special case of a perfect fluid with time-like 4-Velocity $U$, energy density $\rho$, and zero pressure $p$, described by the energy-momentum tensor $T_{\mu\nu} = \rho U^\mu U^\nu$. Because there is no pressure gradient, the fluid elements of the dust follow time-like geodesics.

Null-Dust corresponds to the limit in which the 4-Velocity $U$ becomes null, and is described by $T_{(null dust)} = \rho K \otimes K$, $K \cdot K = 0$, with $p > 0$ and trace[$T$] = 0. The elements of dust follow null geodesics.

Null-Dust is interpreted as a coherent zero-rest-mass field propagating at the speed of light ($c$) in the null direction $K$. A null-dust can describe propagating electromagnetic (EM) or gravitational waves.

Note: There is an unfortunate slight notational clash between:

4-(Dust)NumberFlux $N = (nc, nu) = n_o U$

4-“Unit”Null $N = (1, \pm \eta)$

Use colored overbars on the “Unit” 4-Vectors $T =$ Temporal : $N =$ Null : $S =$ Spatial

4-Momentum

$P^\mu = E/c = mc, p = nu = m_o U$

4-(Dust)NumberFlux $N = N^\mu = (nc, nu) = n_o U$

4-Velocity

$U = U^\mu c = \gamma(1, u) = c T$

$\gamma = \sqrt{1 - \frac{v^2}{c^2}}$

4-UnitTemporal $T = T^\mu = \gamma(1, \beta)$

$= \gamma(1, u/c) = U/c$

4-“Unit”Null $N = N^\mu = (1, \pm \eta)$

$\sim K_{\text{photonic}} = (|k|, k)$

4-NumberFlux $N^\mu = N_0^\mu$

$\rightarrow \text{Diag}[c^2, 0, 0, 0]_{\text{MCRF}}$

$\rightarrow \text{Diag}[1, 0, 0, 0]_{\text{MCRF}}$

Temporal “Vertical” Projection Tensor

$V^\mu = T^\mu T^\nu$

$\rightarrow \text{Diag}[1, 0, 0, 0]_{\text{MCRF}}$

$\rightarrow \text{Diag}[c^2, 0, 0, 0]_{\text{MCRF}}$

$\sim K_{\text{photonic}} = (|k|, k)$

4-“Unit”Null $N = N^\mu = (1, \pm \eta)$

$\rightarrow \text{Diag}[c^2, 0, 0, 0]_{\text{MCRF}}$
SRQM Study: Physical 4-Tensors

Vacuums vs. Fluids vs. Dusts

Isotropy Group SO(3)=Ordinary Rotation Group
Segre Type {1,(1,1)}

Lambda Vacuum
\[ T^{\mu \nu} \rightarrow (\rho_{eo})\eta^{\mu \nu} \rightarrow (\text{MCRF}) \]

Perfect Fluid Stress-Energy
\[ T^{\mu \nu} \rightarrow (\rho_{eo})V^{\mu \nu} + (-\rho_{eo})H^{\mu \nu} \rightarrow (\text{MCRF}) \]

Fluids (have pressure \( p \))

Dusts (no pressure, \( p=0 \))

Null-Dust: "Gravitational Wave"
Incoherent EM Mixture
\[ T^{\mu \nu} \rightarrow \Phi_0 N^\mu N^\nu = \Phi_0 N^\mu \rightarrow (\text{MCRF}, z=0 \text{ rest frame}) \]

Stress-Energy 4-Tensor
Symmetric, Null

Equation of State
\[ EoS[T^{\mu \nu}] = \rho = \rho_{eo} \]

Trace\[ T^{\mu \nu} = \eta_{\mu \nu} T^{\mu \nu} = T^{\mu \nu} \rightarrow T = VV = V \cdot V = (\mathbf{v})^2 \cdot (\mathbf{v}) = (\mathbf{v})^2 \]

SR 4-Tensor

(2,0)-Tensor \( T^{\mu \nu} \)
(1,0)-Tensor \( V^\nu = \mathbf{V} = (\mathbf{v}^\nu) \)
(0,1)-Tensor \( V_\nu = (\mathbf{v}_\nu) \)

SR 4-Vector

(0,0)-Tensor Recommended for 4-Vector\( V^\nu = \mathbf{V} = (\mathbf{v}^\nu) \)

SR 4-Scalar

(0,0)-Tensor \( S_\nu = (\mathbf{s}_\nu) \)

Lorentz Scalar

\[ \text{SRQM Study of Physical 4-Vectors} \]
SRQM Diagram:
4-Tensors and 4-Scalars
generated from 4-Vectors

All SR 4-Tensors can be generated from SR 4-Vectors:

- $F^{\mu\nu} = \partial^A \partial^\mu A^\nu - \partial^\nu A^\mu$ : Faraday EM 4-Tensor (from the 4-Gradient & 4-EMVectorPotential)
- $M^{\mu\nu} = X^P \partial^\mu P^\nu - X^\nu \partial^\nu P^\mu$ : 4-AngularMomentum 4-Tensor (from the 4-Position & 4-Momentum)
- $\eta^{\mu\nu} = \partial[R] = \partial^\mu [R^\nu]$ : SR:Minkowski Metric 4-Tensor (from the 4-Gradient & 4-Position)
- $V^{\mu\nu} = T \otimes T = T^\nu T^\mu$ : (V)ertical:Temporal Projection 4-Tensor (from the 4-UnitTemporal)
- $H^{\mu\nu} = \eta^{\mu\nu} - V^{\mu\nu} = \eta^{\mu\nu} - T^\nu T^\mu$ : (H)orizontal:Spatial Projection 4-Tensor (from previously made 4-Tensors above)
- $T_{\text{cold dust}}^{\mu\nu} = P \otimes N = P^{\mu} N^{\nu}$ : (Cold)Dust Stress-Energy 4-Tensor (from the 4-Momentum & 4-DustNumberFlux)
- $(\rho_{eo}) = T_{\text{Cold dust}}^{\mu\nu} V_{\mu\nu}$ : MCRF EnergyDensity 4-Scalar (from previously made 4-Tensors above)
- $T_{\text{Lambda Vacuum}}^{\mu\nu} = (\rho_{eo}) \eta^{\mu\nu}$ : LambdaVacuum (Dark Energy) Stress-Energy 4-Tensor (from previously made 4-Tensors above)
- $(p_o) = (k)(1/3) T_{\text{Lambda Vacuum}}^{\mu\nu} H_{\mu\nu}$ : MCRF Pressure 4-Scalar (from previously made 4-Tensors above)
- $T_{\text{Perfect Fluid}}^{\mu\nu} = (\rho_{eo}) V^{\mu\nu} + (-p_o) H^{\mu\nu}$ : PerfectFluid Stress-Energy 4-Tensor (from previously made 4-Tensors above)

Equation of State
$EoS[T^{\mu\nu}] = w = p_o/\rho_{eo}$

Trace$[T^{\mu\nu}] = \eta^{\mu\nu} T^{\mu\nu} = T^\nu = T$,
$V \cdot V = V^\nu \eta^{\nu\nu} V^\nu = (v^\mu)^2 - v^\mu v^\mu = (v_0)^2$ = Lorentz Scalar Invariant
SRQM Study: 4D Gauss’ Theorem

4D Gauss' Theorem in Special Relativity
\[ \int_{\Omega} d^4x \left( \partial_{\mu} V^\mu \right) = \oint_{\partial \Omega} dS \left( V \cdot N \right) \]

with:
\( V = V^\mu \) is a 4-Vector field defined in 4D Minkowski Region \( \Omega \)
\( (\partial \cdot V) = (\partial_{\mu} V^\mu) \) is the 4-Divergence of \( V \)
\( (V \cdot N) = (V^\mu N_\mu) \) is the component of \( V \) along the boundary normal \( N \)-direction
\( \Omega \) is a 4D simply-connected region of Minkowski SpaceTime
\( \partial \Omega = S \) is its 3D boundary with its own 3D Volume element \( dS \) and outward pointing normal \( N \).
\( N = N^\mu \) is the outward-pointing normal of the boundary
\( d^4x = (c \, dt)(dx \, dy \, dz) \) is the 4D differential volume element

In vector calculus, the divergence theorem, also known as Gauss's theorem or Ostrogradsky's theorem, is a result that relates the flow (that is, flux) of a vector field through a surface to the behavior of the vector field inside the surface. More precisely, the divergence theorem states that the outward flux of a vector field through a closed surface is equal to the volume integral of the divergence over the region inside the surface. Intuitively, it states that the sum of all sources minus the sum of all sinks gives the net flow out of a region. In vector calculus, and more generally in differential geometry, the generalized Stokes' theorem is a statement about the integration of differential forms on manifolds, which both simplifies and generalizes several theorems from vector calculus.
SRQM Diagram:
Minimal Coupling = (EM)Potential Interaction
Conservation of 4-TotalMomentum

\[ \Delta R = (c \Delta t, \Delta r) \]
\[ dR = (cdt, ddr) \]

4-Displacement

\[ U = \gamma(c, \mathbf{u}) \]
\[ \gamma ddt [\ldots] \]
\[ ddr [\ldots] \]

4-Velocity

\[ \varphi = \frac{\mathbf{U} \cdot \partial}{c^2} \]
\[ \eta = \frac{\mathbf{U} \cdot \partial}{c^2} \]

4-Displacement Potential

\[ \mathbf{P} = (E/c, \mathbf{p}) \]
\[ \mathbf{Q} = (V/c, \mathbf{q}) = q \mathbf{A} \]
\[ \mathbf{A} = (\varphi/c, \mathbf{a}) \]
\[ \mathbf{P}_f = (E/c, \mathbf{p}_f) \]
\[ \mathbf{P}_t = (E/c, \mathbf{p}_t) = (H/c, \mathbf{p}_t) = \sum |\mathbf{P} |_t \]

4-Momentum

\[ \mathbf{P} = \mathbf{P}_f - q \mathbf{A} = (E/c-\varphi/c, \mathbf{p}_f-\mathbf{q}a) \]

Minimal Coupling Relation

\[ \mathbf{P}_f = \mathbf{P} + \mathbf{Q} = \mathbf{P} + q \mathbf{A} \]

Conservation of 4-MomentumIncPotentialField

\[ \mathbf{P}_f = \mathbf{P} + q \mathbf{A} \]
\[ \mathbf{P}_f = (m_0) \mathbf{U} + (q \varphi/c) \mathbf{U} \]
\[ \mathbf{P}_f = (E/c^2) \mathbf{U} + (q \varphi/c^2) \mathbf{U} \]
\[ \mathbf{P}_f = ((E+q \varphi/c^2) \gamma(c, \mathbf{u}) \mathbf{U} \]
\[ \mathbf{P}_f = ((E+q \varphi/c) \gamma(c, \mathbf{u}) \mathbf{U} \]
\[ \mathbf{P}_f = ((E+q \varphi)/c, \mathbf{p}+qa) \]

4-MomentumIncPotentialField has a contribution from:

- a Mass "charge" \((m_0)\)
- an EM charge \((q)\) interacting with a potential \((\varphi/c)\)

\[ \mathbf{P}_t = \Sigma |\mathbf{P} |_t \]

Conservation of 4-TotalMomentum

4-TotalMomentum is the Sum over all such 4-Momenta

\[ \mathbf{P}_f = (E/c, \mathbf{p}_f) = (H/c, \mathbf{p}_f) \]

4-TotalMomentum

\[ \mathbf{P}_f = \mathbf{P} + \mathbf{Q} = \mathbf{P} + q \mathbf{A} \]

Minimal Coupling \(\mathbf{P}_f = \mathbf{P} + q \mathbf{A}\)

\[ \text{EM Charge} \]
\[ \mathbf{q} \]

\[ \mathbf{Q} = (U/c, \mathbf{q}) = q \mathbf{A} \]

4-EMPotentialMomentum

\[ \text{Trace} [\mathbf{T}^{\mu\nu}] = \eta^{\mu\nu} T^{\nu\mu} = T^{\nu\mu} \]
\[ V \cdot V = V^{\mu} \eta^{\mu\nu} V^{\nu} = (V^0)^2 - V \cdot V = (V^0)^2 \]

Lorentz Scalar Invariant

\[ \text{SR 4-Tensor} \]
\[ \text{SR 4-Vector} \]
\[ \text{SR 4-CoVector:OneForm} \]
\[ \text{SR 4-Scalar} \]
4-Momentum $P = m_0 U = (E_0/c^2) U$; 4-Vector Potential $A = (\phi_0/c^2) U$

4-Total Momentum $P_T = (P + qA) = (H/c = E_0/c + (q + q\varphi)/c, p_T = p + qa)$

$P \cdot U = \gamma(E - p \cdot u) = E_0 = m_0 c^2; A \cdot U = \gamma(\varphi - a \cdot u) = \varphi_0$

$P_T \cdot U = (P \cdot U + qA \cdot U) = E_0 + q\varphi_0 = m_0 c^2 + q\varphi_0$

$\gamma = 1/\sqrt{1 - \beta^2}$: Relativistic Gamma Identity

$(\gamma - 1/\gamma) = (\gamma\beta \cdot \beta)$: Manipulate into this form... still an identity

$(\gamma - 1/\gamma)(P_T \cdot U) = (\gamma\beta \cdot \beta)(P_T \cdot U)$: Still covariant with Lorentz Scalar

$\gamma(P_T \cdot U) + -(P_T \cdot U)/\gamma = (\gamma\beta \cdot \beta)(P_T \cdot U)$

$\gamma(P_T \cdot U) + -(P_T \cdot U)/\gamma = (\gamma\beta \cdot \beta)(E_0 + q\varphi_0)$

$\gamma(P_T \cdot U) + -(P_T \cdot U)/\gamma = \gamma(E_0/c^2 + q\varphi_0/c^2)(u \cdot u)$

$\gamma(P_T \cdot U) + -(P_T \cdot U)/\gamma = ((\gamma E_0/c + q\varphi_0/c^2)(u \cdot u)$

$\gamma(P_T \cdot U) + -(P_T \cdot U)/\gamma = ((E_0/c^2 + q\varphi_0/c^2)(u \cdot u)$

$\gamma(P_T \cdot U) + -(P_T \cdot U)/\gamma = ((p + qa)\cdot u)$

$\gamma(P_T \cdot U) + -(P_T \cdot U)/\gamma = (p_T \cdot u)$

$\{ H \} + \{ L \} = (p_T \cdot u)$: The Hamiltonian/Lagrangian connection

$H = \gamma(P_T \cdot U) = \gamma((P + qa) \cdot U) = The \ Hamiltonian \ with \ minimal \ coupling$

$L = -(P_T \cdot U)/\gamma = -((P + qa) \cdot U)/\gamma = The \ Lagrangian \ with \ minimal \ coupling$

$H + L = (p_T \cdot u) = \gamma(P_T \cdot U) + -(P_T \cdot U)/\gamma$

4-Vector notation gives a very nice way to find the Hamiltonian/Lagrangian connection: $(H) + (L) = (p_T \cdot u)$, where $H = \gamma(P_T \cdot U)$ & $L = -(P_T \cdot U)/\gamma$
4-Vector SRQM Interpretation of QM

SciRealms.org
John B. Wilson
SciRealms@aol.com
http://scirealm.org/SRQM.pdf

SRQM Study: SRQM Hamiltonian: Lagrangian Connection

\[ H + L = (p_T \cdot u) = \gamma(P_T \cdot U) + -(P_T \cdot U)/\gamma \]

The non-relativistic Hamiltonian \( H \) is an approximation of the relativistic \( H \):
\[ H = (m_c^2 + q\Phi) \]

The non-relativistic Lagrangian \( L \) is an approximation of the relativistic \( L \):
\[ L = -(P_T \cdot U)/\gamma \]

Thus, \( H + L = \gamma(P_T \cdot U) \) & \( L = -(P_T \cdot U)/\gamma \)

Vector notation gives a very nice way to find the Hamiltonian/Lagrangian connection:
\[ (H) + (L) = (P_T \cdot u), \quad H = \gamma(P_T \cdot U) \quad \text{&} \quad L = -(P_T \cdot U)/\gamma \]
SRQM Study: 

SR Lagrangian, Lagrangian Density, and Relativistic Action (S)

Relativistic Action (S) is Lorentz Scalar Invariant

\[
S = \int \left( \frac{L}{c} \right) \left( \gamma \frac{dt}{c} \right) = \int (L_0) (d^4x) \\
S = \int \left( \frac{\mathcal{L}}{n} \right) dt = \int \mathcal{L} (n) dt = \int \mathcal{L} (d^3x) dt = \int (\mathcal{L}/c) (d^3x) (cdt) = \int (\mathcal{L}/c) (d^4x)
\]

Explicitly-Covariant Relativistic Action (S)

<table>
<thead>
<tr>
<th>Particle Form</th>
<th>Density Form (( n_* )Particle)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S = \int L_0 d\tau = -\int H_0 d\tau )</td>
<td>( S = (1/c) \int n_0 L_0 (d^4x) = -(1/c) \int n_0 H_0 (d^4x) )</td>
</tr>
<tr>
<td>( S = -\int (P_T \cdot U) dt )</td>
<td>( S = (1/c) \int (\mathcal{L}/c) (d^4x) )</td>
</tr>
<tr>
<td>( S = -\int (P \cdot dR/d\tau) dt )</td>
<td>( S = \int (\mathcal{L}/c) (d^4x) )</td>
</tr>
</tbody>
</table>

Lagrangian \( \{ L = (p_T \cdot u) - H \} \) is not* Lorentz Scalar Invariant

Rest Lagrangian \( \{ L = \gamma L = -(P_T \cdot U) \} \) is Lorentz Scalar Invariant

Lagrangian Density \( \{ \mathcal{L} = nL = (\gamma n_0) (L_0/\gamma) = n_0 L_0 \} \) is Lorentz Scalar Invariant

\( n = \gamma n_0 = \#/(dx)(dy)(dz) \) = number density
\( dt = \gamma dt \)
\( cdt = n_0 (cdt)(dx)(dy)(dz) = n_0 (d^4x) \)
\( d\tau = (n_0/c)(d^4x) \)

H-L Connection in Density Format for Photonic System (no rest-frame)

\( H + L = (p_T \cdot u) \)
\( nH + nL = n (p_T \cdot u) \), with number density \( n = n_0 \)
\( \mathcal{H} + \mathcal{L} = (g_T \cdot u) \), with
momentum density \( \{ g_T = \epsilon_T \} \)
Hamiltonian density \( \{ \mathcal{H} = nH \} \)

Lagrangian Density \( \{ \mathcal{L} = nL = (\gamma n_0) (L_0/\gamma) = n_0 L_0 \} \)
Lagrangian Density is Lorentz Scalar

for an EM field (photonic):
\[
\mathcal{H} = (1/2) \{ \epsilon \cdot e + \epsilon \cdot e \cdot \mu_b + \mu_b \cdot \mu_b \} = n \epsilon_0 \epsilon + \rho_{\epsilon_0} = EM Field Energy Density
\]
\( \mathcal{L} = (1/2) \{ \epsilon \cdot e + \epsilon \cdot e \cdot \mu_b + \mu_b \cdot \mu_b \} = (-1/4 \mu_b) F_{\mu \nu} F^{\mu \nu} = (-1/4 \mu_b) EM Tensor Inner Product
\]
\( \mathcal{H} + \mathcal{L} = \epsilon \cdot e + (g_T \cdot u) \)
\[
|u| = c \\
|g_T| = \epsilon_0 \epsilon c \\
Poynting Vector \( |s| = |g||c| \rightarrow c \epsilon_0 \epsilon \cdot e \\
\epsilon_0 \mu_0 = 1/c^2 \) : Electric:Magnetic Constant Eqn
SRQM Study:
SR Hamilton-Jacobi Equation and Relativistic Action (S)

Lagrangian \{ L = (p_T \cdot u) - H = -(p_T \cdot u)/\gamma \} is *not* a Lorentz Scalar
Rest Lagrangian \{ L_0 = \gamma L = -(p_T \cdot u) \} is a Lorentz Scalar

Relativistic Action (S) is Lorentz Scalar
S = \int L dt
S = \int(L_0/\gamma)(\gamma d\tau)
S = \int(L_0)(d\tau)

Explicitly Covariant
Relativistic Action (S)
S = \int L_0 dt = \int H_0 d\tau
S = -\int(p_T \cdot U)d\tau
S = -\int(p_T \cdot dR/d\tau)d\tau
S = -\int(p_T \cdot dR)d\tau
S = -\int((P + qA) \cdot U)d\tau
S = -\int(p \cdot U + qA \cdot U)d\tau
S = -\int(E_0 + q\phi_0)d\tau
S = -\int(E_0 + V)d\tau \quad \text{with } V = q\phi_0
S = -\int(m_0 c^2 + V)d\tau
S = -\int(H_0)d\tau

The Hamilton-Jacobi Equation is incredibly simple in 4-Vector form

Hamilton-Jacobi Equation
\[ \partial[S] = -\partial[S] = P_T \]

4-Vectors
Relativistic Hamilton-Jacobi Equation
Differential Format
\[ \partial[S] = -(E_0 + q\phi_0)(\tau + \text{const}) \]
\[ \partial[S] = (E_0 + q\phi_0)(\tau + \text{const}) \]
\[ \partial[S] = (E_0 + q\phi_0)/\gamma \]
\[ \partial[S] = (E_0 + q\phi_0)/c^2 \]
\[ \partial[S] = (m_0 + q\phi_0/c^2)U \]
\[ \partial[S] = P_T + qA \]
\[ \partial[S] = \text{Verified!} \]

4-TotalMomentum
\[ P_T = (E/c = H/c, p_T) \]
\[ = -\partial[S_{action}] \]
\[ = \partial/[S_{action}], \nabla[S_{action}] \]

Inverse
\[ R \cdot U = c^2 \tau : \tau = R \cdot U/c^2 \]

SRQM: A treatise of SR → QM by John B. Wilson (SciRealm@aol.com)
SRQM Diagram: Relativistic Hamilton-Jacobi Equation

\[ P_T = -\partial[S] \] Differential Format: 4-Vectors

\[ \partial(t)R = 4 \text{Displacement} \]
\[ \Delta R = (c\Delta t, \Delta r) \]
\[ dR = (cdt, dr) \]
\[ 4-Displacement \]
\[ d\tau = (1/c)\sqrt{dR \cdot dR} \]
\[ U \cdot R = c^2 \tau \]
\[ \text{ProperTime Derivative} \]
\[ d\tau = (1/c)\sqrt{dR \cdot dR} \]
\[ 4-Velocity \]
\[ U = \gamma(c,u) \]
\[ n_0 \]
\[ \rho_0 \]
\[ E_0 / c^2 \]
\[ m_0 \]

\[ 4-NumberFlux \]
\[ N = (nc, n) = \gamma(c, u) \]
\[ 4-ChargeFlux \]
\[ J = (pc, j) = \rho(c, u) \]
\[ 4-CurrentDensity \]
\[ J = \mathbf{V} \cdot \mathbf{S} = \rho \mathbf{V} \cdot \mathbf{E} \]

\[ 4-ChargeFlux \]
\[ J = (pc, j) = \rho \mathbf{V} \cdot \mathbf{E} \]
\[ 4-CurrentDensity \]
\[ J = \mathbf{V} \cdot \mathbf{S} = \rho \mathbf{V} \cdot \mathbf{E} \]

\[ 4-Vector \]
\[ (E/c, p) = \gamma(E/c, p) \]
\[ 4-Momentum \]
\[ P = (mc, p) = (E/c, p) \]
\[ 4-EMVectorPotential \]
\[ A = (\varphi/c, a) \]

\[ 4-EMPotentialMomentum \]
\[ Q = (U/c, q) = qA \]
\[ 4-Force \]
\[ F = \gamma(E/c, f) = qA \]

\[ 4-TotalMomentum \]
\[ P_T = (E/c, p) = (H/c, p) \]
\[ 4-TotalMomentumIncField \]
\[ P_T = (E/c, p) = (E + U)/c, p + qa \]

\[ 4-Force \]
\[ F = \gamma(E/c, f) = qA \]

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\[ P_T = (E/c, p) = (E + U)/c, p + qa \]

\[ 4-Force \]
\[ F = \gamma(E/c, f) = qA \]
SRQM Diagram:

Relativistic Euler-Lagrange Equation

The Easy Derivation \( \mathbf{U}=(\frac{d}{d\tau})\mathbf{R} \rightarrow (\partial_R=(\frac{d}{d\tau})\partial_\mathbf{U}) \)

Note Similarity:

4-Velocity is ProperTime Derivative of 4-Position
\[ \mathbf{U}=(\frac{d}{d\tau})\mathbf{R} \quad [\text{m/s}] = [1/s]^*[\text{m}] \]

Relativistic Euler-Lagrange Eqn
\[ \partial_R= (\frac{d}{d\tau})\partial_\mathbf{U} \quad [1/m] = [1/s]^*[s/m] \]

The differential form just inverses the dimensional units, so the placement of the \( \mathbf{R} \) and \( \mathbf{U} \) switch.

That is it: so simple! Much, much easier than how I was taught in Grad School.

To complete the process and create the Equations of Motion, one just applies the base form to a Lagrangian.

This can be: a classical Lagrangian, a relativistic Lagrangian, a Lorentz scalar Lagrangian, or a quantum Lagrangian.
SRQM Diagram:

Relativistic Euler-Lagrange Equation: Alternate Forms: Particle vs. Density

4-Velocity $\mathbf{U}$ is ProperTime Derivative of 4-Position $\mathbf{R}$. The Euler-Lagrange Eqn can be generated by taking the differential form of the same equation.

Relativistic 4-Vector Kinematical Eqn

$\mathbf{U} = (d/d\tau)\mathbf{R}$

$\mathbf{U} \cdot \mathbf{K} = (d/d\tau) \mathbf{R} \cdot \mathbf{K}$

Relativistic Euler-Lagrange Eqns (uses gradient-type 4-Vectors)

$\partial_R = (d/d\tau) \partial_U$: {particle format}

$\partial_R \cdot \partial_U = (d/d\tau) \partial_U [\mathbf{R} \cdot \mathbf{K}]$

$\partial_R \cdot \partial_U = (d/d\tau) \partial_U [\mathbf{U} \cdot \mathbf{K}]$

$\partial_R \cdot \partial_U = (d/d\tau) \partial_U [\mathbf{U} \cdot \mathbf{K}]$

$\partial_R \cdot \partial_U = (d/d\tau) \partial_U [\mathbf{U} \cdot \mathbf{K}]$

$\partial_R \cdot \partial_U = (d/d\tau) \partial_U [\mathbf{U} \cdot \mathbf{K}]$

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$\partial_R \cdot \partial_U = (d/d\tau) \partial_U [\mathbf{U} \cdot \mathbf{K}]$

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$\partial_R \cdot \partial_U = (d/d\tau) \partial_U [\mathbf{U} \cdot \mathbf{K}]$
SRQM Diagram:  
Relativistic Euler-Lagrange Equation of Motion (EoM) for EM particle

\[ \gamma = 1/\sqrt{1 - \beta^2}; \text{ Relativistic Gamma Identity} \]
\[ (\gamma - 1/\gamma) = (\gamma^{-1} - 1) \]
Manipulate into this form... still an identity
\[ \gamma(P_U + P_T) = \gamma - (P_U + P_T) \]
\[ \gamma(P_U + P_T) = (P_U - P_T) \]
\( H = \{ H \} + \{ L \} \)
The Hamiltonian/Lagrangian connection
\( H = \gamma H_U = \gamma(P_U + P_T) \)
\( L = L_U = -L \)
\( H = \{ H \} \)
\( L = \{ L \} \)

4-Vector SRQM Interpretation of QM

\[ H = \gamma H_U = \gamma(P_U + P_T) \]
The Hamiltonian with minimal coupling
\[ L = L_U = -L \]
The Lagrangian with minimal coupling
\( H = \{ H \} \)
\( L = \{ L \} \)

Rest Lagrangian \( L_o \)
\[ = -(P_T - U) \]
\[ = -\gamma(P_T - U) \]
\[ = -\gamma(P + qA) \]
\[ = -P \cdot U - qA \cdot U \]

4-TotalMomentum
\[ P_T = (E/c, p_T) \]
\[ = (H/c, p_T) \]
\[ = P + qA \]

4-Velocity
\[ U = \gamma(c, u) \]
\[ = \gamma(c, u) \]

4-Velocity Gradient
\[ \partial U = (\partial_U/c, -\nabla U) \]
\[ \gamma/(d/dt) \]
\[ = (\partial_U/c, -\nabla U) \]
\[ \gamma/(d/dt) \]

4-Velocity Gradient part
\[ (d/dt)\partial U[L_0] = \gamma \partial U[L_0] \]
\[ = (d/dt)\partial U[L_0] \]
\[ = (d/dt)\partial U[L_0] \]

Relativistic Euler-Lagrange Eqn
\[ \partial_\alpha = (d/dt)\partial_U \]
\[ = (d/dt)\partial U \]
\[ = (d/dt)\partial U \]
\[ = (d/dt)\partial U \]

Relativistic 4-Vector
\[ U = (d/dt)R \]
\[ = (d/dt)R \]
\[ = (d/dt)R \]

Relativistic 4-Vector
\[ F_m = \gamma((u \cdot e)c, (e) + (u \times b)) \]
\[ = (-\nabla \phi - \partial a) \]
\[ = (\nabla \phi + \partial a) \]

EM Faraday 4-Vector Potential
\[ F_m = \phi \partial_\alpha \wedge \partial_\beta A^\alpha \]
\[ = [0, -e/c, 0, -e/b] \]

4-Fourier \[ \gamma = 1/\sqrt{1 - \beta^2}; \text{ Relativistic Gamma Identity} \]
\[ (\gamma - 1/\gamma) = (\gamma^{-1} - 1) \]
Manipulate into this form... still an identity
\[ \gamma(P_U + P_T) = \gamma - (P_U + P_T) \]
\[ \gamma(P_U + P_T) = (P_U - P_T) \]
\( H = \{ H \} + \{ L \} \)
The Hamiltonian/Lagrangian connection
\( H = \gamma H_U = \gamma(P_U + P_T) \)
The Hamiltonian with minimal coupling
\( L = L_U = -L \)
The Lagrangian with minimal coupling
\( H = \{ H \} \)
\( L = \{ L \} \)

Rest Lagrangian \( L_o \)
\[ = -(P_T - U) \]
\[ = -\gamma(P_T - U) \]
\[ = -\gamma(P + qA) \]
\[ = -P \cdot U - qA \cdot U \]

4-TotalMomentum
\[ P_T = (E/c, p_T) \]
\[ = (H/c, p_T) \]
\[ = P + qA \]

4-Velocity
\[ U = \gamma(c, u) \]
\[ = \gamma(c, u) \]

4-Velocity Gradient
\[ \partial U = (\partial_U/c, -\nabla U) \]
\[ \gamma/(d/dt) \]
\[ = (\partial_U/c, -\nabla U) \]
\[ \gamma/(d/dt) \]

4-Velocity Gradient part
\[ (d/dt)\partial U[L_0] = \gamma \partial U[L_0] \]
\[ = (d/dt)\partial U[L_0] \]
\[ = (d/dt)\partial U[L_0] \]

Relativistic Euler-Lagrange Eqn
\[ \partial_\alpha = (d/dt)\partial_U \]
\[ = (d/dt)\partial_U \]
\[ = (d/dt)\partial_U \]

Relativistic 4-Vector
\[ U = (d/dt)R \]
\[ = (d/dt)R \]
\[ = (d/dt)R \]

Relativistic 4-Vector
\[ F_m = \gamma((u \cdot e)c, (e) + (u \times b)) \]
\[ = (-\nabla \phi - \partial a) \]
\[ = (\nabla \phi + \partial a) \]

EM Faraday 4-Vector Potential
\[ F_m = \phi \partial_\alpha \wedge \partial_\beta A^\alpha \]
\[ = [0, -e/c, 0, -e/b] \]

4-Fourier
SRQM Diagram:
Relativistic Hamilton’s Equations
Equation of Motion (EoM) for EM particle

\[ \gamma = \frac{1}{\sqrt{1 - \beta^2}}: \text{Relativistic Gamma Identity} \]
\[ (\gamma - 1/v^2) = (\gamma \beta) : \text{Manipulate into this form... still an identity} \]
\[ \gamma(P_r \cdot U) + (P_r \cdot U) \gamma = (\gamma \beta \cdot \gamma)(P_r \cdot U) \]
\[ \gamma(P_r \cdot U) + (P_r \cdot U) \gamma = (P_r \cdot U) \]

\( \{ H \} + \{ L \} = (P_r \cdot U): \text{The Hamiltonian/Lagrangian connection} \)

\[ H = \gamma H_o = \gamma (P_r \cdot U) = \gamma ((P + qA) \cdot U) = \text{The Hamiltonian with minimal coupling} \]
\[ L = L_o = \{ P_r \cdot U \} = \text{The Lagrangian with minimal coupling} \]

\[ H_o = (P_r \cdot U) = -E_o = \text{Rest Hamiltonian = Total RestEnergy} \]
\[ L_o = -\{ P_r \cdot U \} = -H_o \]

\[ \partial_r [H_o] = \partial_r [U \cdot P_r] = \partial_r [U \cdot P_r] + U \cdot \partial_r [P_r] = 0 \]
\[ \partial_r [H_o] = \partial_r [U \cdot P_r] + U \cdot \partial_r [P_r] = U = \frac{d}{dt}[X] \]

Thus: \( \frac{d}{dt}[X] = \partial_r [H_o] = [\partial_r [P_r]] [H_o] \)

\[ \partial_\gamma [H_o] = \partial_\gamma [U \cdot P_r] = \partial_\gamma [U \cdot P_r] + U \cdot \partial_\gamma [P_r] = 0 \]
\[ \partial_\gamma [H_o] = \partial_\gamma [U \cdot P_r] + U \cdot \partial_\gamma [P_r] = \frac{d}{dt}[P_r] \]

Thus: \( \frac{d}{dt}[P_r] = \partial_\gamma [H_o] = [\partial_\gamma [X]] [H_o] \)

Relativistic Hamilton’s Equations (4-Vector):
\[ (\partial / \partial [X]) = (\partial / \partial [P_r]) [H_o] \]
\[ (\partial / \partial [P_r]) = (\partial / \partial [X]) [H_o] \]

\[ \frac{d}{dt}[X] = \gamma (\partial / \partial [X]) [H_o] = (\partial / \partial [P_r]) [H_o] = (\partial / \partial [X]) [(P + qA) \cdot U] \]
\[ \frac{d}{dt}[P_r] = (\partial / \partial [X]) [H_o] = (\partial / \partial [X]) [(P + qA) \cdot U] = \partial_r [P_r] \]

\[ (\partial / \partial [X]) = (\partial / \partial [P]) [H_o] \]
\[ (\partial / \partial [P]) = (\partial / \partial [X]) [H_o] \]

Taking just the spatial components:
\[ \gamma (\partial / \partial [x]) = (\partial / \partial [P_r]) [H_o] = (\partial / \partial [X]) [H_o] \]
\[ \gamma (\partial / \partial [p_r]) = (\partial / \partial [X]) [H_o] \]

Take the Classical limit (\( \gamma \rightarrow 1) \)

Classical Hamilton’s Equations (3-vector):
\[ (\partial / \partial [x]) = (\partial / \partial [p_r]) [H] \]
\[ (\partial / \partial [p_r]) = (\partial / \partial [x]) [H] \]

Sign-flip difference is interaction of \((\partial / \partial p_r) \) with \( [1/\gamma] \)

\( \partial \)SR 4-Tensor
\[ T^{\mu \nu} \text{ (2,0) } \text{ Tensor } T_{\mu \nu} \text{ or } T_{\nu \mu} \]

\( \partial \)SR 4-Vector
\[ V^\mu = V^\nu = (v^\mu, v^\nu) \]

\( \partial \)SR 4-CoVector:OneForm
\[ (1,0) \text{- Tensor } V_\mu = (v_\mu, v_\nu) \]

\( \partial \)SR 4-Scalar
\[ S_0 \text{ or } S_0 \text{ Lorentz Scalar} \]
SRQM Diagram:

Relativistic Hamilton’s Equations

Equation of Motion (EnM) for EM particle

\[ \gamma = 1/\sqrt{1 - v^2} \]
Relativistic Gamma Identity

\( (\gamma - 1/v) = (v/\sqrt{v^2 - 1}) \)
Manipulate into this form... still an identity

\[ \gamma P_u \] + \( \gamma (P_u + P_t) \gamma = (\gamma v) (P_t + P_u) \gamma = (\gamma v) (P_t + P_u) \gamma \]
Take the Classical Limit \( \gamma \rightarrow 1 \)

\[ \gamma P_u = \gamma (P_t + P_u) \gamma \]
Relativistic Hamilton's Equations of Motion

\[ \frac{d}{dt} \mathbf{P}_a = \mathbf{F}_a + q \mathbf{U} \times \mathbf{A} \]
\[ \mathbf{F}_a = \mathbf{q} \mathbf{U} \times \mathbf{A} \]
\[ \mathbf{F}_a = \mathbf{q} \mathbf{U} \mathbf{A} \]
Lorentz Force Equation

\[ \mathbf{F}_a = \mathbf{q} \mathbf{U} \mathbf{A} \]
\[ \mathbf{F}_a = \mathbf{q} \mathbf{U} \mathbf{A} \]

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\[ \mathbf{F}_a = \mathbf{q} \mathbf{U} \mathbf{A} \]
\[ \mathbf{F}_a = \mathbf{q} \mathbf{U} \mathbf{A} \]
**SRQM Diagram:**

**Relativistic Hamilton’s Equations**

**Equation of Motion (EoM) for Harmonic Oscillator**

\[
\begin{align*}
\mathbf{A} \cdot \mathbf{U} &= \phi_0 : \text{RestScalar-Potential} \\
q\mathbf{A} \cdot \mathbf{U} &= q\phi_0 = V_0 : \text{RestVoltage = Electrical Potential Energy}
\end{align*}
\]

Let \{ \quad q\mathbf{A} \cdot \mathbf{U} = V_0 = -k\mathbf{X} \cdot \mathbf{X}/2 \} \quad \text{then} \quad \{ \quad \mathbf{A} \cdot \mathbf{U} = \phi_0 = -(k/q)\mathbf{X} \cdot \mathbf{X}/2 \}

RestHamiltonian \( H_0 = (P \cdot \mathbf{U}) = \mathbf{P} \cdot \mathbf{U} + q\mathbf{A} \cdot \mathbf{U} = \mathbf{P} \cdot \mathbf{U} - k\mathbf{X} \cdot \mathbf{X}/2 \)

**Classical limit:**

\[
\gamma \to 1 : v \ll c \\
d/dt[a] \to 0 : 3\text{-vector-potential changes very slowly}
\]

\[
\{ \quad \mathbf{A} \cdot \mathbf{U} = \phi_0 = -(k/q)\mathbf{X} \cdot \mathbf{X}/2 = -(k/2q)(ct)^2 - \mathbf{x} \cdot \mathbf{x} \}
\]

For \( t = 0 \)

\[
\phi_0 = -(k/2q)(-\mathbf{x} \cdot \mathbf{x}) = (k/2q)(\mathbf{x} \cdot \mathbf{x})
\]

\[
V_0 = -(k/2)(-\mathbf{x} \cdot \mathbf{x}) = (k/2)(\mathbf{x} \cdot \mathbf{x}) : \text{PE of classical harmonic oscillator}
\]

\[
f = -k\mathbf{x} : \text{Spring Force acting on classical harmonic oscillator}
\]

RestHamiltonian

\[
H_0 = (P \cdot \mathbf{U}) = \mathbf{P} \cdot \mathbf{U} + q\mathbf{A} \cdot \mathbf{U} = \mathbf{P} \cdot \mathbf{U} - k\mathbf{X} \cdot \mathbf{X}/2
\]

\[
H_0 = \mathbf{P} \cdot \mathbf{U} - k\mathbf{X} \cdot \mathbf{X}/2 : \text{Covariant Relativistic version}
\]

\[
H_0 = E_0 + (k/2)(\mathbf{x} \cdot \mathbf{x}) : \text{Classical}
\]

\[
\text{Tot} = \text{Rest + Potential}
\]

RestLagrangian

\[
L_0 = -(P \cdot \mathbf{U}) = -\mathbf{P} \cdot \mathbf{U} + q\mathbf{A} \cdot \mathbf{U} = -\mathbf{P} \cdot \mathbf{U} + k\mathbf{X} \cdot \mathbf{X}/2
\]

(d/dt[\delta_0]L_0) = (\delta_0 L_0)/t, Relativistic Euler-Lagrange Eqn

\[
(d/dt)[\delta_0 (-\mathbf{P} \cdot \mathbf{U} + k\mathbf{X} \cdot \mathbf{X}/2)] = \delta_0(-\mathbf{P} \cdot \mathbf{U} + k\mathbf{X} \cdot \mathbf{X}/2)
\]

\[
(d/dt)[\delta_0 (-\mathbf{P} \cdot \mathbf{U})] = \delta_0(k\mathbf{X} \cdot \mathbf{X}/2)
\]

\[
(d/dt)[-\mathbf{P}] = k\mathbf{X}
\]

\[
[-F] = k\mathbf{X}
\]

\[
F = -k\mathbf{X}
\]
Lorentz EM Force Equation:

\[ F^\alpha = q(F^{\alpha\beta})U_\beta \]

\[ F^\alpha = (\varepsilon^\alpha\beta - \varepsilon^\alpha\gamma)U_\beta \]

Examine just the spatial components of 4-Force \( F \):

\[ F^i = q(\varepsilon^\alpha\beta - \varepsilon^\alpha\gamma)U_\beta \]

\[ F^i = q(\varepsilon^\alpha\beta - \varepsilon^\alpha\gamma)U_\beta \]

\[ \gamma f = q(-\nabla[\phi] - (\varepsilon^{\alpha\beta\gamma})a)(yc) + q(-\nabla[a\cdot u] - u\cdot \nabla[a])\gamma \]

\[ f = q(-\nabla[\phi] - (\varepsilon^{\alpha\beta\gamma})a)(c) + q(u\cdot \nabla[a] - \nabla[a\cdot u]) \]

\[ f = q(-\nabla[\phi] - \varepsilon^{\alpha\gamma}a + u\cdot \nabla[a] - \nabla[a\cdot u]) \]

\[ f = q(-\nabla[\phi] - \varepsilon^{\alpha\gamma}a + u\cdot b) \]

Take the limit of \( \{ | \nabla[\phi] | >> | \varepsilon^{\alpha\gamma}a - u\cdot b \} \)

\[ f \sim q(-\nabla[\phi]) = -\nabla[\phi] = -\nabla[U] = -\text{Grad}[\text{Potential}] \]

The Classical Force = -Grad[Potential]

when \( \{ | \nabla[\phi] | >> | \varepsilon^{\alpha\gamma}a - u\cdot b \} \) or when \( \{ a = 0 \} \)

The majority of non-gravitational, non-nuclear potentials dealt with in CM are those mediated by the EM potential.

ex. Spring Potential \( U = kx^2/2 \), then \( f = -\nabla[kx^2/2] = -kx \) Hooke’s Law
The Speed-of-Light (c) is THE connection between Time and Space: \( \text{d}R = (\text{cdt}, \text{d}r) \)

This physical constant appears in several seemingly unrelated places. You don’t notice these cool relations when you set \( c \rightarrow 1 \). Also notice that the set of all these relations definitely rules out a variable speed-of-light. (c) is an Invariant Lorentz Scalar constant.

The Speed-of-Light (c) is THE connection between Time and Space: \( \text{d}R = (\text{cdt}, \text{d}r) \)

\[ \text{U}\cdot\text{U} = \gamma^2 (\text{c}^2 - \text{u} \cdot \text{u}) = c^2 \]

Speed of all things into the Future

\[ (E/mc^2) = (E/\text{E}'_\text{m}c^2) = \text{c}^2 \]

Mass is concentrated Energy, \( E = mc^2 \)

\[ |\text{v} \times \text{v}|^2 |\text{\lambda}^2 |^2 = \lambda^2 (f-\text{fo})^2 = c^2 \]

Wavelength-Frequency Relation: \( \lambda f = c \) for photons

\[ (1/c \cdot \text{u} \cdot \text{u}) = \text{c}^2 \]

Electric (\( \varepsilon_0 \)) and Magnetic (\( \mu_0 \)) EM Field Constants

\[ (-\text{h}^2/m_0^2)(\partial \cdot \partial) = \text{c}^2 \]

Relativistic Quantum Wave Equation

\[ (-\text{h}/m_0 \text{c}^2)(\partial \cdot \partial) = \text{c}^2 \]

Klein-Gordon (spin 0), Proca (spin 1), Maxwell (spin 1, m=0) Factors to Dirac (spin \( \frac{1}{2} \)) Classical-limit (\( |v|<<c \)) to Schrödinger

\[ (\text{h}/\text{mc})^2 = \text{c}^2 \]

Reduced Compton Wavelength: \( \lambda_c = (\text{h}/\text{mc}) \)

\( 2\text{GM/R}_S = \text{c}^2 \)

GR Black Hole Equation

\( R_S = \text{Schwarzschild Radius} \)

\( G = \text{GR Gravitational Const.} \)

\( M = \text{BH Mass} \)

\[ 8\pi G/\text{c}^4 = \text{c}^2 \]

GR Einstein Curvature Constant: \( \kappa = 8\pi G/\text{c}^4 \)

\[ (\text{c}^2 \times \text{scalar}, \text{3-vector}) = 4\text{-Vector} \]

Every Physical 4-Vector has a (c) factor to maintain equivalent dimensional units across the whole 4-Vector
SRQM: The Speed-of-Light (c)  
\(c^2\) Invariant Relations (part 2)

The Speed-of-Light (c) is THE connection between Time and Space: 
\[
dt = c \frac{dr}{d\tau} 
\]

This physical constant appears in several seemingly unrelated places. You don’t notice these cool relations when you set \(c \rightarrow 1\).  Also notice that the set of all these relations definitely rules out a variable speed-of-light.  (c) is an Invariant Lorentz Scalar constant.

\[
\eta_{\mu\nu} \cdot \delta_{\mu\nu} \text{ Minkowski Metric}
\]

\[
\text{4-Vector Scalar Product}
\]

\[
\text{EM} \\
\frac{(e \cdot b)^2}{\det[F_{\mu\nu}]} \\
\text{Electric:Magnetic} \\
\frac{1}{\epsilon_0 \mu_0} = c^2 \\
\frac{E}{m} = c \\
\frac{\omega}{m_0} \approx \frac{\omega}{m} \approx c \\
\frac{\kappa}{\lambda^2} = \frac{8\pi G}{\lambda^2} \\
\frac{8\pi G/c^2}{\text{SRQM}}
\]

\[
\text{Invariant 4-Velocity} \\
\frac{U \cdot U}{c^2} = \frac{P \cdot P}{m_0^2} \\
\frac{E_0^2}{P \cdot P} \\
\frac{\omega^2}{K \cdot K} \\
\frac{-(h/m_0)^2 \partial \cdot \partial}{c^2} \\
\text{SRQM}
\]

\[
\text{Trace}[T'] = \eta_{\mu\nu} T^\mu_{\nu} = T^\mu_{\mu} = T \\
V \cdot V = \left(\frac{V}{c}\right)^2 - V \cdot V = \left(\frac{V}{c}\right)^2 \\
\text{Lorentz Scalar Invariant}
\]

\[
\text{Energy:Mass} \\
\frac{E}{m} = c^2 \\
\frac{\omega}{m_0} \approx \frac{\omega}{m} \approx c \\
\frac{\kappa}{\lambda^2} = \frac{8\pi G}{\lambda^2} \\
\frac{8\pi G/c^2}{\text{SRQM}}
\]

\[
\text{Intensity} \\
\frac{R \cdot R}{\tau^2} \\
\text{dR} \cdot \text{dR}/\text{d}t^2 \\
\text{ProperTime Differential} \\
\frac{-S_{\text{action,free}}}{(m_0) (d\tau)} \\
\frac{8\pi G/c^2}{\text{GR}} \\
\frac{2GM/R_S}{c^2}
\]

\[
\text{Every Physical 4-Vector has a (c) factor to maintain equivalent dimensional units across the whole 4-Vector}
\]

\[
\text{(c)^2 \cdot scalar, 3-vector} = \text{4-Vector}
\]

\[
\text{SR 4-Tensor} \\
(2,0) - \text{Tensor} T^{\mu\nu}_{\text{IV}} \\
(1,1) - \text{Tensor} T_{\text{IV}}, \text{ or } T_{\nu}^\mu \\
(0,2) - \text{Tensor} T_{\text{P}}^\nu \\
\text{SR 4-Scalar} \\
(0,0) - \text{Tensor} S \text{ or } S_0 \\
\text{Lorentz Scalar}
\]

\[
\text{SR 4-Vector} \\
(1,0) - \text{Tensor} V^\nu = V = (v^\nu, v) \\
\text{SR 4-Vector:OneForm} \\
(0,1) - \text{Tensor} V_\mu = (v_\mu - v)
\]

\[
\text{SR 4-Scalar} \\
(0,0) - \text{Tensor} S_0 \text{ or } S_0 
\]

\[
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\text{SciRealm@aol.com} \\
\text{http://scirealm.org/SRQM.pdf}
\]
The 4-ThermalVector is used in Relativistic Thermodynamics.

My prime motivation for the form of this 4-Vector is that the probability distributions calculated by statistical mechanics ought to be covariant functions since they are based on counting arguments.

\[ F(\text{state}) \sim e^{^\frac{\chi}{k_B T}} = e^{^\frac{\beta E}{k_B T}}, \text{ with } \beta = 1/k_B T, \text{ (not } v/c) \]

A covariant way to get this is the Lorentz Scalar Product of the 4-Momentum \( P \) with the 4-ThermalVector \( \Theta = (\theta, \Theta) = (c/k_B T, u/k_B T) = (\theta/c) U \)

This also gets Boltzmann’s constant \( (k_B) \) out there with the other Lorentz Scalars like \((c)\) and \((\Theta)\).

see (Relativistic) Maxwell-Jüttner distribution

\[ f(P) = N_o/(2\pi m_o)^3 K_{\theta}^2[m_o c \Theta]^{2} * e^{^\frac{(\theta^2 c^2 - P^2)}{4 k_B T \Theta}} \]

It is possible to find this distribution written in multiple ways because many authors don’t show constants, which is quite annoying. Show the damn constants people! \((k_B),(c),(\Theta)\) deserve at least that much respect.

\[ \text{SRQM 4-Vector Study:} \]

\[ \textbf{4-ThermalVector} \]

\[ \text{Relativistic Thermodynamics} \]
The 4-ThermalVector is used in Relativistic Thermodynamics. It can be used in a partial derivation of Unruh-Hawking Radiation (up to a numerical constant).

Let a "Unruh-DeWitt thermal detector" be in the Momentarily-Comoving Rest Frame (MCRF) of a constant spatial acceleration (a), in which |u|→0, γ→1, γ′→0.

4-Acceleration MCRF = A_{MCRF}^\mu = (0, a)^{\mu}_{MCRF}

Take the Lorentz Scalar Product with the 4-ThermalVector
A_{MCRF}^\mu \Theta = (0, a)_{MCRF}^\mu (c/k_B T, u/k_B T) = (-a \cdot u/k_B T) = Lorentz Scalar Invariant

The (u) here is part of the 4-ThermalVector: the 3-velocity of the thermal radiation. (not from A_{MCRF})

Let the thermal radiation be photonic: EM in nature, so |u| = c, and in a direction opposing the acceleration of the "thermal detector", which removes the minus sign.

A_{MCRF}^\mu \Theta_{\text{radiation}} = (ac/k_B T) = Invariant Lorentz Scalar

Use Dimensional Analysis to find appropriate Lorentz Scalar Invariant with same units:
Invariant Units = [m/s]^4 / [kg·m^2/s] = [1/kg·s] ~ c^2/h [kg·m^2/s] / [kg·m^2/s]

A_{MCRF}^\mu \Theta_{\text{radiation}} = (ac/k_B T) = Invariant ~ c^2/h

Temperature T ~ ha/k_Bc, (from EM radiation, only from the dir. of acceleration)

Further methods give the constant of proportionality (1/2m):
See (Imaginary Time, Euclideanization, Wick Rotation, Matsubara Frequency)
See (Thermal QFT, Bogoliubov transformation)

\[ T_{\text{Unruh}} = \frac{ha}{2\pi k_Bc} \] (due to constant Minkowski-hyperbolic acceleration)

\[ T_{\text{Hawking}} = \frac{hc}{2\pi k_Bc} \] (due to gravitational acceleration a=q)

\[ T_{\text{Schwarzschild BH}} = \frac{hc}{8\pi GM/c^2} \] (Temp at BH Event Horizon, g=GM/R_s^2, R_s=2GM/c^2)

\[ T_{\text{SR}} = \frac{ha(u \cdot a)}{2\pi k_Bc^2} \] (correct version from 4-Vector derivation A_{MCRF}^\mu \Theta_{\text{radiation}} = 2\pi c^2/h)

4-Acceleration
A = A^\mu = \gamma(c \cdot u + a) / \gamma\cdot dU/dt = d^2R/d\tau^2

4-Acceleration MCRF = A_{MCRF}^\mu = (0, a)_{MCRF}^\mu

\[ \Theta(\theta, \theta) = (c/k_B T, u/k_B T) = (\theta/c) U = (1/k_B T_0) U \]

\[ P \cdot \Theta = (E/c, p) \cdot (c/k_B T, \theta) = (E/k_B T \cdot p \cdot \theta) = (E/k_B T_0) = \text{Invariant}_{\text{dimensionless}} \]

\[ P = (mc, p) = (E/c) = m c \cdot U \]

\[ P \cdot P = (m c^2)^2 = (E/c)^2 \]

\[ \text{Trace}[T^\tau] = n_{\\text{in}} T^\tau = T_{\\text{in}} \cdot T = T \]

\[ V \cdot V = V_{\\text{in}}^2 + V_{\\text{in}} = (V^\mu V_\mu)^2 - (V_\mu V_\mu) = (V^\mu V_\mu)^2 \]

\[ = \text{Lorentz Scalar Invariant} \]

Note that the temperature here is relativistically direction-specific, unlike in the classical use of temperature.
SRQM 4-Vector Study: 4-ThermalVector
Unruh-Hawking Radiation

Temperature $T \sim h a/k_B$, (from EM radiation, only from the dir. of acceleration)

Further methods give the constant of proportionality $(1/2\pi)$:
See (Imaginary Time, Euclideanization, Wick Rotation, Matsubara Frequency)
See (Thermal QFT, Bogoliubov transformation)

$T_{\text{Unruh}} = h a/(2\pi k_B) \{\text{due to constant Minkowski-hyperbolic acceleration}\}$

$T_{\text{Hawking}} = h a/(2\pi k_B) \{\text{due to gravitational acceleration } a=g\}$

$T_{\text{Schwarzschild BH}} = h c^3/(8\pi G M k_B) \{\text{Temp at BH Event Horizon, } g=G M/R_s^2, R_s=2G M/c^2\}$

$T_{\text{SR}} = h (a \cdot u)/(2\pi k_B c)^2 \{\text{correct version from 4-Vector derivation } A_{\text{MCRF}} \Theta_{\text{radiation}} = 2\pi c^3/h\}$

Alternate forms:

$T = (1/kT) \{\text{due to gravitational acceleration } a=g\}$

$T_{\text{Schwarzschild BH}} = h c^3/(8\pi G M k_B) \{\text{Temp at BH Event Horizon, } g=G M/R_s^2, R_s=2G M/c^2\}$

$T_{\text{SR}} = h (a \cdot u)/(2\pi k_B c)^2 \{\text{correct version from 4-Vector derivation } A_{\text{MCRF}} \Theta_{\text{radiation}} = 2\pi c^3/h\}$

The $2\pi$ factor is interesting

There are cases when the dimensional units must match.
See 4-Momentum related to 4-WaveVector:

$P = h K \rightarrow [J/s/m] = [J/s/\text{rad}]$ [rad/m]

$h = h/2\pi \rightarrow [J/s/\text{rad}] = [J/s][2\pi \text{ rad}]$

And other where the $2\pi$ factor doesn’t seem to use [rad] units.
See Circles & Spheres:

$C = 2\pi r \rightarrow [m] = [2\pi][m]$  \quad A = \pi r^2 \rightarrow [m^2] = [\pi][m]^2$

$V = (4/3)\pi r^3 \rightarrow [m^3] = [(4/3)\pi][m]^3$

$A_{\text{MCRF}} = (2\pi c^2) K = (2\pi c^2) P$

$(dP/d\tau)_{\text{MCRF}} \Theta_{\text{radiation}} = 2\pi\omega_0$

$F_{\text{MCRF}} \Theta_{\text{radiation}} = 2\pi\omega_0 : \{\text{for } m_c = \text{constant}\}$

SR 4-Tensor
$(2,0)$-Tensor $T^{\mu\nu}$
$(1,1)$-Tensor $T^{\mu\nu}$, or $T^\nu_\nu$
$(0,2)$-Tensor $T_{\mu\nu}$

SR 4-Vector
$(0,1)$-Vector $V = (\phi, v)$

SR 4-CoVector: OneForm
$(0,1)$-Tensor $V = (\phi, v)$

SR 4-Scalar
$(0,0)$-Tensor $S$ or $S_0$
Lorentz Scalar

Note that the temperature here is relativistically direction-specific, unlike in the classical use of temperature.
SRQM 4-Vector Study:

4-ThermalVector

Wick Rotations, Matsubara Freqs

In the Matsubara Formalism, the basic idea (due to Felix Bloch) is that the expectation values of operators in a canonical ensemble:

\[ <A> = \frac{\text{Tr} \left[ e^{-\beta H} A \right]}{\text{Tr} \left[ e^{-\beta H} \right]} \]

may be written as expectation values in ordinary quantum field theory (QFT), where the configuration is evolved by an imaginary time \( \tau = -i t \) (\( 0 \leq \tau \leq \beta \)).

One can therefore switch to a spacetime with Euclidean signature, where the above trace (Tr) leads to the requirement that all bosonic and fermionic fields be periodic and antiperiodic, respectively, with respect to the Euclidean time direction with periodicity \( \beta = \frac{\hbar}{k_B T} \).

This allows one to perform calculations with the same tools as in ordinary quantum field theory, such as functional integrals and Feynman diagrams, but with compact Euclidean time.

Note that the definition of normal ordering has to be altered. In momentum space, this leads to the replacement of continuous frequencies by discrete imaginary (Matsubara) frequencies:

Bosonic \( \omega_n = \left( n + \frac{1}{2} \right) \frac{2\pi}{\beta} \)

Fermionic \( \omega_n = \left( n + \frac{1}{2} \right) \frac{2\pi}{\beta} \)

and, through the de Broglie relation \( E = \hbar \omega \), to a discretized EM thermal energy spectrum \( E_n = \hbar \omega_n = n \frac{2\pi k_B T}{\beta} \).

The QM/QFT↔SM Correspondence, via the Wick Rotation

The operator which governs how a quantum system evolves in time, the time evolution operator, and the density operator, a time-independent object which describes the statistical state of a many-particle system in an equilibrium state (with temperature T) can be related via arithmetic substitutions:

Quantum Mechanics (QM)

\[ e^{\frac{\tau}{\hbar}} \left( -i \mathbf{P} \cdot \mathbf{X} / \hbar \right) \]

\[ = e^{\frac{i}{\hbar} \text{action}} \]

\[ = e^{\frac{i}{\hbar} \mathbf{H} \cdot \mathbf{t} / \hbar} \]

Wick Rotation \( \tau \rightarrow -i \tau \)

Statistical Mechanics (SM)

\[ e^{\frac{-\beta \mathbf{H}}{\hbar}} \]

\[ = e^{\frac{-\beta \mathbf{H}}{k_B T}} \]

\[ = e^{\frac{-\mathbf{H} \cdot \mathbf{t} / \hbar}{k_B T}} \]

Euclidean Time \( \tau \) \( \rightarrow \beta = \frac{\hbar}{k_B T} \)

Imaginary Time \( \tau \)

(\( 0 \leq \tau \leq +\beta \))

where \( \tau \), called Euclidean Time (Imaginary Time) is cyclic with period \( \beta \), (\( 0 \leq \tau \leq +\beta \)).

In Quantum Mechanics (or Quantum Field Theory), the Hamiltonian \( H \) acts as the generator of the Lie group of time translations while in Statistical Mechanics the role of the same Hamiltonian \( H \) is as the Boltzmann weight in an ensemble.

Time Evolution Operator

\[ U(t) = \sum_{n=0}^{\infty} \left[ e^{\frac{i}{\hbar} \left( E_n \frac{t}{\beta} \right)} \right] | n \rangle \langle n | = e^{\frac{i}{\hbar} \mathbf{H} \cdot \mathbf{t} / \hbar} \]

Partition Function (time-independent function of state)

\[ Z = \sum_{n=0}^{\infty} \left[ e^{\frac{i}{\hbar} \left( E_n / k_B T \right)} \right] = \text{Trace} \left[ e^{\frac{i}{\hbar} \mathbf{H} \cdot \mathbf{t} / \hbar} \right] \]

Note that the temperature here is relativistically direction-specific, unlike in the classical use of temperature.
SRQM 4-Vector Study:

4-ThermalVector Covariant Wick Rotation

Note that the temperature here is relativistically direction-specific, unlike in the classical use of temperature.

The operator which governs how a quantum system evolves in time, the time evolution operator, and the density operator, a time-independent object which describes the statistical state of a many-particle system in an equilibrium state (with temperature $T$) can be related via arithmetic substitutions:

$$ e^{i\frac{P_T \cdot R}{\hbar}} = e^{\frac{1}{\hbar}[-i(H_0 + i\beta \omega_0)]} $$

where $\tau$, called Euclidean Time (Imaginary Time) is cyclic with period $\beta$, $(0 \leq \tau \leq +\beta)$.

In Quantum Mechanics (or Quantum Field Theory), the Hamiltonian $H$ acts as the generator of the Lie group of time translations while in Statistical Mechanics the role of the same Hamiltonian $H$ is as the Boltzmann weight in an ensemble.

$$ S_{\text{action}} = -\frac{1}{\hbar}[-(P_T \cdot R)] - \frac{1}{\hbar}[-(P_T \cdot \theta)] $$

$$ = \int \gamma(H_0 - p \cdot u) \, d\tau $$

$$ = \frac{1}{\hbar}[(E_0/c, p) \cdot (c/k_BT, \theta)] = \frac{1}{\hbar}((E_0/c, p) \cdot (c/k_BT, \theta) \cdot (c/k_BT, \theta)) $$

$$ = \frac{1}{\hbar}((E_0/c, p) \cdot (c/k_BT, \theta)) $$

$$ = \frac{1}{\hbar}((E_0/c, p) \cdot (c/k_BT, \theta)) $$

$$ = \frac{1}{\hbar}((E_0/c, p) \cdot (c/k_BT, \theta)) $$

where $i$, called Euclidean Time (Imaginary Time) is cyclic with period $\beta$, $(0 \leq \tau \leq +\beta)$.

In Quantum Mechanics (or Quantum Field Theory), the Hamiltonian $H$ acts as the generator of the Lie group of time translations while in Statistical Mechanics the role of the same Hamiltonian $H$ is as the Boltzmann weight in an ensemble.
SRQM 4-Vector Study:
Deep Symmetries: Schrödinger Relations &
Cyclic Imaginary Time ↔ Inv Temp

4-Momentum:
\[ P = (mc, p) = (mc, mu) = m_u U = (E/c, p) = (E/c^2) U \]

4-Position:
\[ R = R(\mathbf{r}) = ct, r, <\text{Event}> \rightarrow (ct, x, y, z) \]
alt. notation: \( X = X^\mu \)

4-ImaginaryPosition:
\[ R_m = R_{(m)} = i(\mathbf{ct}, r) = (ict, ir) = (ct, ir) \]

4-Gradient:
\[ \partial = \partial R = \partial R_m = \partial U = (\partial / c, -\mathbf{V}) \]
\[ \rightarrow (\partial / c, -\partial_x, -\partial_y, -\partial_z) \]
\[ = (\partial / c t, \partial / \partial x, \partial / \partial y, \partial / \partial z) \]

Einstein-de Broglie:
\[ P = \hbar K \rightarrow \{ E = \hbar \omega : p = \hbar k \} \]

Complex Plane-Wave:
\[ K = i\omega \]

Schrödinger Relations:
\[ \mathbf{P} = i\hbar \partial \rightarrow \{ E = i\hbar \omega : p = -i\hbar \mathbf{V} \} \]

Wick Rotation:
\[ \mathbf{R} = -i\mathbf{R}_m \rightarrow \{ t = -i r : r = -i(\mathbf{r}) \} \]

Cyclic Temp:
\[ \mathbf{R}_m \rightarrow \hbar \Theta \rightarrow \{ t = \hbar k_\Sigma T : r = \hbar k \mathbf{u} / k_\Sigma T \} \]

Time Temp:
\[ \mathbf{R} = -i\hbar \Theta \rightarrow \{ t = -i\hbar k_\Sigma T : r = -i\hbar \mathbf{u} / k_\Sigma T \} \]

Boltzmann Distribution:
\[ \mathbf{P} \cdot \mathbf{\Theta} = (E/c, p) \rightarrow (c/k_\Sigma T, \mathbf{u}) = (\theta_\Sigma / c) \mathbf{U} = (1/k_\Sigma T) \mathbf{U} = (1/k_\Sigma T_0) \mathbf{U} = (\mathbf{U} / k_\Sigma T_0) \]

Trace of T^n = m^2 = T^\nu_\nu = T \rightarrow \mathbf{V} \cdot \mathbf{V} = V^\nu V_\nu = (v^\nu) - (v \cdot v) \rightarrow (v^\nu) \rightarrow \text{Lorentz Scalar Invariant}
SRQM 4-Vector Study:
Deep Symmetries:
Schrödinger Relations &
Cyclic Imaginary Time ↔ Inv Temp

A Tensor Study of Physical 4-Vectors

4-Position
R=R^μ=(ct,r)=<Event>
→(ct,x,y,z)
alt. notation X=X^μ

SpaceTime Dimension
\[ \partial R = \partial^\mu R^\mu = 4 \]

4-Gradient
\[ \partial^\mu = \partial / \partial R^\mu = \partial / (\partial c, -\nabla) \]
\[ = (\partial c, -\partial_x, -\partial_y, -\partial_z) \]
\[ = (\partial c, -i\partial_x, -i\partial_y, -i\partial_z) \]

Minkowski Metric
\[ \delta^{\mu\nu} = \eta^{\mu\nu} \]

4-WaveVector
\[ \mathbf{K} = \mathbf{K}^\mu = (\omega/c, \mathbf{K}) = (\omega/c^2) \mathbf{U} \]
\[ = (\omega/c, \omega/\nu/\text{phase}) = (1/cT, \eta/\lambda) \]

Covariant Wick Rotation
\[ \mathbf{R} = -i \mathbf{R}^\mu \]

Inverses
\( \{ K^\mu, R^\mu \} \)
\( \{ \mathbf{K}^\mu, \mathbf{R}^\mu \} \)
\( \{ K^\mu, R^\mu \} \)
\( \{ \mathbf{K}^\mu, \mathbf{R}^\mu \} \)

Complex Plane-Waves
\( \mathbf{K} = i \lambda \)

ProperTime Derivative
\[ U \cdot \partial = \gamma d/dt = d/d\tau \]

Einstein de Broglie
\( P = \hbar K = (\hbar/c) \mathbf{U} \)

1/KβTβ

Energy Factors
\( E = mc^2 = k_j T_j \)

4-Momentum
\( P = P^\mu = (mc, \mathbf{p}) = (mc, \mu) = m_0 \mathbf{U} \)
\( = (E/c, \mathbf{p}) = (E/c^2) \mathbf{U} \)

Boltzmann Distribution
\( P \cdot (c_k, p) = (E/c_k T, 0) = (E/c_k T_0) \)

Covariant Time ~ Inv Temp
\( \mathbf{R} = i \hbar \Theta \)
\( (ct, r) = i \hbar (c/k_0 T, u/k_0 T) \)

4-Velocity
\( U = U^\mu = \gamma(c, \mathbf{u}) = dR/d\tau = cT \)

Inverses
\( \{ \mathbf{P}^\mu, \Theta^\mu \} \)
\( \{ \mathbf{P}^\mu, \Theta^\mu \} \)

1/κβTβ

4-ThermalVector
4-Inverse Temperature
\( \Theta = \Theta^\mu = (\Theta^0, \mathbf{0}) = (c/k_0 T, u/k_0 T) = (0, \mathbf{c}) \mathbf{U} \)
\( = (1/k_0 T)(c, \mathbf{u}) = (1/k_0 T_0) U = (1/k_0 T_0) \mathbf{U} \)

Trace[T^μν] = η_μνT^μν = T^νν = T
\( \mathbf{V} \cdot \mathbf{V} = V^\mu V^\nu = V^\mu V^\nu = (v^\mu v^\nu) = (v^\mu v^\nu)^2 \)

= Lorentz Scalar Invariant

Note that the temperature here is relativistically direction-specific, unlike in the classical use of temperature.
The 4-EntropyVector is used in Relativistic Thermodynamics.

Pure Entropy is a Lorentz Scalar in all frames.
Up to this point, we have mostly been exploring the SR aspects of 4-Vectors.

It is now time to show how RQM and QM fit into the works...

This is SRQM, [ SR → QM ]

RQM & QM are derivable from principles of SR

Let that sink in...

Quantum Mechanics is derivable from Special Relativity

GR → SR → RQM → QM → {CM & EM}
The basic idea is to show that Special Relativity plus a few empirical facts lead to Relativistic Wave Equations, and thus RQM, without using any assumptions or axioms from Quantum Mechanics.

Start only with the concepts of SR, no concepts from QM:
(1) SR provides the ideas of Invariant Intervals and \(c\) as a Physical Constant, as well as: Poincaré Invariance, Minkowski 4D SpaceTime, ProperTime, ProperLength, Physical SR 4-Vectors.

Note empirical facts which can relate the SR 4-Vectors from the following:
(2a) Elementary matter particles each have RestMass, \(m_0\), a physical constant which can be measured by experiment: eg. in collisions, cyclotrons, Compton Scattering, etc.

(2b) There is a physical constant, \(\hbar\), which can be measured by classical experiment – eg. the Photoelectric Effect, the inverse Photoelectric Effect, LED's=Injection Electroluminescence, Duane-Hunt Law in Bremsstrahlung, the Watt/Kibble-Balance, etc. All known particles obey this constant.

(2c) The use of complex numbers \(i\) and differential operators \(\{\frac{\partial}{\partial t}, \nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})\}\) in wave-type equations comes from pure mathematics: not necessary to assume any QM Axioms.

These few things are enough to derive the RQM Klein-Gordon equation, the most basic of the relativistic wave equations. Taking the low-velocity limit \(|v| \ll c\) (a standard SR technique) leads to the Schrödinger Equation, the basic QM equation.
If one has a Relativistic Wave Equation, such as the Klein-Gordon equation, then one has RQM, and thence QM via the low-velocity limit \( \{|v| \ll c\} \).

The physical and mathematical properties of QM, usually regarded as axiomatic, are inherent in the Klein-Gordon RWE itself.

QM Principles emerge not from \( \{\text{QM Axioms} + \text{SR} \rightarrow \text{RQM}\} \), but from \( \{\text{SR} + \text{Empirical Facts} \rightarrow \text{RQM}\} \).

The result is a paradigm shift from the idea of \( \{\text{SR and QM as separate theories}\} \) to \( \{\text{QM derived from SR}\} \) – leading to a new interpretation of QM: 
*The SRQM or [SR→QM] Interpretation.*

GR \( \rightarrow \) (low-mass limit = \{curvature \( \sim \) 0\} limit) \( \rightarrow \) SR
SR \( \rightarrow \) (+ a few empirical facts giving Lorentz Invariant Scalars) \( \rightarrow \) RQM
RQM \( \rightarrow \) (low-velocity limit \( \{|v| \ll c\}\)) \( \rightarrow \) QM

The results of this analysis will be facilitated by the use of SR 4-Vectors
## SRQM 4-Vector Study:
### Basic 4-Vectors on the path to QM

<table>
<thead>
<tr>
<th>SR 4-Vector</th>
<th>Dimens. Units (SI)</th>
<th>Definition Component Notation</th>
<th>Unites</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-Position</td>
<td>[m]</td>
<td>$R = R^\mu = (r^\mu) = (r^0, r^i) = \langle \text{Event} \rangle$</td>
<td>Time, Space</td>
</tr>
<tr>
<td></td>
<td></td>
<td>= $(ct, r) \rightarrow (ct, x, y, z)$</td>
<td><em>(when, where)</em> = SR location of \langle \text{Event} \rangle*</td>
</tr>
<tr>
<td>4-Velocity</td>
<td>[m/s]</td>
<td>$U = U^\mu = (u^\mu) = (u^0, u^i) = \gamma(c, u)$</td>
<td>Temporal velocity, Spatial velocity</td>
</tr>
<tr>
<td></td>
<td></td>
<td>nothing real faster than $c$</td>
<td></td>
</tr>
<tr>
<td>4-Momentum</td>
<td>[kg·m/s]</td>
<td>$P = P^\mu = (p^\mu) = (p^0, p^i) = (E/c, p)$</td>
<td>Mass:Energy, Momentum</td>
</tr>
<tr>
<td></td>
<td></td>
<td>used in 4-Momenta Conservation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sum P_{\text{final}} = \sum P_{\text{initial}}$</td>
<td></td>
</tr>
<tr>
<td>4-WaveVector</td>
<td>[{rad}/m]</td>
<td>$K = K^\mu = (k^\mu) = (k^0, k^i) = \left(\omega/c, k\hat{n}/v_{\text{phase}}\right)$</td>
<td>Angular Frequency, WaveNumber</td>
</tr>
<tr>
<td></td>
<td></td>
<td>= $(1/cT, \hat{n}/\lambda) = 2\pi(1/cT, \hat{n}/\lambda)$</td>
<td>used in Relativistic Doppler Shift</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\omega_{\text{obs}} = \omega_{\text{emit}} / \left[\gamma(1 - \beta \cos[\theta])\right]$,</td>
<td>$k = \omega/c$ for photons</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k = \omega/c$</td>
<td></td>
</tr>
<tr>
<td>4-Gradient</td>
<td>[1/m]</td>
<td>$\partial = \partial^\mu = (\partial^0, \partial^i) = (\partial/c, -\nabla) \rightarrow (\partial/c, -\partial_x, -\partial_y, -\partial_z)$</td>
<td>Temporal Partial, Spatial Partial</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\rightarrow (\partial/\partial ct, -\partial/\partial x, -\partial/\partial y, -\partial/\partial z)$</td>
<td>used in SR Continuity Eqns., ProperTime</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\partial \cdot A = 0$ means $A$ is conserved</td>
<td></td>
</tr>
</tbody>
</table>

All of these are standard SR 4-Vectors, which can be found and used in a totally relativistic context, with no mention or need of QM.

I want to emphasize that these objects are ALL relativistic in origin.
SR + A few empirical facts:

**SRQM Overview**

<table>
<thead>
<tr>
<th>SR 4-Vector</th>
<th>Empirical Fact</th>
<th>What it means in SR...</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-Position $\mathbf{R} = (ct, r)$; alt. $\mathbf{X} = (ct, x)$</td>
<td>$\mathbf{R} = \langle \text{Event} \rangle$; alt. $\mathbf{X}$</td>
<td>Location of 4D Spacetime $\langle \text{Event} \rangle$</td>
</tr>
<tr>
<td>4-Velocity $\mathbf{U} = \gamma(c, u)$</td>
<td>$\mathbf{U} = \frac{d\mathbf{R}}{d\tau}$</td>
<td>Motion of 4D Spacetime $\langle \text{Event} \rangle$</td>
</tr>
<tr>
<td>4-Momentum $\mathbf{P} = (E/c, p) = (mc, p)$</td>
<td>$\mathbf{P} = m_0 \mathbf{U}$</td>
<td>$\langle \text{Events} \rangle$ described as Particles</td>
</tr>
<tr>
<td>4-WaveVector $\mathbf{K} = (\omega/c, k)$</td>
<td>$\mathbf{K} = \frac{\mathbf{P}}{\hbar}$</td>
<td>$\langle \text{Events} \rangle$ described as Waves</td>
</tr>
<tr>
<td>4-Gradient $\partial = \left( \frac{\partial}{c}, \nabla \right)$</td>
<td>$\partial = -i\mathbf{K}$</td>
<td>Alteration of 4D Spacetime $\langle \text{Event} \rangle$</td>
</tr>
</tbody>
</table>

The Axioms of SR, which is actually a GR limiting-case, lead us to the use of Minkowski SpaceTime and Physical 4-Vectors, which are elements of Minkowski Space (4D SpaceTime).

Empirical Observation leads us to the transformation relations between the components of these SR 4-Vectors, and to the chain of relations between the 4-Vectors themselves. These relations all turn out to be Lorentz Invariant Constants, whose values are measured empirically. They are manifestly invariant relations, true in all reference frames...

The combination of these SR objects and their relations is enough to derive RQM.
SRQM 4-Vector Study:
SR Lorentz Scalar Invariants

<table>
<thead>
<tr>
<th>SR 4-Vector</th>
<th>Lorentz Scalar Invariant</th>
<th>What it means in SR...</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-Position</td>
<td>( R \cdot R = (ct)^2 - r \cdot r = (ct_o)^2 = (ct)^2 )</td>
<td>SR Invariant Interval</td>
</tr>
<tr>
<td>4-Velocity</td>
<td>( U \cdot U = \gamma^2(c^2 - u \cdot u) = c^2 )</td>
<td>(&lt;\text{Event}&gt;) Motion Invariant Magnitude (c)</td>
</tr>
<tr>
<td>4-Momentum</td>
<td>( P \cdot P = (E/c)^2 - p \cdot p = (E_o/c)^2 )</td>
<td>Einstein Invariant Mass:Energy Relation</td>
</tr>
<tr>
<td>4-WaveVector</td>
<td>( K \cdot K = (\omega/c)^2 - k \cdot k = (\omega_o/c)^2 )</td>
<td>Wave/Dispersion Invariance Relation</td>
</tr>
<tr>
<td>4-Gradient</td>
<td>( \partial \cdot \partial = (\partial t/c)^2 - \nabla \cdot \nabla = (\partial \tau/c)^2 )</td>
<td>The d'Alembert Invariant Operator</td>
</tr>
</tbody>
</table>

All 4-Vectors have invariant magnitudes, found by taking the scalar product of the 4-Vector with itself. Quite often a simple expression can be found by examining the case when the spatial part is zero. This is usually found when the 3-velocity is zero. The temporal part is then specified by its “rest” value.

For example: \( P \cdot P = (E/c)^2 - p \cdot p = (E_o/c)^2 = (m_o c)^2 \)
\( E = \sqrt{[ (E_o)^2 + p \cdot p c^2 ]} \), from above relation
\( E = \gamma E_o \), using \( \gamma = 1/\sqrt{1-\beta^2} = \sqrt{1+\gamma^2 \beta^2} \) and \( \beta=v/c \)
meaning the relativistic energy \( E \) is equal to the relative gamma factor \( \gamma \) * the rest energy \( E_o \)
SRQM Chart:

Special Relativity $\rightarrow$ Quantum Mechanics

SRQM: The [SR$\rightarrow$QM] Interpretation of Quantum Mechanics

Special Relativity (SR) Axioms: Invariant Interval + LightSpeed ($c$) as Universal Physical Constant lead to SR, although technically SR is itself the Minkowski-SpaceTime low-curvature: "flat" limiting-case of GR.

{$c, \tau, m_0, \hbar, i = \{c: \text{SpeedOfLight}, \tau: \text{ProperTime}, m_0: \text{RestMass}, \hbar: \text{Dirac/PlanckReducedConstant} \ (\hbar = h/2\pi), \ i: \text{ImaginaryNumber} \sqrt{-1} \}$: are all Empirically-Measured SR Lorentz-Invariant Physical and/or Mathematical Constants = (0,1)

Standard SR 4-Vectors:
- 4-Position $R = (ct, r)$
- 4-Velocity $U = \gamma (c, u)$
- 4-Momentum $P = (E/c, p)$
- 4-WaveVector $K = (\omega/c, k)$
- 4-Gradient $\partial = (\partial_t / c, \nabla)$

Related by these SR Lorentz Invariants:
- $(R \cdot R) = (ct)^2$
- $(U \cdot U) = (c)^2$
- $(P \cdot P) = (m_0 c)^2$
- $(K \cdot K) = (m_0 c/h)^2$
- $(\partial \cdot \partial) = (-i m_0 c/h)^2 = -(m_0 c/h)^2 = \text{QM Relation} \rightarrow \text{RQM} \rightarrow \text{QM}$

SR + Empirically Measured Physical Constants lead to RQM via the (KG) Klein-Gordon Quantum Eqn, and thence to QM via the low-velocity limit { $|v| << c$ }, giving the Schrödinger Eqn. This fundamental KG Relation also leads to the other Quantum Wave Equations:

- RQM (massless, no rest-frame, Lorentzian): { $|v| = c : m_0 = 0$ }
- RQM (with non-zero mass, Lorentzian): { $0 <= |v| < c : m_0 > 0$ }
- QM (limit-case from RQM, Galilean): { $0 <= |v| << c : m_0 > 0$ }

Quantum Wave Equations:
- spin=0 boson field = 4-Scalar: Free Scalar Wave (Higgs)
- spin=1/2 fermion field = 4-Spinor: Weyl
- spin=1 boson field = 4-Vector: Maxwell (EM photonic)

SRQM: A treatise of SR$\rightarrow$QM by John B. Wilson (SciRealm@aol.com)
SRQM Diagram: 
RoadMap of SR (4-Vectors)

4-Position 
\( R = (ct, r) \)  
\( = \langle \text{Event} \rangle \)

4-Velocity 
\( U = \gamma (c, u) \)

4-WaveVector 
\( K = (\omega / c, k) \)

4-Momentum 
\( P = (mc, p) = (E/c, p) \)

4-Gradient 
\( \partial = (\partial / c, -\nabla) \)

SR 4-Tensor
(2,0)-Tensor \( T^{\mu \nu} \)  
(1,1)-Tensor \( T^\mu_\nu \), or \( T^\nu_\mu \)  
(0,2)-Tensor \( T_{\mu \nu} \)

SR 4-Vector
(1,0)-Tensor \( V^\mu = V = (\nu^0, \nu) \)  
SR 4-CoVector: OneForm  
(0,1)-Tensor \( V_\mu = (\nu^0, -\nu) \)

SR 4-Scalar
(0,0)-Tensor \( S \) or \( S_0 \)  
Lorentz Scalar

\[ \text{Trace}[T^\mu_\nu] = \eta_{\mu \nu} T^{\mu \nu} = T^{\mu}_\mu = T \]
\[ V \cdot V = V^\mu \eta_{\mu \nu} V^\nu = (\nu^0)^2 - \nu \cdot \nu = (\nu^0)^2 \]
\( = \) Lorentz Scalar Invariant
SRQM Diagram: RoadMap of SR (Connections)

4-Velocity
\[ U = \gamma(c, u) \]

4-Momentum
\[ P = (mc, p) = (E/c, p) \]

4-WaveVector
\[ K = (\omega/c, k) \]

4-Gradient
\[ \partial = (\partial/c, \cdot V) \]

4-Position
\[ R = (ct, r) = \langle \text{Event} \rangle \]

4-Velocity SRQM Interpretation of QM

\[ \text{SciRealm.org} \]

John B. Wilson
SciRealm@aol.com

http://scirealm.org/SRQM.pdf

\[ \text{SR} \rightarrow \text{QM} \]

Physics

A Tensor Study of Physical 4-Vectors

4-Vector SRQM Interpretation of QM

\[ \text{SR 4-Tensor} \]
\[ (2,0)-\text{Tensor} \ T^{\mu \nu} \]
\[ (1,1)-\text{Tensor} \ T^\mu_\nu \text{ or } T_{\nu}^\mu \]
\[ (0,2)-\text{Tensor} \ T_{\mu \nu} \]

\[ \text{SR 4-Scalar} \]
\[ (0,0)-\text{Tensor} \ S \text{ or } S_0 \]
\[ \text{Lorentz Scalar} \]

\[ \text{SR 4-Vector} \]
\[ (1,0)-\text{Tensor} \ V^\mu = V = (v^0, v) \]
\[ (0,1)-\text{Tensor} \ V_\mu = (v_0, -v) \]

\[ \text{SR 4-CoVector: OneForm} \]

\[ \text{Trace}[T^{\mu \nu}] = \eta_{\mu \nu} T^{\mu \nu} = T_{\nu}^\mu = T \]
\[ V \cdot V = V^\mu \eta_{\mu \nu} V^\nu = (v^0)^2 - v \cdot v = (v^0)^2 \]

\[ = \text{Lorentz Scalar Invariant} \]
SRQM Diagram: RoadMap of SR (Free Particle)

A Tensor Study of Physical 4-Vectors

4-Gradient = Alteration of SR <Events>
SR SpaceTime Dimension = 4
SR SpaceTime 4D Metric
SR Lorentz Transforms
SR Action → 4-Momentum
SR Proper Time
SR & QM Waves

SR Wave <Events> have 4-WaveVector = Substantiation oscillations proportional to mass:energy & 3-momentum

SR Particle <Events> have 4-Momentum = Substantiation mass:energy & 3-momentum

SR 4-Gradient = \( \frac{\partial}{\partial c} \)

\( \partial[R^\mu] = \eta^{\mu \nu} \) Minkowski Metric
\( \partial[R^\mu] = \Lambda^\nu_\nu \) Lorentz Transform

\( \partial-R=4 \) Space Time Dim

\( \partial-R=4 \)

\( \partial=R=4 \)

\( U \cdot \partial = \gamma \frac{d}{dt} \)

\( \partial \mu \) [ \( R \nu \) ] = \( \eta_{\mu \nu} \)

\( \text{Minkowski Metric} \)

\( -\partial \) [ \( \Phi \) ] phase,free = \( K \)

\( -\partial \) [ \( S \) ] action,free = \( P \)

\( -\partial \) [ \( \Phi \) ] phase = \( K \)

\( -\partial \) [ \( S \) ] action = \( P \)

\( \omega \) _o / \( E \) _o / \( c^2 \)

\( m_o / c^2 \)

\( E = mc^2 = \gamma E_o \)

SR 4-Vector

\( (2,0) \)-Tensor \( T^{\mu \nu} \)
\( (1,1) \)-Tensor \( V^\mu = V = (v^0, v) \)
\( (0,2) \)-Tensor \( T_{\mu \nu} \)

SR 4-CoVector: OneForm

\( (0,1) \)-Tensor \( V_\mu = (v_0, -v) \)

SR 4-Scalar

\( (0,0) \)-Tensor \( S \) or \( S_0 \)

Lorentz Scalar

Trace[\( T^{\mu \nu} \)] = \( \eta_{\mu \nu} T^{\mu \nu} = T^\nu_\nu = T \)
\( V \cdot V = V^\mu \eta_{\mu \nu} V^\nu = (v^\mu)^2 = v \cdot v \)

Lorentz Scalar Invariant
SR QM Waves
SR Proper Time → 4-WaveVector
SR Action → 4-Momentum
SR Lorentz Transforms
SR SpaceTime Dimension=4

4-Gradient=
mass:energy & 3-momentum oscillations proportional to
4-WaveVector=
of Physical 4-Vectors

4-Gradient=Alteration of SR <Events>
SR SpaceTime Dimension=4
SR SpaceTime 4D Metric
SR Lorentz Transforms
SR Action → 4-Momentum
SR Phase → 4-WaveVector
SR Proper Time
SR & QM Waves

4-Gradient
∂= (∂/c, -∇)
Minkowski Metric

∂[R']= Λv' 
Lorentz Transform

4-Vector SR

4-WaveVector
K=(ω/c, k)

∂-∂ = (∂/c)^2 - ∇·∇
= (∂/c)^2

d'Alembertian Particle Wave Equation in EM Potential

∂[R']= Λv' 
Lorentz Transform

-∂[Φ phase,free]= K
-∂[Φ phase]= K_T

Plane-Waves
K_T = -∂[Φ]

4-WaveVector=Substantiation of SR Wave <Events>
oscillations proportional to mass:energy & 3-momentum

K·K = (ω/c)^2·k·k
= (K_T(qω/E_0)A)·(K_T(qω/E_0)A) = (ω/c)^2

SR Particle <Events> have 4-Momentum=Substantiation mass:energy & 3-momentum

-∂[S action,free]= P
-∂[S action]= P_T

Hamilton-Jacobi
P_T = -∂[S]

Wave Velocity
ν group= c^2

ω/c^2

4-Vector SRQM

4-Momentum
P=(mc,p)=(E/c,p)

ω_o/E_o

4-TotalMomentum
P_T=(E/c, p, ν)

EM Faraday
A=(φ/c, a)

4-TotalMomentum
Q=(V/c, q)=q(φ/c, a)

EM Charge
q

4-EMVectorPotential
A=(φ/c, a)

4-Tensor
T

1,1-Tensor T^μν, or T_μν
4-CoVector:OneForm
0,1-Tensor V_μ = (ν_μ)

SR 4-Scalar
(0,0)-Tensor S or S_0
Lorentz Scalar

Trace[T^μν] = η_{μν}T^μν = T
V·V = V^μV_μ = (V_μ)^2 - (V·V) = (V^μ)^2
= Lorentz Scalar Invariant
SRQM Diagram:

Special Relativity $\rightarrow$ Quantum Mechanics

RoadMap of SR $\rightarrow$ QM (w/ EM Potential)

4-Gradient $\rightarrow$ Alteration of SR $\leftarrow$ Events

SR SpaceTime Dimension=4
SR Lorentz Transforms
SR Action $\rightarrow$ 4-Momentum
SR Phase $\rightarrow$ 4-WaveVector
SR Proper Time Derivative
SR & QM Invariant Waves
SR $\rightarrow$ QM Klein-Gordon Relativistic Quantum Particle in EM Potential

SR → QM

4-Gradient $\delta^\mu$ $\delta=(\partial/c,-\nabla)$ $=-k\hat{c}$

4-WaveVector $K^\mu$
$K=(\omega/c,k)$

4-Momentum $P^\mu$ $P=(mc,p)=(E/c,p)=m_o U$

SR 4-Vector
$(2,0)$-Tensor $T^{\mu\nu}$
$(1,1)$-Tensor $T^{\nu}$, or $T^{\nu}_\nu$
$(0,2)$-Tensor $T_{\mu\nu}$

SR 4-Vector
$(1,0)$-Tensor $V=V=(v^\nu,\nu)$

SR 4-CoVector:OneForm
$(0,1)$-Tensor $V_{\nu}=(\nu_\mu \nu)$

SR 4-Scalar
$(0,0)$-Tensor $S$ or $S_0$

Explicit SR Rules Quantum Principles

Existing SR Rules

Motion of SR $\leftarrow$ Events in In Space Time as both particles & waves

4-EM Vector Potential $A^\mu$ $A=(\phi/c,a)=(\phi/c^2)U$

EM Faraday $\mu

4-Tensor

4-SpaceTime Dim $R=(ct, r)$

ProperTime Derivative

SR Phase Lorentz Transforms

ProperTime $\partial\sigma/d\tau=\gamma d/dt$

Derivative

4-WaveVector Plane-Waves $K_{\nu}\rightarrow \Phi_{\nu}$

SR Phase Lorentz Scalars

Phase & Action

SR Action

SR 4-Tensor

4-Gradient Lorentz Transforms

Minkowski Metric

Alteration

SR Phase $\rightarrow$ 4-WaveVector

SR Action $\rightarrow$ 4-Momentum

SR Lorentz Transforms

SR $\rightarrow$ QM Klein-Gordon

Relativistic Quantum

SR $\rightarrow$ RQM

Klein-Gordon

4-GT

4-Potential

4-Momentum

4-Tensor

EM Charge

4-Vector SRQM Interpretation of QM

SciRealm.org

John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

4-Momentum $P^\mu$ $P=(mc,p)=(E/c,p)=m_o U$

4-TotMoment Conservation

$P^\nu_{\tau}=(P^{\nu}+Q)^\nu=(P^{\nu}+Q^\nu)$

Trace $T^\nu_\nu=\eta^\nu_\nu T^\nu_\nu=T^\nu_\nu=T$

$V^\nu V_\nu=\left((V^\nu)^2-\nu V\right)=(\nu V)^2$ Lorentz Scalar Invariant
# SRQM Study: The Empirical 4-Vector Facts

<table>
<thead>
<tr>
<th>SR 4-Vector</th>
<th>Empirical Fact</th>
<th>Discoverer</th>
<th>Physics</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-Position</td>
<td>( R = &lt;\text{Event}&gt; )</td>
<td>Newton+ Einstein</td>
<td>([ t &amp; r] ) Time &amp; Space (&lt;\text{time}&gt;) &amp; (&lt;\text{location}&gt;) [ R=(ct, r) ] SpaceTime as (4D=(1+3)D)</td>
</tr>
<tr>
<td>4-Velocity</td>
<td>( U = \frac{dR}{d\tau} )</td>
<td>Newton Einstein</td>
<td>([ v=\dot{r}=\frac{dr}{dt}] ) Calculus of motion [ U=\gamma(c, u)=\frac{dR}{d\tau} ] Gamma &amp; Proper Time</td>
</tr>
<tr>
<td>4-Momentum</td>
<td>( P = m_0 U )</td>
<td>Newton Einstein</td>
<td>([ p=mv ] ) Classical Mechanics [ P=(E/c, p)=m_0 U ] SR Mechanics</td>
</tr>
<tr>
<td>4-WaveVector</td>
<td>( K = \frac{P}{\hbar} )</td>
<td>Planck Einstein</td>
<td>([ h ] ) Photon Thermal Distribution [ E=h\nu=\hbar\omega ] Photoelectric Effect ((\hbar=h/2\pi))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>de Broglie</td>
<td>([ p=\hbar k ] ) Matter Waves [ P=(E/c, p)=\hbar K=\hbar(\omega/c, k) ] as 4-Vector Math</td>
</tr>
<tr>
<td>4-Gradient</td>
<td>( \partial = -iK )</td>
<td>Schrödinger</td>
<td>([ \omega=i\partial, k=-i\nabla ] ) (SR) Wave Mechanics [ P=(E/c, p)=i\hbar\partial=i\hbar(\partial/c, -\nabla) ] (QM) 4-Vector</td>
</tr>
</tbody>
</table>

1. The SR 4-Vectors and their components are related to each other via constants.
2. We have not taken any 4-vector relation as axiomatic, the constants come from experiment.
3. \(c, \tau, m_0, \hbar\) come from physical experiments, \((-i)\) comes from the general mathematics of waves.
SRQM Study: 4-Vector Relations Explained

<table>
<thead>
<tr>
<th>SR 4-Vector</th>
<th>Empirical Fact</th>
<th>What it means in SRQM...</th>
<th>Lorentz Invariant</th>
</tr>
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<tr>
<td>4-Position $\mathbf{R} = (ct, \mathbf{r})$</td>
<td>$\mathbf{R} = \langle \text{Event} \rangle$</td>
<td>SpaceTime as Unified Concept</td>
<td>$c = \text{LightSpeed}$</td>
</tr>
<tr>
<td>4-Velocity $\mathbf{U} = \gamma(c, \mathbf{u})$</td>
<td>$\mathbf{U} = \frac{d\mathbf{R}}{d\tau}$</td>
<td>Velocity is ProperTime Derivative</td>
<td>$\tau = t_o = \text{ProperTime}$</td>
</tr>
<tr>
<td>4-Momentum $\mathbf{P} = (E/c, \mathbf{p})$</td>
<td>$\mathbf{P} = m_o \mathbf{U}$</td>
<td>Mass:Energy-Momentum Equivalence</td>
<td>$m_o = \text{RestMass}$</td>
</tr>
<tr>
<td>4-WaveVector $\mathbf{K} = (\omega/c, \mathbf{k})$</td>
<td>$\mathbf{K} = \mathbf{P}/\hbar$</td>
<td>Wave-Particle Duality</td>
<td>$\hbar = \text{UniversalAction}$</td>
</tr>
<tr>
<td>4-Gradient $\partial = (\partial /c, -\nabla)$</td>
<td>$\partial = -i\mathbf{K}$</td>
<td>Unitary Evolution, Operator Formalism</td>
<td>$i = \text{ComplexSpace}$</td>
</tr>
</tbody>
</table>

Three old-paradigm QM Axioms: Particle-Wave Duality [(\(\mathbf{P})=\hbar(\mathbf{K})\)], Unitary Evolution [(\(\partial=-i\mathbf{K}\)], Operator Formalism [(\(\partial=-i\mathbf{K}\)] are actually just empirically-found constant relations between known SR 4-Vectors.

Note that these constants are in fact all Lorentz Scalar Invariants.

Minkowski Space and 4-Vectors also lead to idea of Lorentz Invariance. A Lorentz Invariant is a quantity that always has the same value, independent of the motion of inertial observers.

Lorentz Invariants can typically be derived using the scalar product relation.

$\mathbf{U} \cdot \mathbf{U} = c^2$, $\mathbf{U} \cdot \partial = d/d\tau$, $\mathbf{P} \cdot \mathbf{U} = m_o c^2$, etc.

A very important Lorentz invariant is the Proper Time $\tau$, which is defined as the time displacement between two points on a worldline that is at rest wrt. an observer. It is used in the relations between 4-Position $\mathbf{R}$, 4-Velocity $\mathbf{U} = d\mathbf{R}/d\tau$, and 4-Acceleration $\mathbf{A} = d\mathbf{U}/d\tau$. 
SRQM: The SR Path to RQM

Follow the Invariants...

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<td>SR Invariant Interval</td>
</tr>
<tr>
<td>4-Velocity</td>
<td>$\mathbf{U} \cdot \mathbf{U} = \gamma^2(c^2 - \mathbf{u} \cdot \mathbf{u}) = c^2$</td>
<td>Events move into future at magnitude c</td>
</tr>
<tr>
<td>4-Momentum</td>
<td>$\mathbf{P} \cdot \mathbf{P} = (m_o c)^2$</td>
<td>Einstein Mass:Energy Relation</td>
</tr>
<tr>
<td>4-WaveVector</td>
<td>$\mathbf{K} \cdot \mathbf{K} = (m_o c/\hbar)^2 = (\omega_o/c)^2$</td>
<td>Matter-Wave Dispersion Relation</td>
</tr>
<tr>
<td>4-Gradient</td>
<td>$\mathbf{\partial} \cdot \mathbf{\partial} = (-im_o c/\hbar)^2 = -(m_o c/\hbar)^2$</td>
<td>The Klein-Gordon Equation $\rightarrow$ RQM!</td>
</tr>
</tbody>
</table>

$\mathbf{U} = \frac{d\mathbf{R}}{d\tau}$

Remember, everything after 4-Velocity was just a constant times the last 4-vector, and the Invariant Magnitude of the 4-Velocity is itself a constant $\mathbf{P} = m_o \mathbf{U}$, $\mathbf{K} = \mathbf{P}/\hbar$, $\mathbf{\partial} = -i\mathbf{K}$, so e.g. $\mathbf{P} \cdot \mathbf{P} = m_o \mathbf{U} \cdot m_o \mathbf{U} = m_o^2 \mathbf{U} \cdot \mathbf{U} = (m_o c)^2$

The last equation is the Klein-Gordon RQM Equation, which we have just derived without invoking any QM axioms, only SR plus a few empirical facts.
SRQM: Some Basic 4-Vectors

4-Momentum, 4-WaveVector, 4-Position, 4-Velocity, 4-Gradient, Wave-Particle

\( \{ P = (E/c, p) = -\partial[S] = (-\partial c/\partial[S], \nabla[S]) \} \)
- **temporal component** \( E = -\partial_t [S] = -\partial[S] \)
- **spatial component** \( p = \nabla [S] \)

**Note** This is the Action \( S_{action} \) for a free particle. Generally Action is for the 4-TotalMomentum \( P_\tau \) of a system.

See SR Wave Definition for info on the Lorentz Scalar Invariant SR WavePhase. 
\( \{ K = (\omega/c, k) = -\partial[\Phi] = (-\partial/c\partial[\Phi], \nabla[\Phi]) \} \)
- **temporal component** \( \omega = -\partial_t [\Phi] = -\partial[S] \)
- **spatial component** \( k = \nabla [\Phi] \)

**Note** This is the Phase \( \Phi \) for a single free plane-wave. Generally WavePhase is for the 4-TotalWaveVector \( K_\tau \) of a system.
The 4-Vector Wave-Particle relation is inherent in all particle types: Einstein-de Broglie \( P = \frac{E}{c}, \mathbf{p} = \hbar K = \hbar \left( \frac{\omega}{c}, \mathbf{k} \right) \).

All waves can superpose, interfere, diffract: Water waves, gravitational waves, photonic waves of all frequencies, etc. In all cases: experiments using single particles build the diffraction/interference pattern over the course many iterations.

**Photon/light Diffraction:** Photonic particles diffracted by matter particles. Photons of any frequency encounter a translucent “solid=matter” object, grating, or edge. Most often encountered are diffraction gratings and the famous double-slit experiment.

**Matter Diffraction:** Matter particles diffracted by matter particles. Electrons, neutrons, atoms, small molecules, buckyballs (fullerenes), macromolecules, etc. have been shown to diffract through crystals. Crystals may be solid single pieces or in powder form.

**Kapitsa-Dirac Diffraction:** Matter particles diffracted by photonic standing waves. Electrons, atoms, super-sonic atom beams have been diffracted from resonant standing waves of light.

**Photonic-Photonic Diffraction:** Delbruck scattering & Light-by-light scattering. Light-by-light scattering/two-photon physics/gamma-gamma physics. Normally, photons do not interact, but at high enough relative energy, virtual particles can form which allow interaction.
SRQM: Hold on, aren't you getting the “\(\hbar\)" from a QM Axiom?

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<th>SR Empirical Fact</th>
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<td>(K = (\omega/c, k) = (\omega/c, \omega \hat{n}/v_{\text{phase}}) = (\omega_0/c^2)U)</td>
<td>Wave-Particle Duality</td>
</tr>
</tbody>
</table>

(h) is actually an empirically measurable quantity, just like (e) or (c). It can be measured classically from the photoelectric effect, from the inverse photoelectric effect, from LED’s (injection electroluminescence), from Atomic Line Spectra (Bohr Atom), from the Duane-Hunt Law in Bremsstrahlung, from Electron Diffraction in crystals, from the Watt/Kibble-Balance, from Incandescent Blackbody Intensity-Temperature Relations, from Compton Scattering, from the Stern-Gerlach experiment, etc. See [http://scirealm.org/Physics-PlanckConstantViaLEDs.html](http://scirealm.org/Physics-PlanckConstantViaLEDs.html)

For the LED experiment, one uses several different LED’s, each with its own characteristic wavelength. For the LED experiment, one uses several different LED’s, each with its own characteristic wavelength. (\(\text{http://scirealm.org/SRQM.pdf}\) & SciRealm@ao.com)

The spatial component (due to De Broglie) follows naturally from the temporal component (due to Einstein) via to the nature of 4-Vector (Tensor) mathematics.

This is also derivable from pure SR 4-Vector (Tensor) arguments: \(P = m_u U = (E/c^2)U\) and \(K = (\omega_0/c^2)U\)

Since \(P\) and \(K\) are both Lorentz Scalar proportional to \(U\), then by the rules of tensor mathematics, \(P\) must also be Lorentz Scalar proportional to \(K\) i.e. Tensors obey certain mathematical structures:

Transitivity (if \(a \sim b\) and \(b \sim c\), then \(a \sim c\)) & Euclideaness: \(\{a \sim c\} \text{ and } \{b \sim c\} \Rightarrow \{a \sim b\}\)

This invariant proportional constant is empirically measured to be (h)

\(\gamma \rightarrow \gamma = \sqrt{1 - \frac{v^2}{c^2}}\)

Particle: \(\frac{E}{c} = \frac{m_0}{c} \cdot \frac{\omega_0}{c}\)

Wave: \(\frac{4V}{c} = \frac{(\omega_0)}{c}\)

4-Momentum: \(P = (m_c)^2\)

4-WaveVector: \(K = (\omega_0/c_k)\)

Also from standard SR Lorentz 4-Vector Scalar Products:

\(P \cdot U = E/c = m_0 \cdot \omega_0\)

\(K \cdot U = \omega_0\)

\(P \cdot K = m_0 \cdot \omega_0\)

\(P \cdot P = (m_0)^2\)

\(K \cdot K = (\omega_0)^2\)

\(P \cdot U = E/c = m_0 \cdot \omega_0\)

\(K \cdot U = \omega_0\)

\(P \cdot K = m_0 \cdot \omega_0\)

\(P \cdot P = (m_0)^2\)

\(K \cdot K = (\omega_0)^2\)

\(P \cdot P = (m_0)^2\)

\(K \cdot K = (\omega_0)^2\)
SRQM:
Hold on, aren't you getting the “K” from a QM Axiom?

<table>
<thead>
<tr>
<th>SR 4-Vector</th>
<th>SR Empirical Fact</th>
<th>What it means...</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-WaveVector</td>
<td>( \mathbf{K} = \left( \frac{\omega}{c}, \mathbf{k} \right) = \left( \frac{\omega}{c}, \omega \hat{n} / \nu_{\text{phase}} \right) = \left( \frac{\omega}{c^2} \right) \mathbf{U} )</td>
<td>Wave-Particle Duality</td>
</tr>
</tbody>
</table>

\( \mathbf{K} \) is a standard SR 4-Vector, used in generating the SR formulae:

**Relativistic Doppler Effect:**
\[
\omega_{\text{obs}} = \frac{\omega_{\text{emit}}}{\gamma(1 - \beta \cos[\theta])}, \quad |\mathbf{k}| = k = \frac{\omega}{c} \text{ for photons}
\]

**Relativistic Aberration Effect:**
\[
\cos[\theta_{\text{obs}}] = \frac{(\cos[\theta_{\text{emit}}] + |\beta|)}{(1 + |\beta| \cos[\theta_{\text{emit}}])}
\]

The 4-WaveVector \( \mathbf{K} \) can be derived in terms of periodic motion, where families of surfaces move through space as time increases, or alternately, as families of hypersurfaces in SpaceTime, formed by all events passed by the wave surface. The 4-WaveVector is everywhere in the direction of propagation of the wave surfaces.

\[
\mathbf{K} = -\partial \Phi_{\text{phase, planewave}}
\]

From this structure, one obtains relativistic/wave optics without ever mentioning QM.
SRQM:
Hold on, aren't you getting the “-i” from a QM Axiom?

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<td>4-Gradient</td>
<td>( \partial = (\partial t/c, -\nabla) = -iK )</td>
<td>Unitary Evolution of States Operator Formalism</td>
</tr>
</tbody>
</table>

\[ \partial = -iK \] gives the sub-equations \[ \partial_t = -i\omega \] and \[ \nabla = iK \], and is certainly the main equation that relates QM and SR by allowing Operator Formalism. But, this is a basic equation regarding the general mathematics of plane-waves; not just quantum-waves, but anything that can be mathematically described by plane-waves and superpositions of plane-waves… This includes purely SR waves, an example of which would be EM plane-waves (i.e. photons)...

\( \psi(t,r) = ae^{i(k \cdot r - \omega t)} \): Standard mathematical plane-wave equation

\( \partial[\psi(t,r)] = \partial[ae^{i(k \cdot r - \omega t)}] = (-i\omega)[ae^{i(k \cdot r - \omega t)}] = (-i\omega)\psi(t,r), \text{ or } [\partial_t = -i\omega] \nabla[\psi(t,r)] = \nabla[ae^{i(k \cdot r - \omega t)}] = (iK)[ae^{i(k \cdot r - \omega t)}] = (iK)\psi(t,r), \text{ or } [\nabla = iK] \)

In the more economical SR notation:

\( \partial[\psi(R)] = \partial[ae^{-iK \cdot R}] = (-iK)[ae^{-iK \cdot R}] = (-iK)\psi(R), \text{ or in 4-Vector shorthand } [\partial = -iK] \)

This one is more of a mathematical empirical fact, but regardless, it is not axiomatic. It can describe purely SR waves, again without any mention of QM.
SRQM:
Hold on, aren't you getting the “∂” from a QM Axiom?

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<td>( \partial = (\partial_t/c, -\nabla) = -iK )</td>
<td>4D Gradient Operator</td>
</tr>
</tbody>
</table>

\[ \partial = (\partial_t/c, -\nabla) \] is the SR 4-Vector Gradient Operator. It occurs in a purely relativistic context without ever mentioning QM.

\[
\partial \cdot X = \partial_{\mu} \eta_{\mu\nu} X^\nu = (\partial_t/c, -\nabla) \cdot (ct, x) = (\partial_t/c[ct] - (-\nabla \cdot x)) = (\partial_t + \nabla \cdot x) (1) + (3) = 4
\]

The 4-Divergence of the 4-Position gives the dimensionality of SpaceTime.

\[
\partial[J] = \partial^\mu [X^\nu] = (\partial_t/c, -\nabla)[(ct, x)] = (\partial_t/c[ct], -\nabla[x]) = \text{Diag}[1, -I(3)] = \eta^{\mu\nu}
\]

The 4-Gradient acting on the 4-Position gives the Minkowski Metric Tensor.

\[
\partial \cdot J = \partial_{\mu} \eta_{\mu\nu} J^\nu = (\partial_t/c, -\nabla) \cdot (\rho c, j) = (\partial_t/c[\rho c] - (-\nabla \cdot j)) = (\partial_t[\rho] + \nabla \cdot j) = 0
\]

The 4-Divergence of the 4-CurrentDensity is equal to 0 for a conserved current. It can be rewritten as \( \partial_\mu \rho = -\nabla \cdot j \), which means that the time change of ChargeDensity is balanced by the space change or divergence of CurrentDensity. It is a Continuity Equation, giving local conservation of ChargeDensity. It is related to Noether's Theorem.
SRQM: 
Hold on, doesn’t using “∂” in an Equation of Motion presume a QM Axiom?

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<td>4-(Position)Gradient</td>
<td>( \partial_R = \partial = (\partial_t/c, -\nabla) = -iK )</td>
<td>4D Gradient Operator</td>
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Klein-Gordon Relativistic Quantum Wave Equation
\[
\partial \cdot \partial[\Psi] = -(m_0c/\hbar)^2[\Psi] = -\left(\frac{\omega_0}{c}\right)^2[\Psi]
\]

Relativistic Euler-Lagrange Equations
\[
\partial_R[L] = \frac{d}{d\tau}\partial_U[L]: \{\text{particle format}\}
\]
\[
\partial_{\{\phi\}}[L] = (\partial_R) \partial_{[\partial R(\phi)]}[L]: \{\text{density format}\}
\]

\([\partial = (\partial_t/c, -\nabla)]\) is the SR 4-Vector (Position)Gradient Operator.
It occurs in a purely relativistic context without ever mentioning QM.
There is a long history of using the gradient operator on classical physics functions, in this case the Lagrangian. And, in fact, it is another area where the same mathematics is used in both classical and quantum contexts.
The QM Schrödinger Relation

This is derived from the combination of:

- The Einstein-de Broglie Relation
  \[ P = \hbar k \]

- Complex Plane-Waves
  \[ K = i \frac{\partial}{\partial t} = -i \nabla \]

These are the standard QM Schrödinger Relations.

It is this Lorentz Scalar Invariant relation (\( \hbar \)) which connects the 4-Momentum to the 4-Gradient, making it into a QM operator.

Note that these 4-Vectors are already connected in multiple ways in standard SR.

SR 4-Vector

\[ \mathbf{V} = (v^0, \mathbf{v}) \]

SR 4-Scalar

\[ \mathbf{S} = \mathbf{V} \cdot \mathbf{V} = \mathbf{V} \cdot \mathbf{V} = (v^0)^2 - \mathbf{v} \cdot \mathbf{v} \]

\( = \text{Lorentz Scalar Invariant} \)

SR 4-Tensor

\[ T_{\mu \nu} \]

Trace[\( T_{\mu \nu} \)] = \( \eta_{\mu \nu} T^{\mu \nu} = T^\nu_\nu = T \)

\[ \mathbf{V} \cdot \mathbf{V} = \mathbf{V} \cdot \mathbf{V} = (v^0)^2 - \mathbf{v} \cdot \mathbf{v} \]

\( = \text{Lorentz Scalar Invariant} \)
SRQM: Review of SR 4-Vector Mathematics

4-Gradient \( \partial = (\partial/c, -\nabla) \)
4-Position \( X = (ct, x) \)
4-Velocity \( U = \gamma(c, u) \)
4-Momentum \( P = (E/c, p) = (E_0/c^2)U \)
4-WaveVector \( K = (\omega/c, k) = (\omega_0/c^2)U \)

\[ \partial \cdot X = (\partial/c, -\nabla) \cdot (ct, x) = (\partial/c[ct] - (-\nabla \cdot x)) = 1 - (-3) = 4: \]
\[ U \cdot \partial = \gamma(c, u) \cdot (\partial/c, -\nabla) = \gamma(\partial + u \cdot \nabla) = \gamma(d/dt) = d/d\tau: \]
\[ \partial[X] = (\partial/c, -\nabla)(ct, x) = (\partial/c[ct], -\nabla[x]) = \text{Diag}[1, -1] = \eta^{iv}: \]
\[ \partial[K] = (\partial/c, -\nabla)(\omega/c, k) = (\partial/c[\omega/c], -\nabla[k]) = [[0]] \]
\[ K \cdot X = (\omega/c, k) \cdot (ct, x) = (\omega t - k \cdot x) = \phi: \]
\[ \partial[K \cdot X] = \partial[K] \cdot X + K \cdot \partial[X] = K = -\partial[\phi]: \]

\[ (\partial \cdot \partial)[K \cdot X] = ((\partial/c)^2 - \nabla \cdot \nabla)(\omega t - k \cdot x) = 0 \]
\[ (\partial \cdot \partial)[K \cdot X] = \partial \cdot (\partial[K \cdot X]) = \partial \cdot K = 0: \]

let \( f = ae^b(K \cdot X) \):
then \( \partial[f] = (-iK)ae^{-i(K \cdot X)} = (-iK)f: (\partial = -iK): \)
and \( \partial \partial[f] = (-i)^2(K \cdot K)f = -(\omega_0/c)^2f: \)
\[ (\partial \cdot \partial) = (\partial/c)^2 - \nabla \cdot \nabla = -(\omega_0/c)^2: \]
Dimensionality of SpaceTime
Derivative wrt. ProperTime is Lorentz Scalar
The Minkowski Metric
Phase of SR Wave
Neg 4-Gradient of Phase gives 4-WaveVector
Wave Continuity Equation, No sources or sinks
Standard mathematical plane-waves if \{ b = -i \}
Unitary Evolution, Operator Formalism
The Klein-Gordon Equation → RQM

Note that no QM Axioms are assumed: This is all just pure SR 4-vector (tensor) manipulation
Klein-Gordon Equation: \( \partial \cdot \partial = (\partial / c)^2 - \nabla \cdot \nabla = -(m_0 c / \hbar)^2 = -(\omega_0 / c)^2 = -(1/\lambda c)^2 \)

Let \( X_T = (ct + c\Delta t, x) \), then \( \partial[X_T] = (\partial / c, -\nabla)(ct + c\Delta t, x) = \text{Diag}[1, -I(\eta)] = \partial[X] = \eta^{\mu\nu} \)
so \( \partial[X_T] = \partial[X] \) and \( \partial[K] = [0] \)
let \( f = ae^{-i(K \cdot X_T)} \), the time translated version
\[
(\partial \cdot \partial)[f] \\
\partial \cdot (\partial[f]) \\
\partial \cdot (\partial[e^{-i(K \cdot X_T)}]) \\
\partial \cdot (e^{-i(K \cdot X_T)}\partial[-i(K \cdot X_T)]) \\
-i\partial \cdot (f\partial[K \cdot X_T]) \\
-i\partial[f]\partial[K \cdot X_T] + \Psi(\partial \cdot \partial)[K \cdot X_T] \\
(-i)^2f(\partial[K \cdot X_T])^2 + 0 \\
(-i)^2f(\partial[K] \cdot X_T + K \cdot \partial[X_T])^2 \\
(-i)^2f(0 + K \cdot \partial[X])^2 \\
(-i)^2f(K)^2 \\
-(K \cdot K)f \\
-(\omega_0 / c)^2f \]
What does the Klein-Gordon Equation give us? A lot of RQM!

Relativistic Quantum Wave Equation: \( \partial \cdot \partial = (\partial_t/c)^2 - \nabla \cdot \nabla = -(m_o c^2/\hbar)^2 = (im_o c/\hbar)^2 = -(\omega_o/c)^2 \)

The Klein-Gordon Eqn is itself the Relativistic Quantum Equation for spin=0 particles (4-Scalars)
Factoring the KG Eqn leads to the RQM Dirac Equation for spin=1/2 particles (4-Spinors)
Applying the KG Eqn to a 4-Vector field leads to the RQM Proca Equation for spin=1 particles (4-Vectors)

Taking the low-velocity-limit of the KG leads to the standard QM non-relativistic Schrödinger Eqn, for spin=0
Taking the low-velocity-limit of the Dirac leads to the standard QM non-relativistic Pauli Eqn, for spin=1/2

Setting RestMass \( m_o \rightarrow 0 \) leads to the RQM Free Wave Eqn., Weyl Eqn., and Free Maxwell (Standard EM) Eqn.

In all of these cases, the equations can be modified to work with various potentials by using more
SR 4-Vectors, and more empirically found relations between them, e.g. the Minimal Coupling Relations:
4-TotalMomentum \( P_T = P + qA \), where \( P \) is the particle 4-Momentum, \( q \) is a charge, and \( A \) is a 4-VectorPotential, typically the 4-EMVectorPotential.

Also note that generating QM from RQM (via a low-energy limit) is much more natural than attempting to “relativize or
generalize” a given NRQM equation. Facts assumed from a non-relativistic equation may or may not be applicable to
a relativistic one, whereas the relativistic facts are still true in the low-velocity limiting-cases. This leads to the idea
that QM is an approximation only of a more general RQM, just as SR is an approximation only of GR.
## SRQM:
### Relativistic Quantum Wave Eqns.

| Spin-(Statistics) | Relativistic Light-like Mass = 0 | Relativistic Matter-like Mass > 0 | Non-Relativistic Limit (|v|<<c) Mass >0 | Field Representation |
|-------------------|----------------------------------|-----------------------------------|-----------------------------------|---------------------|
| Bose-Einstein=n   | Free Wave N-G Bosons             | Klein-Gordon                      | Schrödinger                      | Scalar              |
|                    | (∂·∂)Ψ = 0                       |                                   | Common NRQM Systems              | (0-Tensor)          |
|                    |                                  |                                   |                                   | Ψ = Ψ[K_{x0}x^4]   |
|                    |                                  |                                   |                                   | = Ψ[Φ]              |
|                    |                                  |                                   |                                   |                     |
|                    |                                  |                                   |                                   | Spinor              |
|                    |                                  |                                   |                                   | Ψ = Ψ[K_{x0}x^4]   |
|                    |                                  |                                   |                                   | = Ψ[Φ]              |
|                    |                                  |                                   |                                   |                     |
|                    |                                  |                                   |                                   | 4-Vector            |
|                    |                                  |                                   |                                   | Ψ = Ψ[K_{x0}x^4]   |
|                    |                                  |                                   |                                   | = Ψ[Φ]              |
| Fermi-Dirac=n/2    |                                  |                                   |                                   |                     |

### Field Representation

0-(Boson)
- **Maxwell**
- Photons/Gluons
- \((∂·∂)A = 0\) free
- \((∂·∂)A = μ_eJ\) where current src
- \((∂·∂)A = \mu_e \overline{\jmath} γ_\mu γ_\nu \psi\) QED

1/2-(Fermion)
- **Weyl**
- Idealized Matter Neutinos
- \((σ·∂)Ψ = 0\)
- factored to Right & Left Spinors
- \((σ·∂)Ψ_R = 0, (σ·∂)Ψ_L = 0\)
- \(L = i\overline{Ψ}_R σ_\mu δ_{\mu\nu}Ψ_L, L = i\overline{Ψ}_L σ_\mu δ_{\mu\nu}Ψ_R\)

1-(Fermion)
- **Dirac**
- Matter Leptons/Quarks
- \((iγ·∂ - m_0c/ℏ)Ψ = 0\)
- \((γ·∂ + im_0c/ℏ)Ψ = 0\)
- \(L = i\overline{Ψ}_R σ_\mu δ_{\mu\nu}Ψ_L, m_0c^2\overline{Ψ}_R Ψ_L\)

3/2-(Fermion)
- **Gravitino?**
- \((ε_{μνρσγ} 5γ_ν∂_ρ)ψ_σ = 0\)
- \(Rarita-Schwinger\)
- \((ε_{μνρσγ} 5γ_ν∂_ρ + m_0σ_μσ)ψ_σ = 0\)

2-(Boson)
- **Einstein**
- Tensor
- \(G_{μν} = \overline{Ψ} μ_ν + μ_ν \overline{Ψ}\)

### 4-Vector SRQM Interpretation of QM

\(\psi = \gamma_\mu \psi^μ\)

SciRealm.org

John B. Wilson
SciRealm@aol.com

http://scirealm.org/SRQM.pdf
Klein-Gordon Equation: \( \partial \cdot \partial = (\partial_t/c)^2 - \nabla \cdot \nabla = -(m_o c/\hbar)^2 \)

Since the 4-vectors are related by constants, we can go back to the 4-Momentum description/representation:

\( (\partial_t/c)^2 - \nabla \cdot \nabla = -(m_o c/\hbar)^2 \)
\( (E/c)^2 - p \cdot p = (m_o c)^2 \)
\( E^2 - c^2 p \cdot p - (m_o c^2)^2 = 0 \)

Factoring: \[ E - c \alpha \cdot p - \beta m_o c^2 \] \[ E + c \alpha \cdot p + \beta m_o c^2 \] = 0

E & p are quantum operators, \( \alpha \) & \( \beta \) are matrices which must obey \( \alpha \beta = -\beta \alpha, \alpha \alpha = -\alpha \alpha, \alpha^2 = \beta^2 = I \)
The left hand term can be set to 0 by itself, giving...
\[ E - c \alpha \cdot p - \beta m_o c^2 \] = 0, which is the momentum-representation form of the Dirac equation

Remember: \( P^\mu = (p^0, p) = (E/c, p) \) and \( \alpha^\mu = (\alpha_0, \alpha) \) where \( \alpha_0 = I_{2(2)} \)

\[ E - c \alpha \cdot p - \beta m_o c^2 \] = \[ c \alpha_0 p^0 - c \alpha \cdot p - \beta m_o c^2 \] = \[ c \alpha_0 P^\mu - \beta m_o c^2 \] = 0
\[ \alpha^\mu P_\mu - \beta (m_o c) \] = \[ i \hbar \alpha^\mu \partial_\mu - \beta (m_o c) \] = 0
\( \alpha^\mu \partial_\mu = -\beta (im_o c/\hbar) \)

Transforming from Pauli Spinor (2 component) to Dirac Spinor (4 component) form:
Dirac Equation: \( (\gamma^\mu \partial_\mu) \psi = -(im_o c/\hbar)\psi \)

Thus, the Dirac Eqn is guaranteed by construction to be one solution of the KG Eqn

The KG Equation is at the heart of all the various relativistic wave equations, which differ based on mass and spin values, but all of them respect \( E^2 - c^2 p \cdot p - (m_o c^2)^2 = 0 \)
SRQM Study:
Lots of Relativistic Quantum Wave Equations
A lot of RQM!

Relativistic Quantum Wave Equation: \( \partial \cdot \partial = (\partial / c)^2 - \nabla \cdot \nabla = -(m_0 c / \hbar)^2 = (i m_0 c / \hbar)^2 = -(\omega_0 / c)^2 \)

The Klein-Gordon Eqn is itself the Relativistic Quantum Equation for spin=0 particles \{Higgs\} (4-Scalars)
Factoring the KG Eqn ("square root method") leads to the RQM Dirac Equation for spin=1/2 particles (4-Spinors)
Applying the KG Eqn to a SR 4-Vector field leads to the RQM Proca Equation for spin=1 particles (4-Vectors)

Setting RestMass \( m_0 \rightarrow 0 \) leads to the:
RQM Free Wave (4-Scalar massless)
RQM Weyl (4-Spinor massless)
Free Maxwell Eqns (4-Vector massless) = Standard EM

So, the same Relativistic Quantum Wave Equation is simply applied to different SR Tensorial Quantum Fields
See Mathematical_formulation_of_the_Standard_Model at Wikipedia:

4-Scalar (massive)  Higgs Field \( \phi \)  \[ \partial \cdot \partial = -(m_0 c / \hbar)^2 \phi \]  Free Field Eqn→Klein-Gordon Eqn  \( \partial \cdot \partial \phi = -(m_0 c / \hbar)^2 \phi \)
4-Vector (massive)  Weak Field \( Z^\mu, W^\mu \)  \[ \partial \cdot \partial = -(m_0 c / \hbar)^2 Z^\mu \]  Free Field Eqn→Proca Eqn  \( \partial \cdot \partial Z^\mu = -(m_0 c / \hbar)^2 Z^\mu \)
4-Vector (massless \( m_0 = 0 \))  Photon Field \( A^\mu \)  \[ \partial \cdot \partial = 0 \]  Free Field Eqn→EM Wave Eqn  \( \gamma \cdot \partial A^\mu = 0 \)
4-Spinor (massive)  Fermion Field \( \psi \)  \[ \gamma \cdot \partial = -i m_0 c / \hbar \psi \]  Free Field Eqn→Dirac Eqn  \( \gamma \cdot \partial \psi = -i m_0 c / \hbar \psi \)

*The Fermion Field is a special case, the Dirac Gamma Matrices \( \gamma^\mu \) and 4-Spinor field \( \psi \) work together to preserve Lorentz Invariance.
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4-Vector SRQM Study:
Lots of Relativistic Quantum Wave Equations
A lot of RQM!

Relativistic Quantum Wave Equation: \( \partial \cdot \partial = (\partial_v/c)^2 - \nabla \cdot \nabla = -(m_o c/\hbar)^2 = -(\omega_o/c)^2 \)
\( \partial \cdot \partial = -(m_o c/\hbar)^2 \)

\((\partial \cdot \partial)A^\nu = 0\): The Free Classical Maxwell EM Equation {no source, no spin effects}

\((\partial \cdot \partial)A^\nu = \mu_o J^\nu\): The Classical Maxwell EM Equation {with 4-Current J source, no spin effects}

\((\partial \cdot \partial)A^\nu = q(\bar{\psi} \gamma^\nu \psi)\): The QED Maxwell EM Spin-1 Equation {with QED source, including spin effects}

So, the same Relativistic Quantum Wave Equation is simply applied to different SR Tensorial Quantum Fields

See Mathematical formulation of the Standard Model at Wikipedia:

4-Scalar (massive) Higgs Field \( \phi \) [\( \partial \cdot \partial = -(m_o c/\hbar)^2 \phi \)] Free Field Eqn→Klein-Gordon Eqn \( \partial \cdot \partial \phi = -(m_o c/\hbar)^2 \phi \)

4-Vector (massive) Weak Field \( Z^\nu, W^\mu \) [\( \partial \cdot \partial = -(m_o c/\hbar)^2 Z^\nu \)] Free Field Eqn→Proca Eqn \( \partial \cdot \partial Z^\nu = -(m_o c/\hbar)^2 Z^\nu \)

4-Vector (massless \( m_o =0 \)) Photon Field \( A^\mu \) [\( \partial \cdot \partial = 0 \)\( A^\mu \)] Free Field Eqn→EM Wave Eqn \( \partial \cdot \partial A^\mu = 0 \)

4-Spinor (massive) Fermion Field \( \psi \) \( [\gamma \cdot \partial = -im_o c/\hbar] \psi \) Free Field Eqn→Dirac Eqn \( \gamma \cdot \partial \psi = -(im_o c/\hbar) \psi \)

*The Fermion Field is a special case, the Dirac Gamma Matrices \( \gamma^\mu \) and 4-Spinor field \( \psi \) work together to preserve Lorentz Invariance.

We can also do the same physics using Lagrangian Densities.

Proca Lagrangian Density \( L = -\frac{1}{2}(\partial \cdot B^\nu - \partial \cdot B^\nu)(\partial^\nu B^\nu - \partial^\nu B^\nu) + (m_o c/\hbar)^2 B^\nu B^\nu \) : with \( B^\nu = (\phi/c, a)[(ct, r)] \) is a generalized complex 4-(Vector)Potential

KG Lagrangian Density \( L = -\eta \psi^\dagger (\partial \cdot \psi^\dagger - \partial \cdot \psi) - (m_o c/\hbar)^2 \psi^\dagger \psi \) : with \( \psi = \psi[R] = \psi[(ct, r)] \)

Dirac Lagrangian Density \( L = \bar{\psi}(\gamma \cdot \partial - m_o c/\hbar)\psi : with \psi = a spinor \psi[(ct, r)] \)

QED Lagrangian Density \( L = \bar{\psi}(i\hbar \gamma^\mu D^\mu - m_o c)\psi - (1/4)F^\mu \nu F^\mu \nu \) : with \( D^\mu = \partial^\mu + iqA^\mu + iqB^\mu \) and \( A^\mu = \text{EM field of the } e^-; B^\mu = \text{external source EM field} \)}
In relativistic quantum mechanics and quantum field theory, the Bargmann–Wigner equations describe free particles of arbitrary spin $j$, an integer for bosons ($j = 1, 2, 3 ...$) or half-integer for fermions ($j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} ...$). The solutions to the equations are wavefunctions, mathematically in the form of multi-component spinor fields.

Bargmann–Wigner equations: 
$$(-\gamma^\mu P_\mu + mc)_{\alpha r \cdots \alpha r \cdots \alpha 2j} \psi_{\alpha 1 \cdots \alpha r \cdots \alpha 2j} = 0$$

In relativistic quantum mechanics and quantum field theory, the Joos–Weinberg equation is a relativistic wave equations applicable to free particles of arbitrary spin $j$, an integer for bosons ($j = 1, 2, 3 ...$) or half-integer for fermions ($j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} ...$). The solutions to the equations are wavefunctions, mathematically in the form of multi-component spinor fields. The spin quantum number is usually denoted by $s$ in quantum mechanics, however in this context $j$ is more typical in the literature.

Joos–Weinberg equation: 
$$[\gamma^\mu_{\nu_1 \cdots \nu_2 \cdots \nu_2j} P_{\mu_1} P_{\mu_2} \cdots P_{\mu_2j} + (mc)^2] \Psi = 0$$

The primary difference appears to be the expansion in either the wavefunctions for (BW) or the Dirac Gamma's for (JW)

For both of these: A state or quantum field in such a representation would satisfy no field equation except the Klein-Gordon equation.

Yet another form is the Duffin-Kemmer-Petiau Equation vs Dirac Equation

DKP Eqn {spin 0 or 1}: 
$$(i\hbar\beta^a \partial_a - m_c)\Psi = 0$$, with $\beta^a$ as the DKP matrices

Dirac Eqn (spin $\frac{1}{2}$): 
$$(i\hbar\gamma^a \partial_a - m_c)\Psi = 0$$, with $\gamma^a$ as the Dirac Gamma matrices
SRQM: A treatise of SR→QM by John B. Wilson (SciRealm@aol.com)

4-Vector SRQM Interpretation of QM

### SR 4-Vector Definition

<table>
<thead>
<tr>
<th>SR 4-Vector</th>
<th>Definition</th>
<th>Unites</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-Position</td>
<td>$R = (c t, r)$; alt. $X = (c t, x)$</td>
<td>Time, Space</td>
</tr>
<tr>
<td>4-Velocity</td>
<td>$U = \gamma(c, u)$</td>
<td>Gamma, Velocity</td>
</tr>
<tr>
<td>4-Momentum</td>
<td>$P = (E/c, p) = (mc, p)$</td>
<td>Energy:Mass, Momentum</td>
</tr>
<tr>
<td>4-WaveVector</td>
<td>$K = (\omega/c, k) = (\omega/c, \omega\hat{n}/v_{phase})$</td>
<td>Frequency, WaveNumber</td>
</tr>
<tr>
<td>4-Gradient</td>
<td>$\partial = (\partial t/c, -\nabla)$</td>
<td>Temporal Partial, Space Partial</td>
</tr>
<tr>
<td>4-VectorPotential</td>
<td>$A = (\phi/c, a)$</td>
<td>Scalar Potential, Vector Potential</td>
</tr>
<tr>
<td>4-TotalMomentum</td>
<td>$P_{tot} = (E/c+q\phi/c, p+qa)$</td>
<td>Energy-Momentum inc. EM fields</td>
</tr>
<tr>
<td>4-TotalWaveVector</td>
<td>$K_{tot} = (\omega/c+(q/\hbar)\phi/c, k+(q/\hbar)a)$</td>
<td>Freq-WaveNum inc. EM fields</td>
</tr>
<tr>
<td>4-CurrentDensity</td>
<td>$J = (c\rho, j) = qJ_{prob}$</td>
<td>Charge Density, Current Density</td>
</tr>
<tr>
<td>4-ProbabilityCurrentDensity</td>
<td>$J_{prob} = (c\rho_{prob}, j_{prob})$</td>
<td>QM Probability (Density, Current Density)</td>
</tr>
</tbody>
</table>

- SR 4-Tensor
  - $(2,0)$-Tensor $T^{iv}$
  - $(1,1)$-Tensor $T^{v}$, or $T^{\nu}$
  - $(0,2)$-Tensor $T_{iv}$
- SR 4-CoVector: OneForm
  - $(0,1)$-Tensor $V_{\nu} = (V^{i}, v)$
- SR 4-Scalar
  - $(0,0)$-Tensor $S$ or $S_{0}$ Lorentz Scalar

---

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http://scirealm.org/SRQM.pdf
# SRQM: More SR 4-Vectors Explained

<table>
<thead>
<tr>
<th>SR 4-Vector</th>
<th>Empirical Fact</th>
<th>What it means...</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-Position</td>
<td>$\mathbf{R} = (ct, \mathbf{r})$</td>
<td>SpaceTime as Single United Concept</td>
</tr>
<tr>
<td>4-Velocity</td>
<td>$\mathbf{U} = d\mathbf{R}/d\tau$</td>
<td>Velocity is Proper Time Derivative</td>
</tr>
<tr>
<td>4-Momentum</td>
<td>$\mathbf{P} = m_o \mathbf{U} = (E_o/c^2)\mathbf{U}$</td>
<td>Mass-Energy-Momentum Equivalence</td>
</tr>
<tr>
<td>4-WaveVector</td>
<td>$\mathbf{K} = \mathbf{P}/\hbar = (\omega_o/c^2)\mathbf{U}$</td>
<td>Wave-Particle Duality</td>
</tr>
<tr>
<td>4-Gradient</td>
<td>$\partial = -i\mathbf{K}$</td>
<td>Unitary Evolution of States</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Operator Formalism, Complex Waves</td>
</tr>
<tr>
<td>4-VectorPotential</td>
<td>$\mathbf{A} = (\phi/c, \mathbf{a}) = (\phi_o/c^2)\mathbf{U}$</td>
<td>Potential Fields...</td>
</tr>
<tr>
<td>4-TotalMomentum</td>
<td>$\mathbf{P}_{tot} = \mathbf{P} + q\mathbf{A}$</td>
<td>Energy-Momentum inc. Potential Fields</td>
</tr>
<tr>
<td>4-TotalWaveVector</td>
<td>$\mathbf{K}_{tot} = \mathbf{K} + (q/\hbar)\mathbf{A}$</td>
<td>Freq-WaveNum inc. Potential Fields</td>
</tr>
<tr>
<td>4-CurrentDensity</td>
<td>$\mathbf{J} = \rho_o \mathbf{U} = q\mathbf{J}_{prob}$ [\partial \cdot \mathbf{J} = 0]</td>
<td>ChargeDensity-CurrentDensity Equivalence</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CurrentDensity is conserved</td>
</tr>
<tr>
<td>4-Probability</td>
<td>$\mathbf{J}<em>{prob} = (\mathbf{c}\rho</em>{prob}, \mathbf{j}<em>{prob})$ [\partial \cdot \mathbf{J}</em>{prob} = 0]</td>
<td>QM Probability from SR</td>
</tr>
<tr>
<td>CurrentDensity</td>
<td></td>
<td>Probability Worldlines are conserved</td>
</tr>
</tbody>
</table>
Minimal Coupling = Potential Interaction

Klein-Gordon Eqn → Schrödinger Eqn

\[ P = P + Q = P + qA \]
\[ K = i\partial \]
\[ P = \hbar K \]
\[ P = ih\partial \]

\[ P = (E/c, p) = P - qA = (E/c - q\phi/c, p - qa) \]
\[ \partial = (\partial/c, -\nabla) = \partial + (iq/\hbar)A = (\partial/c + (iq/\hbar)\phi/c, -\nabla + (iq/\hbar)a) \]

\[ \partial\cdot\partial = (\partial/c)^2 - \nabla^2 = -(m_0c/\hbar)^2 : \]
\[ P\cdot P = (E/c)^2 - p^2 = (m_0c)^2 : \]

\[ (E_T - q\phi)^2 = (m_0c^2)^2 + c^2(p_T - qa)^2 : \]
\[ E_T - q\phi \sim [ (m_0c^2)^2 + (p_T - qa)^2/2m_o ] : \]

\[ (i\hbar\partial_T - q\phi)^2 = (m_0c^2)^2 + c^2(-i\hbar\nabla_T - qa)^2 : \]
\[ (i\hbar\partial_T - q\phi) \sim [ (m_0c^2)^2 + (-i\hbar\nabla_T - qa)^2/2m_o ] : \]

\[ (i\hbar\partial_T)\Psi \sim [ V - (h\nabla_T)^2/2m_o ]\Psi : \]

The better statement is that the Schrödinger Eqn is the limiting low-velocity case of the more general KG Eqn, not that the KG Eqn is the relativistic generalization of the Schrödinger Eqn.
Once one has a Relativistic Wave Eqn...

Klein-Gordon Equation: \( \partial \cdot \partial = (\partial_\tau / c)^2 - \nabla \cdot \nabla = (-i m_0 c / \hbar)^2 = -(m_0 c / \hbar)^2 = -(\omega_0 / c)^2 \)

Once we have derived a RWE, what does it imply?

The KG Eqn. was derived from the physics of SR plus a few empirical facts. It is a 2nd order, linear, wave PDE that pertains to physical objects of reality from SR.

Just being a linear wave PDE implies all the mathematical techniques that have been discovered to solve such equations generally: Hilbert Space, Superpositions, \(<\text{Bra}|,|\text{Ket}>\) notation, wavevectors, wavefunctions, etc. These things are from mathematics in general, not only and specifically from an Axiom of QM.

Therefore, if one has a physical RWE, it implies the mathematics of waves, the formalism of the mathematics, and thus the mathematical Principles and Formalism of QM. Again, QM Axioms are not required – they emerge from the physics and math...
SRQM: Once one has a Relativistic Wave Eqn…
Examine Photon Polarization

From the Wikipedia page on [Photon Polarization]

Photon polarization is the quantum mechanical description of the classical polarized sinusoidal plane electromagnetic wave. An individual photon can be described as having right or left circular polarization, or a superposition of the two. Equivalently, a photon can be described as having horizontal or vertical linear polarization, or a superposition of the two.

The description of photon polarization contains many of the physical concepts and much of the mathematical machinery of more involved quantum descriptions and forms a fundamental basis for an understanding of more complicated quantum phenomena. Much of the mathematical machinery of quantum mechanics, such as state vectors, probability amplitudes, unitary operators, and Hermitian operators, emerge naturally from the classical Maxwell's equations in the description. The quantum polarization state vector for the photon, for instance, is identical with the Jones vector, usually used to describe the polarization of a classical wave. Unitary operators emerge from the classical requirement of the conservation of energy of a classical wave propagating through lossless media that alter the polarization state of the wave. Hermitian operators then follow for infinitesimal transformations of a classical polarization state.

Many of the implications of the mathematical machinery are easily verified experimentally. In fact, many of the experiments can be performed with two pairs (or one broken pair) of polaroid sunglasses.

The connection with quantum mechanics is made through the identification of a minimum packet size, called a photon, for energy in the electromagnetic field. The identification is based on the theories of Planck and the interpretation of those theories by Einstein. The correspondence principle then allows the identification of momentum and angular momentum (called spin), as well as energy, with the photon.
SRQM: Principle of Superposition: From the mathematics of waves

Klein-Gordon Equation: \( \partial \cdot \partial = (\partial / c)^2 - \nabla \cdot \nabla = (-i m_0 c / \hbar)^2 = -(m_0 c / \hbar)^2 = -(\omega_0 / c)^2 \)

The Extended Superposition Principle for Linear Equations

Suppose that the non-homogeneous equation, where L is linear, is solved by some particular \( u_p \)
Suppose that the associated homogeneous problem is solved by a sequence of \( u_i \).
L(\( u_p \)) = C ;  L(\( u_0 \)) = 0 ,  L(\( u_1 \)) = 0 ,  L(\( u_2 \)) = 0 ...
Then \( u_p \) plus any linear combination of the \( u_n \) satisfies the original non-homogeneous equation:
L(\( u_p + \Sigma a_n u_n \)) = C,
where \( a_n \) is a sequence of (possibly complex) constants and the sum is arbitrary.

Note that there is no mention of partial differentiation. Indeed, it's true for any linear equation, algebraic or integro-partial differential-whatever.

QM superposition is not axiomatic, it emerges from the mathematics of the Linear PDE
The Klein-Gordon Equation is a 2\(^{nd}\)-order LINEAR Equation.
This is the origin of superposition in QM.
**Klein-Gordon obeys Principle of Superposition**

Klein-Gordon Equation: \( \partial \cdot \partial = (\partial_t/c)^2 - \nabla \cdot \nabla = (-im_o c/\hbar)^2 = -(m_o c/\hbar)^2 = -(\omega_o/c)^2 \)

- \( K \cdot K = (\omega/c)^2 - k \cdot k = (\omega_o/c)^2 \): The particular solution (w rest mass)
- \( K_n \cdot K_n = (\omega_n/c)^2 - k_n \cdot k_n = 0 \): The homogeneous solution for a (virtual photon?) microstate \( n \)

Note that \( K_n \cdot K_n = 0 \) is a null 4-vector (photonic)

Let \( \Psi_p = Ae^{-i(K \cdot X)} \), then \( \partial \cdot \partial[\Psi_p] = (-i)^2(K \cdot K)\Psi_p = -(\omega_o/c)^2\Psi_p \)

which is the Klein-Gordon Equation, the particular solution...

Let \( \Psi_n = A_n e^{-i(K_n \cdot X)} \), then \( \partial \cdot \partial[\Psi_n] = (-i)^2(K_n \cdot K_n)\Psi_n = (0)\Psi_n \)

which is the Klein-Gordon Equation homogeneous solution for a microstate \( n \)

We may take \( \Psi = \Psi_p + \Sigma_n \Psi_n \)

Hence, the Principle of Superposition is not required as an QM Axiom, it follows from SR and our empirical facts which lead to the Klein-Gordon Equation. The Klein-Gordon equation is a linear wave PDE, which has overall solutions which can be the complex linear sums of individual solutions – i.e. it obeys the Principle of Superposition. This is not an axiom – it is a general mathematical property of linear PDE's.

This property continues over as well to the limiting case \( \{ |v| \ll c \} \) of the Schrödinger Equation.
Klein-Gordon Equation: $\partial \cdot \partial = (\partial t/c)^2 - \nabla \cdot \nabla = (-im_0c/\hbar)^2 = -(m_0c/\hbar)^2 = -(\omega_0/c)^2$

Hilbert Space (HS) representation:
if $|\Psi\rangle \in HS$, then $c|\Psi\rangle \in HS$, where $c$ is complex number
if $|\Psi_1\rangle$ and $|\Psi_2\rangle \in HS$, then $|\Psi_1\rangle + |\Psi_2\rangle \in HS$
if $|\Psi\rangle = c_1|\Psi_1\rangle + c_2|\Psi_2\rangle$, then $<\Phi|\Psi\rangle = c_1<\Phi|\Psi_1\rangle + c_2<\Phi|\Psi_2\rangle$ and $<\Psi| = c_1^*<\Psi_1| + c_2^*<\Psi_2|$
$<\Phi|\Psi\rangle = <\Psi|\Phi>$
$<\Psi|\Psi\rangle \geq 0$
if $<\Psi|\Psi\rangle = 0$, then $|\Psi\rangle = 0$
etc.

Hilbert spaces arise naturally and frequently in mathematics, physics, and engineering, typically as infinite-dimensional function spaces. They are indispensable tools in the theories of partial differential equations, Fourier analysis, signal processing, heat transfer, ergodic theory, and Quantum Mechanics.

The QM Hilbert Space emerges from the fact that the KG Equation is a linear wave PDE – Hilbert spaces as solutions to PDE’s are a purely mathematical phenomenon – no QM Axiom is required.

Likewise, this introduces the $<\text{bra}|,|\text{ket}>$ notation, wavevectors, wavefunctions, etc.

**Note:**

One can use Hilbert Space descriptions of Classical Mechanics using the Koopman-von Neumann formulation. One can not use Hilbert Space descriptions of Quantum Mechanics by using the Phase Space formulation of QM.
Standard QM Canonical Commutation Relation:

\[ [x^i, p^k] = i\hbar \delta^{ik} \]

The Standard QM Canonical Commutation Relation is simply an axiom in standard QM. It is just given, with no explanation. You just had to accept it.

I always found that unsatisfactory.

There are at least 4 parts to it:

Where does the commutation (\([ , ]\)) come from?
Where does the imaginary constant (\(i\)) come from?
Where does the Dirac: reduced-Planck constant (\(\hbar\)) come from?
Where does the Kronecker Delta (\(\delta^{ik}\)) come from?

See the next page for SR enlightenment...
The SR Metric is the source of “quantization”.
A Tensor Study

Physics

\[\mathbf{\Delta X} = \mathbf{\Delta t} = \mathbf{\Delta x} = (\mathbf{\Delta t}, \mathbf{\Delta x})\]

\[\mathbf{dX} = (\mathbf{dt}, \mathbf{dx})\]

\[\mathbf{4-Displacement}\]

\[\begin{vmatrix} \text{4-Dimensional SpaceTime Dimension} \\ \end{vmatrix}\]

\[\text{Position: Momentum \( \mathbf{p} = \mathbf{\hbar} \mathbf{\delta}^{jk} \)}

\[\text{Time: Energy \( \mathbf{E} = \mathbf{mc}^2 \)}

\[\text{4-Velocity \( \mathbf{U} = \gamma (c, \mathbf{u}) \)}

\[\begin{vmatrix} \text{4-WaveVector} \\ \end{vmatrix}\]

\[\text{Einstein de Broglie \( \mathbf{P} = \hbar \mathbf{K} \)}

\[\text{Non-Zero Commutation Relation via SR 4-Momentum} \]

\[\{ \mathbf{P} = \hbar \mathbf{K} \}\text{ and \( \{ \mathbf{K} = i \mathbf{\delta} \) are empirical SR relations} \]

\[\text{Non-Zero Commutation Relation via SR 4-WaveVector} \]

\[\begin{vmatrix} \text{Complex Plane-waves} \quad \mathbf{K} = i \mathbf{\delta} \\ \end{vmatrix}\]

\[\begin{vmatrix} \text{Complex Non-Zero Commutation Relation} \\ \end{vmatrix}\]

\[\begin{vmatrix} \text{4-Vector SRQM Interpretation of QM} \\ \text{SciRealm.org} \quad \text{John B. Wilson} \quad \text{SciRealm@aol.com} \quad \text{http://scirealm.org/QRM.pdf} \\ \end{vmatrix}\]
Standard QM Canonical Commutation Relation:

\[
[x^i, p^k] = i\hbar \delta^{ik}
\]

As we have seen, this relation is generated from simple SR math.

\[
[\partial_\mu, X^\nu] = \delta^{\mu\nu}
\]

\[
[P_{\mu}, X^\nu] = i\hbar \delta^{\mu\nu}
\]

This is the more general 4D version, with the Standard QM version being just the spatial part.

One of the great misconceptions on modern physics is that since QM is about “tiny” things, that ALL things should be built up from it. That paradigm of course works well for many things:
- Compounds are built-up from smaller molecules.
- Molecules are built-up from smaller elements.
- Elements are built-up from smaller atoms.
- Atoms are built-up from smaller protons, neutrons, and electrons.
- Protons and neutrons are built-up from smaller quarks.

And all experiments to-date show that electrons and quarks appear to be point-like, with wave-type properties giving extent.

So, one can mistakenly think that “SpaceTime” must be made up of smaller “quantum” stuff as well. However, that is not what the math says. The “quantization” paradigm doesn’t apply to SpaceTime itself, just to <events>. All of the “quantum”-sized things above, electrons and quarks, are material things, <events>, which move around “within” SpaceTime. Their “quantization” comes about from the properties of the math and metric of SR.

The math does *NOT* say that SpaceTime itself is “quantized”. It says that SR Minkowski SpaceTime is the source of “quantization.”
SRQM Study: 4-Position and 4-Gradient

4-Displacement
\[ \Delta R = (c\Delta t, \Delta r) \]
\[ dR = (cdt, dr) \]
4-Position
\[ R = R^\mu = (ct, r) \]

Invariant Interval
\[ R \cdot R = (ct)^2 - r \cdot r = (ct)^2 \]

Invariant Calculus
\[ dR \cdot \partial = (cdt, dr) \cdot (\partial / c, -\nabla) \]
\[ dR^\mu \eta_{\mu\nu} (\partial^\nu) = dR^\mu (\partial / c) = dR^\nu (\partial/\partial R^\mu) = (dt \partial_t + dx \partial_x + dy \partial_y + dz \partial_z) \]
Total Derivative Chain Rule

SR: Lorentz Transform
\[ \partial[R^\mu] = \partial R^\nu / \partial R^\nu = \Lambda^\nu_\nu \]
\[ \Lambda^\mu_\nu \Lambda^\nu_\beta = \eta_{\alpha\beta} \]
\[ (\text{Det}[\Lambda])^2 = 1 \]
\[ \Lambda^\mu_\nu = (\Lambda^{-1})^{\mu}_\nu \]
\[ \Lambda^\mu_\nu \Lambda^\nu_\mu = 4 \]
Rotations
Boosts
CPT

SRQM: Tensor Zero Exterior Product
\[ \partial^\nu R = \partial_\nu R^\mu = 4 \]
Dimension

SpaceTime
\[ \partial \cdot R = \partial_\mu R^\mu = 4 \]

SRQM: Non-Zero Commutation
\[ [\partial, R] = \{ \partial_\mu, R^\nu \} = \partial_\mu R^\nu - \partial_\nu R^\mu = \eta^{\mu\nu} - \eta^{\nu\mu} = 0^{\mu\nu} \]

SR 4-Tensor
(2,0)-Tensor \( T^{\mu\nu} \)
(1,1)-Tensor \( T^\mu \) or \( T_\mu \)
(0,2)-Tensor \( \Theta^{\mu\nu} \)
SR 4-Vector
\( V = (V^0, V) \)
\( V \cdot V = (V^0)^2 - V \cdot V = (V^0)^2 = \text{Lorentz Scalar Invariant} \)

SR 4-CoVector: OneForm
\( V_\mu = (V_\mu, V) \)
\( \text{Trace}[T^{\mu\nu}] = \eta_{\mu\nu} T^{\mu\nu} = T^\mu_\mu = T \)

SR 4-Scalar
(0,0)-Tensor \( S \) or \( S_0 \)

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4-Vector SRQM Interpretation of QM

Heisenberg Uncertainty Principle: Viewed from SRQM

The commutator is \([A,B] = AB-BA\), where \(A\) & \(B\) are functional "measurement" operators. The Operator Formalism arose naturally from our SR → QM path: \(\partial = -i\mathbf{K}\).

The Generalized Uncertainty Relation: \(\sigma_f^2 \sigma_g^2 \geq \frac{1}{2}|\langle [F,G] \rangle|\)

The Cauchy–Schwarz inequality asserts that (for all vectors \(f\) and \(g\) of an inner product space, with either real or complex numbers):
\[\sigma_f^2 \sigma_g^2 = |\langle f | f \cdot g | g \rangle| \geq |\langle f | g \rangle|^2\]

But first, let's back up a bit; Using standard complex number math, we have:
\[z = a + ib\]
\[z^* = a - ib\]
\[\text{Re}(z) = a = (z + z^*)/(2)\]
\[\text{Im}(z) = b = (z - z^*)/(2i)\]
\[z^2 = |z|^2 = a^2 + b^2 = |\text{Re}(z)|^2 + |\text{Im}(z)|^2 = [(z + z^*)(2)]^2 + [(z - z^*)(2i)]^2\]

Now, generically, based on the rules of a complex inner product space we can arbitrarily assign:
\[z = \langle f | g \rangle , z^* = \langle g | f \rangle\]

Which allows us to write:
\[|\langle f | g \rangle|^2 = |(\langle f | g \rangle + \langle g | f \rangle)\langle 2 \rangle|^2 + |(\langle f | g \rangle - \langle g | f \rangle)/(2i)|^2\]

We can also note that:
\[|\langle f | F \rangle| \text{ and } |\langle g | G \rangle|\]

Thus,
\[|\langle f | g \rangle|^2 = |\langle \langle F | G \rangle | \Psi \rangle + \langle \langle G^* | F \rangle | \Psi \rangle)/(2)|^2 + |\langle \langle F^* | G \rangle | \Psi \rangle - \langle \langle G^* | F \rangle | \Psi \rangle)/(2i)|^2\]

For Hermetian Operators...
\[F^* = +F, G^* = +G\]

For Anti-Hermetian (Skew-Hermetian) Operators...
\[F^* = -F, G^* = -G\]

Assuming that \(F\) and \(G\) are either both Hermetian, or both anti-Hermetian...
\[|\langle f | g \rangle|^2 = |(\langle \langle F | G \rangle | \Psi \rangle + \langle \langle G | F \rangle | \Psi \rangle)/(2)|^2 + |(\langle \langle F^* | G \rangle | \Psi \rangle - \langle \langle G^* | F \rangle | \Psi \rangle)/(2i)|^2\]

We can write this in commutator and anti-commutator notation...
\[|\langle f | g \rangle|^2 = |(\langle \langle F | G \rangle | \Psi \rangle)\langle 2 \rangle|^2 + |(\langle \langle G^* | F \rangle | \Psi \rangle)/(2i)|^2\]

Due to the squares, the \((z)^2\)'s go away, and we can also multiply the commutator by an \((i)^2\)

\[|\langle f | g \rangle|^2 = |(\langle \langle F | G \rangle | \Psi \rangle)\langle 2 \rangle|^2 + |(\langle \langle F^* | G \rangle | \Psi \rangle)/(2i)|^2\]

The Cauchy–Schwarz inequality again...
\[\sigma_f^2 \sigma_g^2 = |\langle f | f \cdot g | g \rangle| \geq |\langle f | g \rangle|^2 = |(\langle F | G \rangle \langle 2 \rangle)|^2 + |(\langle F^* | G \rangle)/(2i)|^2\]

Taking the root:
\[\sigma_f^2 \sigma_g^2 \geq (1/2)|\langle F | G \rangle|\]

Which is what we had for the generalized Uncertainty Relation.
Heisenberg Uncertainty Principle: Simultaneous vs Sequential

Heisenberg Uncertainty \(\sigma_A^2\sigma_B^2 \geq (1/2)|<[A,B]>|\) arises from the non-commuting nature of certain operators.

\[
[\partial^\mu, X^\nu] = \partial[X] = \eta^{\mu\nu} = \text{Minkowski Metric}
\]

\[
[P^\mu, X^\nu] = [i\hbar\partial^\mu, X^\nu] = i\hbar[\partial^\mu, X^\nu] = i\hbar\eta^{\mu\nu}
\]

Consider the following:
Operator A acts on System \(|\Psi>\) at SR Event A: \(A|\Psi> \rightarrow |\Psi'>\)
Operator B acts on System \(|\Psi'>\) at SR Event B: \(B|\Psi'> \rightarrow |\Psi''>\)
or \(BA|\Psi> = B|\Psi'> = |\Psi''>\)

If measurement Events A & B are space-like separated, then there are observers who can see \{A before B, A simultaneous with B, A after B\}, which of course does not match the quantum description of how Operators act on Kets.

If Events A & B are time-like separated, then all observers will always see A before B. This does match how the operators act on Kets, and also matches how \(|\Psi>\) would be evolving along its worldline, starting out as \(|\Psi>\), getting hit with operator A at Event A to become \(|\Psi'>\), then getting hit with operator B at Event B to become \(|\Psi''>\).

The Uncertainty Relation here does NOT refer to simultaneous (space-like separated) measurements, it refers to sequential (time-like separated) measurements. This removes the need for ideas about the particles not having simultaneous properties. There are simply no “simultaneous measurements” of non-zero commuting properties on an individual system, a single worldline – they are sequential, and the first measurement places the system in such a state that the outcome of the second measurement will be altered wrt. if the order of the operations had been reversed.
SRQM:
Pauli Exclusion Principle:
Requires SR for the detailed explanation

The Pauli Exclusion Principle is a result of the empirical fact that nature uses identical (indistinguishable) particles, and this combined with the Spin-Statistics theorem from SR, leads to an exclusion principle for fermions (anti-symmetric, Fermi-Dirac statistics) and an aggregation principle for bosons (symmetric, Bose-Einstein statistics). The Spin-Statistics Theorem is related as well to the CPT Theorem.

For large numbers and/or mixed states these both tend to the Maxwell-Boltzmann statistics. In the \( \{kT>>(\epsilon_i-\mu)\} \) limit, Bose-Einstein reduces to Rayleigh-Jeans. The commutation relations here are based on space-like separation particle exchanges. Exchange operator \( P, P^2 = +1 \), Since two exchanges bring one back to the original state. \( P \) thus has two eigenvalues \( (\pm 1) \) and two eigenvectors \{ |Symm\>, |AntiSymm> \}

\[
P|\text{Symm}\> = +|\text{Symm}\>
\]

\[
P|\text{AntiSymm}\> = -|\text{AntiSymm}\>
\]

---

<table>
<thead>
<tr>
<th>Spin-Symmetry</th>
<th>Particle Type</th>
<th>Quantum Statistics</th>
<th>Classical ( { kT&gt;&gt;(\epsilon_i-\mu) } )</th>
</tr>
</thead>
<tbody>
<tr>
<td>spin:(0,1,...,N)</td>
<td>Indistinguishable, Commutation relation ( [a,b] = ab-ba = -[b,a] = \text{constant} ) ( ( ab = ba ) ) if commutes</td>
<td>Bose-Einstein: ( n_i = g_i / \left[ e^{(\epsilon_i-\mu)/kT} -1 \right] ) aggregation principle</td>
<td>Rayleigh-Jeans: ( \text{from } e^x \sim (1 + x +...) ) ( n_i = g_i / \left[ (\epsilon_i-\mu)/kT \right] )</td>
</tr>
<tr>
<td>Multi-particle Mixed</td>
<td>Distinguishable, or high temp, or low density</td>
<td>Maxwell-Boltzmann: ( n_i = g_i / \left[ e^{(\epsilon_i-\mu)/kT} +0 \right] )</td>
<td>Maxwell-Boltzmann: ( n_i = g_i / \left[ e^{(\epsilon_i-\mu)/kT} \right] )</td>
</tr>
<tr>
<td>spin:(1/2,3/2,...,N/2)</td>
<td>Indistinguishable, Anti-commutation relation ( {a,b} = ab+ba = +{b,a} = \text{constant} ) ( ( ab = - ba ) ) if anti-commutes</td>
<td>Fermi-Dirac: ( n_i = g_i / \left[ e^{(\epsilon_i-\mu)/kT} +1 \right] ) exclusion principle</td>
<td></td>
</tr>
</tbody>
</table>

\[ \downarrow \text{Limit as } e^{(\epsilon_i-\mu)/kT} \gg 1 \]

\[ \uparrow \text{Limit as } e^{(\epsilon_i-\mu)/kT} \gg 1 \]
Complex 4-vectors are simply 4-Vectors where the components may be complex-valued

\[ \mathbf{A} = A^\mu = (a^0, a) = (a^0, a^1, a^2, a^3) \rightarrow (a^t, a^x, a^y, a^z) \]
\[ \mathbf{B} = B^\mu = (b^0, b) = (b^0, b^1, b^2, b^3) \rightarrow (b^t, b^x, b^y, b^z) \]

Examples of 4-Vectors with complex components are the 4-Polarization and the 4-ProbabilityCurrentDensity

Minkowski Metric \( g^{\mu\nu} \rightarrow \eta^{\mu\nu} = \eta_{\mu\nu} \rightarrow \text{Diag}[1,-1,-1,-1] = \text{Diag}[1,-1^{(3)}] \), which is the \{curvature\~0 limit = low-mass limit\} of the GR metric \( g^{\mu\nu} \).

Applying the Metric to raise or lower an index also applies a complex-conjugation *

Scalar Product = Lorentz Invariant \( \rightarrow \) Same value for all inertial observers

\[ \mathbf{A} \cdot \mathbf{B} = \eta_{\mu\nu} A^\mu B^\nu = A^*_\nu B^\nu = A^\mu B^*_\mu = (a^0 b^0 - a^* \cdot b) \] using the Einstein summation convention

This reverts to the usual rules for real components

However, it does imply that \( \mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A} \)
SRQM: CPT Theorem
Phase Connection, Lorentz Invariance

The Phase is a Lorentz Scalar Invariant – all observers must agree on its value. 
\[ K \cdot X = (\omega/c)X \cdot (ct, x) = (\omega - k \cdot x) = -\phi \cdot X \]

We take the point of view of an observer operating on a particle at 4-Position \( X \), which has an initial 4-Velocity Vector \( \mathbf{K} \). The 4-Position \( X \) of the particle, the operation’s event, will not change: we are applying the various operations only to the particle’s 4-Momentum \( \mathbf{K} \).

Note that for matter particles, the operations only to the particle’s 4-Momentum at its event, will not change: we are applying the various operations listed below work similarly on the Null 4-Vector.

For photonic particles, the gamma factor is the “Unit”-Null 4-Vector \( \mathbf{N} \), and \( \mathbf{N} \) is a unit-spatial 3-vector. All operations listed above work similarly on the Null 4-Vector.

Do a Time Reversal Operation: \( T \)

Do a Parity Operation (Space Reflection): \( P \)

Do a Charge Conjugation Operation: \( C \)

Charge Conjugation actually changes all internal quantum #’s: charge, lepton #, etc.

Feynman showed this is the equivalent of a world-line reversal & complex-conjuguition:

\[ \mathbf{T}_C = \gamma(1, -\beta) \]

Pairwise combinations:

\[ \begin{align*}
\mathbf{T}_P &= \mathbf{T}_T \mathbf{F} = \gamma(1, -\beta) \\
\mathbf{T}_C &= \mathbf{T}_C \mathbf{T}_T = \gamma(1, -\beta) \\
\mathbf{T}_P &= \mathbf{T}_C \mathbf{F} = \gamma(1, -\beta) \\
\mathbf{T}_T &= \mathbf{T}_T \mathbf{T}_T = \gamma(1, -\beta)
\end{align*} \]

A CP event is mathematically the same as a \( T \) event

\[ \begin{align*}
\mathbf{T}_C \mathbf{T}_P &= \mathbf{T} = \gamma(1, \beta) \\
\mathbf{T}_P \mathbf{T}_C &= \gamma(1, \beta) \\
\mathbf{T}_T &= \gamma(1, \beta)
\end{align*} \]

\[ \gamma(1, \beta) \]

\[ \begin{align*}
\mathbf{T}_P \mathbf{T}_C &= \mathbf{T} = \gamma(1, \beta) \\
\mathbf{T}_T &= \gamma(1, \beta)
\end{align*} \]

4-Position
\[ \mathbf{R} = (ct, r) \]

4-Displacement
\[ \Delta \mathbf{R} = (c \Delta t, \Delta r) \]

4-Velocity
\[ \mathbf{U} = \gamma(c, \mathbf{u}) \]

Proper Time Derivative
\[ \dot{\mathbf{U}} = \gamma \frac{d\mathbf{U}}{d\tau} \]

4-UnitTemporal
\[ \mathbf{T} = \gamma(1, \beta) \]

4-UnitSpatial
\[ \mathbf{S} = (\hat{n} \cdot \beta, \hat{n}) \]

4-UnitNull
\[ \mathbf{N} = (1, \mathbf{n}) \]

\[ \dot{\mathbf{R}} = \mathbf{4} \]

SpaceTime Dimension

\[ \mathbf{U} \cdot \dot{\mathbf{R}} \]

Minkowski Metric

\[ \delta[\mathbf{R}] = \eta^{\mathbf{iv}} \rightarrow \text{Diag}(1, 1, 1, 1) \]

\[ \mathbf{U} \cdot \mathbf{U} = c^2 \]

\[ \mathbf{A} = \gamma(c \dot{\mathbf{u}} + \mathbf{a}) \]

4-Gradient
\[ \partial = (\partial / c, \nabla) \]

SRQM: 4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

\[ \text{Trace}[T^\mu] = \eta_{\mu} \text{Trace}[T]^\mu = T^\mu \rightarrow T^\mu = T \]

\[ V \cdot V = V_\mu V^\mu = (V_\mu V^\mu)_R = (V_\mu V^\mu)_L \]

\[ = \text{Lorentz Scalar Invariant} \]

\[ \text{Phase Connection, Lorentz Invariance} \]
SRQM: CPT Theorem (Charge) vs (Parity) vs (Time)

Classical SR Time-Reversal neglects spin and charge. SRQM includes these effects. Then one gets (CC), (PP), (TT), & (CPT) transforms all leading back to the Identity (I).

Parity-Inverted 4-Vector

\[ A' = A^{\mu} = (a^0, a^\nu) = (a^0, a) \]

Time-Reversed 4-Vector

\[ A' = A^{\mu} = T^{\mu} = (a^0, -a^\nu) \]

Charge-Conjugated 4-Vector

\[ A' = A^{\mu} = C^{\mu} = (a^0, a^\nu) \]

Identity and Space-Parity are Unitary. Time-Reversal and Charge-Conjugation are Anti-Unitary.

Trace of 4-Vector

\[ \text{Trace}[T^{\mu\nu}] = \eta^{\mu\nu}T^{\mu\nu} = T_{\mu\nu} = T \]

Lorentz Scalar

\[ V \cdot V = V^{\mu} \eta_{\mu\nu} V^{\nu} = (V^0)^2 - V^\nu V^\nu = (V^0)^2 \]

SR 4-Tensor

(2,0)-Tensor \( T^{\mu\nu} \)
(1,1)-Tensor \( V_{\mu\nu} = V = (v^0, v^\nu) \)
(0,2)-Tensor \( T_{\mu\nu} \)

SR 4-Vector

(1,0)-Tensor \( V_{\mu} = V = (v^0, v^\nu) \)

SR 4-CoVector: One Form

(0,1)-Tensor \( V^\mu = (v_0, -v) \)

SR 4-Scalar

(0,0)-Tensor \( S \) or \( S^0 \)

Lorentz Scalar
SRQM Transforms: Venn Diagram

Poincaré = Lorentz + Translations

(10)  (6)  (4)

<table>
<thead>
<tr>
<th>Lorentz Transform $\Lambda^{\nu'}_{\mu}$</th>
<th>Translation Transform $\Delta X^{\mu'}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-Tensor {mixed type-(1,1)}</td>
<td>4-Vector</td>
</tr>
<tr>
<td>Discrete</td>
<td>Continuous</td>
</tr>
<tr>
<td>Continuous</td>
<td>Discrete</td>
</tr>
<tr>
<td>Continuous</td>
<td>Continuous</td>
</tr>
</tbody>
</table>

**Poincaré Transformation Group** aka. Inhomogeneous Lorentz Transformation

Lie group of all affine isometries of SR:Minkowski Time-Space (preserve quadratic form $\eta_{\mu\nu}$)

General Linear, Affine Transform $X^{\nu'} = \Lambda^{\nu'}_{\mu}X^{\mu} + \Delta X^{\nu'}$ with $\text{Det}[\Lambda^{\nu'}_{\mu}] = \pm 1$

(6+4=10)

**Lorentz Transform**

<table>
<thead>
<tr>
<th>Time-reversal $\Lambda^{\nu'}<em>{\mu} \rightarrow T^{\nu'}</em>{\nu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0)</td>
</tr>
<tr>
<td>t $\rightarrow$ -t*</td>
</tr>
<tr>
<td>time parity anti-unitary</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parity-Inversion $\Lambda^{\nu'}<em>{\mu} \rightarrow P^{\nu'}</em>{\mu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0)</td>
</tr>
<tr>
<td>r $\rightarrow$ -r</td>
</tr>
<tr>
<td>space parity unitary</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Charge-Conjugation $\Lambda^{\nu'}<em>{\mu} \rightarrow C^{\nu'}</em>{\mu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0)</td>
</tr>
<tr>
<td>R $\rightarrow$ -R*, q $\rightarrow$ -q</td>
</tr>
<tr>
<td>charge parity anti-unitary</td>
</tr>
</tbody>
</table>

**Translation Transform**

<table>
<thead>
<tr>
<th>SpatialFlipCombos $\Lambda^{\nu'}<em>{\mu} \rightarrow F^{\nu'}</em>{\nu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0)</td>
</tr>
<tr>
<td>{x</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Identity $I^{(4)}$ $\Lambda^{\nu'}<em>{\mu} \rightarrow \eta^{\nu'}</em>{\mu}=0^\nu_{\nu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0)</td>
</tr>
<tr>
<td>no mixing unitary</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Boost $\Lambda^{\nu'}<em>{\mu} \rightarrow B^{\nu'}</em>{\mu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3)</td>
</tr>
<tr>
<td>tx</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CPT Symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Charge)</td>
</tr>
<tr>
<td>{Partly}</td>
</tr>
<tr>
<td>(Time)</td>
</tr>
<tr>
<td>symmetry</td>
</tr>
<tr>
<td>same all directions</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Isotropy</th>
</tr>
</thead>
<tbody>
<tr>
<td>(same all points)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Temporal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta X^{\nu'} \rightarrow (c\Delta t,0)$</td>
</tr>
<tr>
<td>(1)</td>
</tr>
<tr>
<td>$\Delta t$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spatial</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta X^{\nu'} \rightarrow (0,\Delta x)$</td>
</tr>
<tr>
<td>(3)</td>
</tr>
<tr>
<td>$\Delta x</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Homogeneity</th>
</tr>
</thead>
<tbody>
<tr>
<td>(same all points)</td>
</tr>
</tbody>
</table>

4-Angular Momentum $M^{\nu\mu} = X^{\mu} \wedge P^{\nu} = X^{(\nu}P^{\mu)} - X^{\nu}P^{\mu}$

= Generator of Lorentz Transformations (6)

= \{ $\Lambda^{\nu'}_{\mu}, R^{\nu'}_{\nu}$, Rotations (3) + $\Lambda^{\nu'}_{\mu}, B^{\nu'}_{\nu}$ Boosts (3) \}

4-Linear Momentum $P^{\mu}$

= Generator of Translation Transformations (4)

= \{ $\Delta X^{\nu'} \rightarrow (c\Delta t,0)$ Time (1) + $\Delta X^{\nu'} \rightarrow (0,\Delta x)$ Space (3) \}

$\text{Det}[\Lambda^{\nu'}_{\mu}] = +1$ for Proper Lorentz Transforms

$\text{Det}[\Lambda^{\nu'}_{\mu}] = -1$ for Improper Lorentz Transforms

Lorentz Matrices can be generated by a matrix $M$ with $\text{Tr}[M]=0$ which gives:

\{ $\Lambda = e^M = e^{+(\theta J - \zeta K)}$ \}

\{ $\Lambda^\dagger = (e^M)^\dagger = e^{-M}$ \}

\{ $\Lambda^{-1} = (e^M)^{-1} = e^{-M}$ \}

$\text{Tr}[\Lambda^{\nu'}_{\mu}] = \{ -\infty, +\infty \}$

= Lorentz Transform Type

SR: Lorentz Transform

\[ \delta[R^{\nu'}] = \partial R^{\nu'}/\partial R^{\nu} = \Lambda^{\nu'}_{\nu} \]

$\Lambda^{\nu'}_{\nu} = (\Lambda^{-1})^{\nu'}_{\nu} : \Lambda^{\nu'}_{\nu}\Lambda^{\mu}_{\nu} = \eta^{\nu}_{\nu} = \delta^{\nu}_{\nu}$

$\eta^{\nu}_{\nu}\Lambda^{\nu'}_{\nu}=\eta_{\nu\nu}$

$\Lambda = e^M = e^{+(\theta J - \zeta K)}$

$\text{Det}[\Lambda^{\nu'}_{\mu}] = \pm 1$

$\Lambda^{\nu'}_{\mu}\Lambda^{\nu}_{\nu} = 4$

Rotations $J = -\epsilon_{\alpha\beta}M^{\alpha\beta}/2$, Boosts $K = M_0$
The Hermitian Generators that lead to translations and rotations via unitary operators in QM...

These all ultimately come from the Poincaré Invariance → Lorentz Invariance that is at the heart of SR and Minkowski Space.

Infintesimal Unitary Transformation
\[ \hat{U}_t(\hat{G}) = I + i\varepsilon \hat{G} \]

Finite Unitary Transformation
\[ \hat{U}_\alpha(\hat{G}) = e^{i\alpha \hat{G}} \]

let \( \hat{G} = \hat{P}/\hbar = K \)
let \( \alpha = \Delta x \)

\[ \hat{U}_{\Delta x}(P/\hbar)\Psi(X) = e^{i\Delta x \cdot P/\hbar}\Psi(X) = e^{i(-\Delta x \cdot \partial)}\Psi(X) = \Psi(X - \Delta x) \]

Time component: \( \hat{U}_{\Delta t}(P/\hbar)\Psi(ct) = e^{i\Delta t E/\hbar}\Psi(ct) = e^{i(-\Delta t \partial_t)}\Psi(ct) = \Psi(ct - c\Delta t) = c\Psi(t - \Delta t) \)

Space component: \( \hat{U}_{\Delta x}(p/\hbar)\Psi(x) = e^{i\Delta x \cdot p/\hbar}\Psi(x) = e^{i(\Delta x \cdot \nabla)}\Psi(x) = \Psi(x + \Delta x) \)

By Noether's Theorem, this leads to \( \partial \cdot K = 0 \)

We had already calculated
\( (\partial \cdot \partial)[K \cdot X] = ((\partial / c)^2 - \nabla \cdot \nabla)(\omega t - k \cdot x) = 0 \)
\( (\partial \cdot \partial)[K \cdot X] = \partial \cdot (\partial [K \cdot X]) = \partial \cdot K = 0 \)

Poincaré Invariance also gives the Casimir invariants of mass and spin, and ultimately leads to the spin-statistics theorem of RQM.
SRQM: A treatise of SR → QM by John B. Wilson (SciRealm@aol.com)

QM Correspondence Principle: Analogous to the GR and SR limits

Basically, the old school QM Correspondence Principle says that QM should give the same results as classical physics in the realm of large quantum systems, i.e. where macroscopic behavior overwhelms quantum effects. Perhaps a better way to state it is when the change of system by a single quantum has a negligible effect on the overall state.

There is a way to derive this limit, by using Hamilton-Jacobi Theory:

$$(i\hbar \partial_t)\Psi \sim [V - (\hbar \nabla T)^2/2m_o ]\Psi : \text{The Schrödinger NRQM Equation for a point particle (non-relativistic QM)}$$

Examine solutions of form $\Psi = \Psi_0 e^{i\Phi}$, where $S$ is the QM Action $\delta[S] = (i/\hbar)\Psi \delta[S] \text{ and } \delta,S = (i/\hbar)\Psi \delta[S] \text{ and } \nabla^2[S] = (i/\hbar)\Psi \nabla^2[S] - (\Psi/\hbar^2)(\nabla[S])^2$

$$(i\hbar)(i/\hbar)\Psi \delta[S] = \nabla \Psi - (\hbar^2/2m_o)((i/\hbar)\Psi \nabla^2[S] - (\Psi/\hbar^2)(\nabla[S])^2)$$

$$(i)(i)\Psi \delta[S] = \nabla \Psi - ((i\hbar/2m_o)\Psi \nabla^2[S] - (\Psi/2m_o)(\nabla[S])^2)$$

$$\delta[S] = -V + (i\hbar/2m_o)\nabla^2[S] - (1/2m_o)(\nabla[S])^2$$

$$\delta[S] + [V+(1/2m_o)(\nabla[S])^2] = (i\hbar/2m_o)\nabla^2[S] \text{ : Quantum Single Particle Hamilton-Jacobi}$$

$$\delta[S] + [V+(1/2m_o)(\nabla[S])^2] = 0 \text{ : Classical Single Particle Hamilton-Jacobi}$$

Thus, the classical limiting case is:

$$\nabla^2[\Phi] \ll (\nabla[\Phi])^2$$

$$\hbar \nabla^2[S] \ll (\nabla[S])^2$$

$$\hbar \nabla \cdot \mathbf{p} \ll (\mathbf{p} \cdot \mathbf{p})$$

$$\nabla \cdot \mathbf{k} \ll (\mathbf{k} \cdot \mathbf{k})$$

$$\langle p \lambda \rangle \nabla \cdot \mathbf{p} \ll (\mathbf{p} \cdot \mathbf{p})$$
QM Correspondence Principle: Analogous to the GR and SR limits

\[ \partial_t [S] + \left[ V + (1/2m) \nabla^2 [S] \right] = 0 \]

\[ \partial_t [S] + \left[ V + (1/2m) \nabla^2 [S] \right] = \frac{i \hbar}{2m} \nabla^2 [S] \] : Quantum Single Particle Hamilton-Jacobi

Thus, the quantum → classical limiting-case is: {all equivalent representations}

\[ \hbar \nabla^2 [S_{\text{action}}] \ll (\nabla [S_{\text{action}}])^2 \]

\[ \nabla^2 [\Phi_{\text{phase}}] \ll (\nabla [\Phi_{\text{phase}}])^2 \]

\[ \nabla \cdot p \ll (p \cdot p) \]

\[ \nabla \cdot k \ll (k \cdot k) \]

with

\[ P = (E/c, p) = -\partial [S_{\text{action}}] = -\left( \partial/c, -\nabla \right)[S_{\text{action}}] = (\partial/c, \nabla) [S_{\text{action}}] \]

\[ K = (\omega/c, k) = -\partial [\Phi_{\text{phase}}] = -\left( \partial/c, -\nabla \right)[\Phi_{\text{phase}}] = (\partial/c, \nabla) [\Phi_{\text{phase}}] \]

It is analogous to GR → SR in limit of low curvature (low mass), or SR → CM in limit of low velocity \( |v| \ll c \).

It still applies, but is now understood as the same type of limiting-case as these others.

*Note* The commonly seen form of \( (c \to \infty, \hbar \to 0) \) as limits are incorrect!

c and \( \hbar \) are universal constants – they never change.

If \( c \to \infty \), then photons (light-waves) would have infinite energy \( E = pc \). This is not true classically.

If \( \hbar \to 0 \), then photons (light-waves) would have zero energy \( E = \hbar \omega \). This is not true classically.

Always better to write the SR Classical limit as \( |v| \ll c \), the QM Classical limit as \( \nabla^2 [\Phi_{\text{phase}}] \ll (\nabla [\Phi_{\text{phase}}])^2 \)

Again, it is more natural to find a limiting-case of a more general system than to try to unite two separate theories which may or may not ultimately be compatible. From logic, there is always the possibility to have a paradox result from combination of arbitrary axioms, whereas deductions from a single true axiom will always give true results.

This page needs some work. Source was from Goldstein.
Conservation of Probability: Probability Current: Charge Current
Consider the following purely mathematical argument (based on Green's Vector Identity):
\[ \partial \cdot (f \partial g - \partial f \cdot g) = f \partial \cdot \partial g - \partial \cdot \partial f g \]
with \((f)\) and \((g)\) as SR Lorentz Scalar functions.

**Proof:**
\[
\partial \cdot (f \partial g - \partial f \cdot g) = \partial \cdot (f \partial g) - \partial \cdot \partial f g
\]
\[
= (f \partial \cdot \partial g) - (\partial f \cdot \partial g) = f \partial \cdot \partial g - \partial f \cdot \partial g
\]

We can also multiply this by a Lorentz Invariant Scalar Constant \(s\):
\[
s \partial \cdot (f \partial g - \partial f \cdot g) = \partial \cdot (f \partial g) - \partial \cdot \partial f g
\]

Ok, so we have the math that we need...

Now, on to the physics… Start with the Klein-Gordon Eqn.
\[ \partial \cdot \partial = (-im_o c/ћ)^2 = -(m_o c/ћ)^2 \partial \cdot \partial + (m_o c/ћ)^2 = 0 \]
Let it act on SR Lorentz Invariant function \(g\):
\[ \partial \cdot \partial g + (m_o c/ћ)^2 g = 0 \]
Then pre-multiply by \(f\):
\[ [f] \partial \cdot \partial [g] + [f] (m_o c/ћ)^2 [g] = [f] 0 [g] \]
Do similarly with SR Lorentz Invariant function \(f\):
\[ \partial \cdot \partial [f] + (m_o c/ћ)^2 [f] = 0 \]
Then post-multiply by \(g\):
\[ \partial \cdot \partial [f] [g] + (m_o c/ћ)^2 [f] [g] = 0 \]

Now, subtract the two equations:
\[ \{[f] \partial \cdot \partial [g] + (m_o c/ћ)^2 [f] [g] = 0\} - \{\partial \cdot \partial [f] [g] + (m_o c/ћ)^2 [f] [g] = 0\}
\[ [f] \partial \cdot \partial [g] - \partial \cdot \partial [f] [g] = 0 \]
As we noted from the mathematical Green’s Vector identity at the start…
\[ [f] \partial \cdot \partial [g] - \partial \cdot \partial [f] [g] = \partial \cdot (f \partial [g] - \partial [f] g) = 0 \]

Therefore,
\[
s \partial \cdot (f \partial [g] - \partial [f] g) = 0
\]
\[ \partial \cdot s( f \partial [g] - \partial [f] g) = 0 \]
Thus, there is a conserved current 4-Vector, \(J_{\text{prob}} = s( f \partial [g] - \partial [f] g)\), for which \(\partial \cdot J_{\text{prob}} = 0\), and which also solves the Klein-Gordon equation.

Let’s choose as before \((\partial = -iK)\) with a plane wave function \(f = ae^{-i(K \cdot x)} = \psi\), and choose \(g = f^* = ae^{i(K \cdot x)} = \psi^*\) as its complex conjugate.

At this point, I am going to choose \(s = (iћ/2m_o)\), which is Lorentz Scalar Invariant, in order to make the probability have dimensionless units and be normalized to unity in the rest case.
4-Vector Quantum Probability

4-ProbabilityFlux, Klein-Gordon RQM Eqn

A Tensor Study of Physical 4-Vectors

Conservation of 4-NumberFlux, a.k.a. 4-ProbabilityFlux
\[ J_{\text{prob}} = (c \rho_{\text{prob}}, \mathbf{J}_{\text{prob}}) = (i\hbar/2m)\left((\mathbf{\psi}^*\partial[\mathbf{\psi}] - \mathbf{\partial}[\mathbf{\psi}^*]\mathbf{\psi}\right) = (\rho_{\text{prob}}) \mathbf{U} = (\rho_{\text{prob}}) (c, \mathbf{U}) = (\gamma \rho_{\text{prob}}) (c, \mathbf{U}) = (\rho_{\text{prob}}) (c, \mathbf{U}) \]

with 4-Divergence of Probability \( \partial \cdot \mathbf{J}_{\text{prob}} = 0 \) by construction via Green’s Vector Identity and the Klein-Gordon RQM Eqn.

The reason for \( s = (i\hbar/2m) \) becomes more clear by examining our diagram:

Start at the 4-Gradient and follow the arrows toward the 4-ProbabilityFlux

You immediately see where the \((i\hbar/m)\) factor comes from.

The \( \rho_{\text{prob}} \) is then a function of the \( \psi \)'s divided by 2.

\[ \partial \cdot \left( f \left[ \mathbf{\partial}[g] - \mathbf{\partial}[f] \mathbf{g} \right] \right) = f \partial \cdot \mathbf{\partial}[g] - \partial \cdot \mathbf{\partial}[f] \mathbf{g} \quad \text{Green’s Vector Identity} \]

\[ \partial \cdot (\mathbf{m}_c/c^2) = 0: \text{KG RQM Eqn} \]

Examine the temporal component, the Relativistic Probability Density
\[ \rho_{\text{prob}} = (i\hbar/2m_c^2)(\mathbf{\psi}^* \partial[\mathbf{\psi}] - \partial[\mathbf{\psi}^*] \mathbf{\psi}) \]

Assume wave solution in following general form:
\[ \psi = A \mathbf{f} [k] e^{-i\omega t} \]
\[ \{ \psi^* = A^* \mathbf{f} [k]^* e^{+i\omega t} \} \]

then
\[ \{ \hat{\partial}[\psi] = (-i\omega)A \mathbf{f} [k] e^{-i\omega t} = (-i\omega)\psi \} \]
\[ \{ \hat{\partial}[\psi^*] = (+i\omega)A^* \mathbf{f} [k]^* e^{+i\omega t} = (+i\omega)\psi^* \} \]

\[ \rho_{\text{prob}} = (i\hbar/2m_c^2)(\psi^* \partial[\mathbf{\psi}] - \partial[\mathbf{\psi}^*] \mathbf{\psi}) \]
\[ \rho_{\text{prob}} = (i\hbar/2m_c^2)(-i\omega)\psi^* \mathbf{\psi}^* \]
\[ \rho_{\text{prob}} = (i\hbar/2m_c^2)(-i\omega)\psi^* \mathbf{\psi}^* \]
\[ \rho_{\text{prob}} = (h\omega/m_c^2)(\psi^* \mathbf{\psi}) \]
\[ \rho_{\text{prob}} = (h\omega/m_c^2)(\psi^* \mathbf{\psi}) \]
\[ \rho_{\text{prob}} = (\gamma)(\psi^* \mathbf{\psi}) = (\gamma) \rho_{\text{prob}} \]

Finally, multiply by charge \( q \) to get standard SR EM
\[ 4\text{-CurrentDensity} = 4\text{-ChargeFlux} = \mathbf{J} = (c\mathbf{p}, j) = q\mathbf{J}_{\text{prob}} = q(c\rho_{\text{prob}} \mathbf{J}_{\text{prob}}) \]

Trace\[ T^\mu_{\nu} = n_m T^{\mu}_{\nu} = T^{\mu}_{\nu} = T \]
\[ V^\nu V^\lambda = V^\nu n_m V^\lambda = (V^\nu)^2 - \nu \mathbf{V} = (V^\nu)^2 \]

= Lorentz Scalar Invariant

4-Vector SRQM Interpretation of QM

4-Vector SRQM

SciRealm.org
John B. Wilson
SciRealm.org
http://scirealm.org/SRQM.pdf
Now have the additional factor: Follow past Born Rule. Follow back past (1/
Minimal Coupling allows passage back to Start at \( J \)
If we include minimal coupling:
with 4-Divergence of Probability \( \partial \cdot J = 0 \) by construction via Green's Vector Identity and the Klein-Gordon RQM Eqn.
We include minimal coupling:
\( J_{\text{prob}} = \left( \frac{\hbar}{2m_0} \right) \left( \psi^* \partial_0 \psi - \partial_0 (\psi^* \psi) + (q/m_0)(\psi^* \psi) \right) \)
Start at A on the chart
Follow past (q) factor to get to Q = qA
Minimal Coupling allows passage back to P with no factors
Follow back past (1/m_0) to get to U
Follow past Born Rule (\( \psi^* \psi \))
Now have the additional factor: (+ (q/m_0)(\psi^* \psi)A)

4-Vector SRQM Interpretation
of QM

John B. Wilson
SciRealm.org
http://scirealm.org/SRQM.pdf
4-Vector Quantum Probability

Newtonian Limit

4-ProbabilityCurrentDensity \( J_{\text{prob}} = (c\rho_{\text{prob}}, J_{\text{prob}}) = (i\hbar/2m_0)(\psi^* \partial[\psi] - \partial[\psi^*]\psi) + (q/m_0)(\psi^*\psi)A \)

Examine the temporal component:
\[ \rho_{\text{prob}} = (i\hbar/2m_0c^2)(\psi^* \partial[\psi] - \partial[\psi^*]\psi) + (q/m_0)(\psi^*\psi)(\phi/c^2) \]
\[ \rho_{\text{prob}} \rightarrow (\gamma)(\psi^*\psi) + (\gamma)(q\phi_0/m_0c^2)(\psi^*\psi) = (\gamma)[1 + q\phi_0/E_0](\psi^*\psi) \]

Typically, the particle EM potential energy \((q\phi_0)\) is much less than the particle rest energy \((E_0)\), else it could generate new particles. So, take \((q\phi_0 << E_0)\), which gives the EM factor \((q\phi_0/E_0) \sim 0\).

Now, taking the low-velocity limit \((\gamma \rightarrow 1)\), \(\rho_{\text{prob}} = \gamma[1 + \sim 0](\psi^*\psi)\), \(\rho_{\text{prob}} \rightarrow (\psi^*\psi) = (\rho_{\text{prob}})\) for \(|v| << c\).

The Standard Born Probability Interpretation, \((\psi^*\psi) = (\rho_{\text{prob}})\), only applies in the low-potential-energy & low-velocity limit.

This is why the \{non-positive-definite\} probabilities and \{|probabilities| > 1\} in the RQM Klein-Gordon equation gave physicists fits, and is the reason why one must regard the probabilities as charge conservation instead.

The original definition from SR is Continuity of Worldlines, \(\partial \cdot J_{\text{prob}} = 0\), for which all is good and well in the RQM version. The definition says there are no external sources or sinks of probability = conservation of probability.

The Born idea that \((\rho_{\text{prob}}) \rightarrow \text{Sum}[(\psi^*\psi)] = 1\) is just the Low-Velocity QM limit. Only the non-EM rest version \((\rho_{\text{prob}}) = \text{Sum}[(\psi^*\psi)] = 1\) is true. It is not a fundamental axiom, it is an emergent property which is valid only in the NRQM limit.

We now multiply by charge \((q)\) to instead get a 4-“Charge”CurrentDensity \( J = (c\rho, J) = qJ_{\text{prob}} = q(c\rho_{\text{prob}}, J_{\text{prob}})\), which is the standard SR EM 4-CurrentDensity.
SRQM 4-Vector Study: The QM Compton Effect

Compton Scattering Derivation: Compton Effect

\[
\mathbf{P} \cdot \mathbf{p} = (m_e c)^2 \] generally \(\rightarrow 0\) for photons \(m_e=0\)

\[
P_{\text{photon}} \cdot P_{\text{photon}} = \mathbf{K} \cdot \mathbf{K} = (h^2 \mathbf{w}_e \cdot \mathbf{w}_e / c^2) = (1 - \mathbf{n} \cdot \mathbf{n}) = (h^2 w_e \cdot w_e / c^2) \]

\[
P_{\text{photon}} \cdot \mathbf{P}_{\text{mass}} = \hbar \mathbf{K} \cdot \mathbf{P}_{\text{mass}} = (h/\mathbf{w}_{\text{ph}}/c) (1 - \mathbf{n} \cdot \mathbf{n}) = (h^2 w_{\text{ph}} \cdot w_{\text{ph}} / c^2) = (h \mu m_e)
\]

\[
P_{\text{photon}} + \mathbf{P}_{\text{mass}} = \mathbf{P}^*_{\text{photon}} + \mathbf{P}_{\text{mass}} \text{ rearrange,}
\]

\[
(P_{\text{photon}} + \mathbf{P}_{\text{mass}} - \mathbf{P}^*_{\text{photon}})^2 = (\mathbf{P}_{\text{mass}})^2 \] square to get scalars

\[
(P_{\text{photon}} + \mathbf{P}_{\text{mass}} - 2 \mathbf{P}^*_{\text{photon}} + \mathbf{P}^*_{\text{photon}}) = (\mathbf{P}_{\text{mass}})^2
\]

\[
(0 + 2 \mathbf{P}^*_{\text{photon}} - 2 \mathbf{P}^*_{\text{photon}} + \mathbf{P}_{\text{mass}} - 2 \mathbf{P}^*_{\text{photon}} + \mathbf{P}^*_{\text{photon}}) = (m_e c)^2
\]

\[
P_{\text{photon}} \cdot \mathbf{P}_{\text{mass}} = \mathbf{P}_{\text{mass}} \cdot \mathbf{P}_{\text{photon}} = \mathbf{P}^*_{\text{photon}} \cdot \mathbf{P}^*_{\text{photon}}
\]

\[
(h \mu m_e) - (h \mu m_e) = (h^2 w_{\text{ph}} \cdot w_{\text{ph}} / c^2) (1 - \cos[\theta])
\]

\[
(\omega - \omega')/\omega' = (h m_e c^2 / \mathbf{w}_{\text{ph}}) (1 - \cos[\theta])
\]

\[
(1/\omega - 1/\omega') = \text{null}
\]

\[
\Delta \lambda = \lambda - \lambda' = (h / m_e c^2) (1 - \cos[\theta]) = \lambda_c (1 - \cos[\theta])
\]

The Compton Effect: Compton Scattering

with

\[
\lambda_c = \lambda c / 2 \pi = (h / m_e c) = \text{Reduced Compton Wavelength}
\]

\[
\lambda_c = (h / m_e c) = \text{Compton Wavelength (not a rest-wavelength, but the wavelength of a photon with the energy equivalent to a massive particle of rest-mass m_e)}
\]

Calculates the wavelength shift of a photon scattering from an electron (ignoring spin)

Proves that light does not have a “wave-only” description, photon 4-Momentum required

\[
E/\omega = \gamma E/\gamma \omega = E/\omega = h
\]

\[
K_{\text{photon}} = (\mathbf{w}_c / c) (1, \mathbf{n}) = 0
\]

\[
\omega_A = \omega \lambda = \lambda c
\]

for photons

\[
\omega = \mathbf{w}_c = \mathbf{v}
\]

\[
\mathbf{K} \cdot \mathbf{K} = (\mathbf{w}_c / c)^2
\]

\[
\mathbf{P} \cdot \mathbf{P} = (m_e c)^2
\]

\[
\mathbf{E} / \omega = \mathbf{c} = \mathbf{v}
\]

\[
\mathbf{K} = (\mathbf{w}_c / c) = (1, \mathbf{n})
\]

\[
\mathbf{E} / \omega_0 = \mathbf{h} c
\]

\[
\mathbf{E} / \omega_0 = \mathbf{c} / \omega_0
\]

\[
\mathbf{E} / \omega_0 = \mathbf{c} / \omega_0
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\]

\[
\mathbf{E} / \omega_0 = \mathbf{c} / \omega_0
\]
SRQM 4-Vector Study: The QM Aharonov-Bohm Effect

**QM Potential**\[ \Delta \Phi_{pot} = -(q/\hbar) \int_{\text{path}} A \cdot dX \]

**Aharonov-Bohm Effect**

The EM 4-Vector Potential gives the Aharonov-Bohm Effect.

\[ \Phi_{pot} = -(q/\hbar) A \cdot X = -K_{pot} \cdot X \]

or taking the differential...

\[ d\Phi_{pot} = -(q/\hbar) A \cdot dX \]

over a path...

\[ \Delta \Phi_{pot} = \int_{\text{path}} d\Phi_{pot} = -(q/\hbar) \int_{\text{path}} A \cdot dX \]

\[ \Delta \Phi_{pot} = -(q/\hbar) \int_{\text{path}} [(\varphi/c)(cdt) - a \cdot dX] \]

\[ \Delta \Phi_{pot} = -(q/\hbar) \int_{\text{path}} (\varphi dt - a \cdot dX) \]

Note that both the Electric and Magnetic effects come out by using the 4-Vector notation.

Electric AB effect: \[ \Delta \Phi_{pot \text{,Elec}} = -(q/\hbar) \int_{\text{path}} (\varphi dt) \]

Magnetic AB effect: \[ \Delta \Phi_{pot \text{,Mag}} = +(q/\hbar) \int_{\text{path}} (a \cdot dX) \]

Proves that the 4-Vector Potential A is more fundamental than e and b fields, which are just components of the Faraday EM Tensor.
**SRQM 4-Vector Study:**

The QM Josephson Junction Effect = SuperCurrent

EM 4-VectorPotential \( A = -(\hbar/q)\partial[\Delta \Phi_{pot}] \)

Josephson Effect

The EM 4-VectorPotential gives the Aharonov-Bohm Effect. Phase \( \Phi_{pot} = -(\hbar/q)A \cdot X = -K_{pot} \cdot X \)

Rearrange the equation a bit:

\[-(\hbar/q)\Delta \Phi_{pot} = A \cdot \Delta X \]

\[A \cdot \Delta X = -(\hbar/q)\Delta \Phi_{pot} \]

\[d/dt[A \cdot \Delta X] = d/dt[-(\hbar/q)\Delta \Phi_{pot}] = d/dt[A] \cdot \Delta X + A \cdot d/dt[\Delta X] = d/dt[A] \cdot \Delta X + A \cdot U \]

Assume that \( d/dt[A] \cdot \Delta X \sim 0 \)

Which explains Josephson Effect criteria:

\[\Delta X \sim 0: \text{small gap} \]

\[d/dt[A] \sim 0: \text{“critical current” & no voltage} \]

\[d/dt[\Delta X] \sim \text{orthogonal: ??} \]

\[A = (\hbar/q)K; \quad K = (\omega/c,k) = (\omega/c)(\phi/c,a) \]

Take the temporal part:

EM ScalarPotential \( \phi = -(\hbar/q)(\partial/\partial t)[\Delta \Phi_{pot}] \); \( \omega = (\hbar/q)\phi \)

If the charge \( q \) is a Cooper-electron-pair: \( q = -2e \)

Voltage \( V(t) = \phi(t) = (\hbar/2e)(\partial/\partial t)[\Delta \Phi_{pot}] \); \( \text{AngFreq} \omega = -2eV/h \)

This is the superconducting phase evolution equation of the Josephson Effect

\((\hbar/2e)\) is defined to be the Magnetic Flux Quantum \( \Phi_o \)

**SR 4-Tensor**

(2,0)-Tensor \( T^{IV}_{\nu \mu} \)

(1,1)-Tensor \( T^{\nu \mu}_\nu = T^{\nu \mu} \)

(0,2)-Tensor \( T^{\mu \nu \lambda}_\lambda \)

**SR 4-Vector**

(1,0)-Tensor \( V = V^{\nu}_\nu = (\nu^\nu, v) \)

SR 4-CoVector:OneForm \( (\nu,0)-Tensor \ V_\nu = (\nu_\nu, -v) \)

**SR 4-Scalar**

(0,0)-Tensor \( S = S_0 \)

Lorentz Scalar

**Existing SR Rules**

**Quantum Principles**
SRQM Symmetries:
Hamilton-Jacobi vs Relativistic Action
Josephson vs Aharonov-Bohm
Differential (4-Vector) vs Integral (4-Scalar)

Differential Formats : 4-Vectors : HJ

SR Hamilton-Jacobi Equation
\[ P_T = P + qA = P + Q = -\partial[\Delta S_{\text{action}}] = -\partial[h\Delta \Phi_{\text{phase, dyn}} + \Delta \Phi_{\text{phase, potential}}] \]
Notice the Symmetry:
Integral Formats : 4-Scalars : Action

SR Action Equation
\[ \Delta S_{\text{action}} = -\int_{\text{path}} P_T \cdot dX = -\int_{\text{path}} (P + qA) \cdot dX = -\int_{\text{path}} (P + Q) \cdot dX = h\Delta \Phi_{\text{phase}} = h(\Delta \Phi_{\text{phase, dyn}} + \Delta \Phi_{\text{phase, potential}}) \]

Josephson Junction Relation
\[ A = -\frac{(\hbar/c)\partial[\Delta S_{\text{act,dynamic}}]}{\hbar} = -\frac{1}{\hbar}[\partial[\Delta S_{\text{act,dynamic}}]}{\hbar} \]

Technically, the standard Josephson Junction uses just the temporal part \{ A = (\phi/c,a) \} & Cooper-pair-electrons \{ q = -2e \} giving \( V(t) = \phi = (\hbar/2e)\partial/\partial t[\Delta \Phi_{\text{pot}}] \).
There should be a spatial part as well.

4-PotentialMomentum
\[ Q = qA = -\partial[\Delta S_{\text{act,potential}}] \]

Inverse

4-Momentum (free part)
\[ P = -\partial[\Delta S_{\text{act,dynamic}}] \]

Inverse

4-WaveVector
\[ K = -\partial[\Delta S_{\text{act,dyn}}]/\hbar \]

Inverse

4-TotMomentum Conservation
\[ P_T = (P + Q) = (P + qA) \]
Minimal Coupling
\[ P = (P_T - qA) = (P_T - Q) \]

Minimal Coupling
\[ P = (P_T - qA) = (P_T - Q) \]

Potential Part

4-TotMomentum Conservation
\[ P_T = (P + Q) = (P + qA) \]
Minimal Coupling
\[ P = (P_T - qA) = (P_T - Q) \]

Potential Part

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4-TotMomentum Conservation
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Potential Part

Minimal Coupling
\[ P = (P_T - qA) = (P_T - Q) \]
SRQM Symmetries:
Schrödinger Relations
Cyclic Imaginary Time ↔ Inv Temp

4-Momentum
\[ P = p = (mc, p) = (mc, mu) = m_u U \]
\[ = (E/c, p) = (E/c^2) U \]

Einstein de Broglie
\[ R = \hbar K \]
\[ = \hbar \left[ \frac{P}{\hbar} \right] \]

Inverses

4-Position
\[ R = R_i = (ct, r) = \langle \text{Event} > \rightarrow (ct, x, y, z) \]
alt. notation \[ X = X^\mu \]

4-ImaginaryPosition
\[ R_m = R_m^\mu = i(ct, r) \]
\[ = (ict, ir) = (ct, ir) \]

Covariant Wick Rotation
\[ R = -iR_m \]

\([i = 1/i] \]

1/\hbar

Complex Plane-Wave
\[ K = K^\mu = (\omega/c, k) = (\omega/c^2) U \]
\[ = (\omega/c, \omega \hat{n}/\text{phase}) = (1/c \hat{T}, \hat{n}/\Lambda) \]

4-Gradient
\[ \partial = \partial_R \partial = \partial_R \partial_R \partial = \partial_R (\partial/c, -\nabla) \]
\[ \rightarrow (\partial/c, -\partial_x, -\partial_y, -\partial_z) \]
\[ = (\partial/c, \partial, \partial_x, \partial_y, \partial_z) \]

1/\hbar

4-Vector SRQM Interpretation of QM

Einstein-de Broglie: \[ P = \hbar K \rightarrow \{ E = \hbar \omega : p = \hbar k \} \]
Complex Plane-Wave: \[ K = i \Theta \rightarrow \{ \omega = i \partial : \Theta = -i \nabla \} \]

Schrödinger Relations: \[ P = i \hbar \Theta \rightarrow \{ E = \hbar \omega : p = -i \nabla \} \]

Wick Rotation: \[ R = -iR_m \rightarrow \{ t = -i r : r = -i(\text{ir}) \} \]
Cyclic Temp: \[ R_m = \hbar \Theta \rightarrow \{ t = \hbar k_u T : r = \hbar u/\kappa T \} \]

Time Temp: \[ R = -i\hbar \Theta \rightarrow \{ t = -i \hbar k_u T : r = -i \hbar u/\kappa T \} \]

Boltzmann Distribution
\[ P = \Theta = (E/c, p) : (c/\kappa T, \Theta) \]
\[ = (E/\kappa T - p \cdot \Theta) = (E/\kappa T) \]

Trace\[ T^\mu = T_{\mu} = T^\mu = T \]
\[ V = V_{\mu} = (v^\mu) \cdot (v^\nu) = (v^\mu)^2 \]
\[ = \text{Lorentz Scalar Invariant} \]

Note that the temperature here is relativistically direction-specific, unlike in the classical use of temperature.
SRQM Symmetries: Wave-Particle

4-Momentum
\[ P = (mc, p) = (E/c, p) \]
\[ P = \delta[S_{\text{action, free}}] \]

4-Gradient
\[ \delta = \left( \frac{\partial}{\partial E}, -\nabla \right) \rightarrow (\frac{\partial}{\partial E}, -\frac{\partial}{\partial \tau}, -\frac{\partial}{\partial x}, -\frac{\partial}{\partial y}) \]
d'Alembertian
\[ \delta^{2} = \left( \frac{\partial}{\partial c}, -\nabla \right) \cdot \left( \frac{\partial}{\partial c}, -\nabla \right) = (\frac{\partial}{\partial c})^{2} - \nabla \cdot \nabla = (\frac{\partial}{\partial c})^{2} \]

4-WaveVector
\[ K = (\omega/c, k) = (\omega/c, \omega \hat{n}/v_{\text{phase}}) \]
\[ K = -\delta[\Phi_{\text{phase, plane}}] \]
\[ K \cdot K = (\omega/c)^{2} \]

See SR Wave Definition for info on the Lorentz Scalar Invariant SR WavePhase.
\{ \Phi = (\omega/c, k) = -\delta[Ф] = (-\delta[c\hat{a}(s), V(S)] \}
\{ temporal component \} \omega = -\delta[a(t)] = -\delta[Ф] \}
\{ spatial component \} k = V(Ф) \}
**Note** This is the Phase (Ф) for a single free plane-wave.

Generally WavePhase is for the 4-TotalWaveVector (K) of a system.

\{ P = (E/c, p) = -\delta[S] = -\delta[c\hat{a}(s), V(S)] \}
\{ temporal component \} E = -\delta[a(t)] = -\delta[Ф] \}
\{ spatial component \} p = V(Ф) \}
**Note** This is the Action (S_{\text{action}}) for a free particle.

Generally Action is for the 4-TotalMomentum (P) of a system.

<table>
<thead>
<tr>
<th>SR 4-Tensor</th>
<th>SR 4-Vector</th>
<th>SR 4-Scalar</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2,0)-Tensor ( T^{\mu \nu} )</td>
<td>( (1,0) )-Tensor ( V^{\nu} = (v^{\nu}, v) )</td>
<td>Lorentz Scalar</td>
</tr>
<tr>
<td>( (1,1) )-Tensor ( T^\nu_v ) or ( T^\nu_v )</td>
<td>( (0,1) )-Tensor ( V_{\nu} = (v_{\nu}, v) )</td>
<td>Lorentz Scalar Invariant</td>
</tr>
</tbody>
</table>

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
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http://scirealm.org/SRQM.pdf
SRQM Symmetries:
Relativistic Euler-Lagrange Equation

The Easy Derivation \((U = (d/d\tau)R) \rightarrow (\partial_R = (d/d\tau)\partial_U)\)

- **Note Similarity:**
  - 4-Velocity is Proper Time Derivative of 4-Position
  - \(U = (d/d\tau)R\) \([\text{m/s}] = [1/s][\text{m}]\)

**Relativistic Euler-Lagrange Eqn**
\(\partial_R = (d/d\tau)\partial_U\) \([1/m] = [1/s][\text{s/m}]\)

- The differential form just inverses the dimensional units, so the placement of the \(R\) and \(U\) switch.

**That is it: so simple!** Much, much easier than how I was taught in Grad School.

To complete the process and create the Equations of Motion, one just applies the base form to a Lagrangian.

This can be:
- a classical Lagrangian
- a relativistic Lagrangian
- Lorentz scalar Lagrangian
- a quantum Lagrangian

**SR 4-Tensor**
- (2,0)-Tensor \(T^{\mu\nu}\)
- (1,1)-Tensor \(T^{\alpha\beta}\) or \(T^\nu\)
- (0,2)-Tensor \(T_{\mu\nu}\)

**SR 4-Vector**
- (1,0)-Tensor \(V^\mu = V = (V^0, v)\)
- (0,1)-Tensor \(V_\nu = (v_0, \vec{v})\)

**SR 4-Scalar**
- (0,0)-Tensor \(S\) or \(S_\mu = (S, \vec{0})\)
- Lorentz Scalar

**4-Vector SRQM Interpretation of QM**
- 
  - Natural 4-Vector (1,0)-Tensor
  - Proper Time Derivative
  - Interestingly, this has it's own similar inverse relations.
    \(\frac{d}{d\tau} = \gamma \frac{d}{dt}\)
  - \(dt = \gamma d\tau\)

**Index-raised One-Form**
- 4-Vector (1,0)-Tensor

**One-Form (0,1)-Tensor**

**Trace**
- \(\eta^\mu_\nu = T^{\mu\nu} = T^\nu_\mu = T\)
- \(\mathbf{V} \cdot \mathbf{V} = V^\mu V_\mu = (V^0)^2 - \mathbf{v} \cdot \mathbf{v} = (\mathbf{v}^0)^2\)
- Lorentz Scalar Invariant
SRQM Symmetries:
Lorentz Transform Connection Map – Trace Identification
CPT, Big-Bang, [Matter↔AntiMatter], Arrow(s)-of-Time

All Lorentz Transforms have Tensor Invariants: Determinant = ±1 and InnerProduct = 4. However, one can use the Tensor Invariant Trace to Identify CPT Symmetry & AntiMatter

<table>
<thead>
<tr>
<th>Transform Type</th>
<th>Trace Invariant</th>
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Two interesting properties of (1,1)-Tensors, of which the Lorentz Transform is an example:
- Trace = Sum (Σ) of EigenValues : Determinant = Product (Π) of EigenValues
- As 4D Tensors, each Lorentz Transform has 4 EigenValues (EV's).
- Create an Anti-Transform which has all EigenValue Tensor Invariants negated.
- The Trace Invariant identifies a “Dual” Negative-Side for all Lorentz Transforms.

SRQM Symmetries:
Lorentz Transform Connection Map – Trace Identification
CPT, Big-Bang, [Matter↔AntiMatter], Arrow(s)-of-Time

All Lorentz Transforms have Tensor Invariants: Determinant = ±1 and InnerProduct = 4. However, one can use the Tensor Invariant Trace to Identify CPT Symmetry & AntiMatter

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SR:Lorentz Transform
\[ \partial [R^\nu] = \partial R^\nu / \partial R^\nu = \Lambda^\nu_v \]
\[ \Lambda^\nu_v = (\Lambda^{-1})^\nu_v : \Lambda^{\nu\alpha} \Lambda^{\nu\beta} = \eta^{\nu\beta} \]
\[ \eta_{\nu\beta} = \Lambda^{\nu\alpha} \Lambda^{\nu\beta} = \eta_{\beta\gamma} \]
\[ \text{Det}[(\Lambda^{\nu\nu})] = \pm 1 \]
\[ \text{Tr}[(\Lambda^{\nu\nu})] = \pm (\Lambda^{\nu\nu}) = \Lambda^{\nu\nu} \]

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The $h$ Connection

$P = hK$: Basic Einstein-de Broglie

$P + Q = P + Q$

$P + Q = hK_{dyn} + hK_{pot}$

$P + Q = h(K_{dyn} + K_{pot})$

Sum over $n$ particles: $P_T = \Sigma_n(P + Q), K_T = \Sigma_n(K_{dyn} + K_{pot})$

$P_T = hK_T$

$P_T \cdot X = hK_T \cdot X$

$\sigma_{action} = -h\Phi_{phase}$

$S_{action} = h\Phi_{phase}$

$\sigma[S_{action}] = -h[\Phi_{phase}]$

$P_T = hK_T$

{SR Hamilton-Jacobi} = $h$\{QM Complex Plane-Waves\}

The SR Hamilton-Jacobi Equation, and the QM idea of Complex Plane-Waves, are related by a simple constant ($h$) relation.
SRQM 4-Vector Study: Dimensionless Physical Objects

There are a number of dimensionless physical objects in SR that can be constructed from Physical 4-Vectors. There are 4-Scalars, 4-Vectors, and 4-Tensors.

- $\delta \cdot X = 4$: SpaceTime Dimension
- $\delta^\nu [X] = \eta^{\nu\nu}$: The SR Minkowski Metric
- $\delta^\nu [X] = \gamma^{\nu\nu}$: Lorentz Transformation Tensor
- $dX\cdot \mathbf{d}X$: Calculus Total Derivative Chain Rule

There are a number of dimensionless physical objects in SR:

- Dimensionless Physical Objects

- 4-Displacement $\Delta X = (c\Delta t, \Delta x)$
- 4-Position $X = (c, x)$
- 4-UnitTemporal $T = (1, \beta)$
- 4-UnitSpatial $S = (\gamma, \beta)$
- 4-WaveVector $K = (\omega/c, k)$
- 4-Momentum $P = (mc, p)$
- 4-ThermalVector $B = (E_{\Theta}/k_B T, 0)$

These objects are used in the interpretation of QM and its relationship to SR. For example, the de Broglie wavelength is related to the momentum of a particle:

$$\lambda = \frac{h}{P} = \frac{h}{mc}$$

where $h$ is the Planck constant. The de Broglie wavelength is also related to the energy of a particle:

$$E = \frac{p^2}{2m} + mc^2$$

This relationship is a key aspect of SRQM and its interpretation of QM.
SRQM: QM Axioms Unnecessary

QM Principles emerge from SR

QM is derivable from SR plus a few empirical facts – the “QM Axioms” aren't necessary. These properties are either empirically measured or are emergent from SR properties...

3 “QM Axioms” are really just empirical constant relations between purely SR 4-Vectors:
- Particle-Wave Duality \([\mathbf{P} = \hbar \mathbf{K}]\)
- Unitary Evolution \([\partial = (-i)\mathbf{K}]\)
- Operator Formalism \([\{\partial\} = -i\mathbf{K}]\)

2 “QM Axioms” are just the result of the Klein-Gordon Equation being a linear wave PDE:
- Hilbert Space Representation (<bra|,|ket>, wavefunctions, etc.) & The Principle of Superposition

3 “QM Axioms” are a property of the Minkowski Metric and the empirical fact of Operator Formalism
- The Canonical Commutation Relation
- The Heisenberg Uncertainty Principle (time-like-separated measurement exchange)
- The Pauli Exclusion Principle (space-like-separated particle exchange)

1 “QM Axiom” only holds in the NRQM case
- The Born QM Probability Interpretation – Not applicable to RQM, use Conservation of Worldlines instead

1 “QM Axiom” is really just another level of limiting cases, just like SR → CM in limit of low velocity
- The QM Correspondence Principle (\(\text{QM} \rightarrow \text{CM in limit of } \{\nabla^2[\phi] << (\nabla[\phi])^2\} \))

SRQM: A treatise of \(\text{SR} \rightarrow \text{QM}\) by John B. Wilson (SciRealm@aol.com)
The SRQM interpretation fits fairly well with Carlo Rovelli's Relational QM interpretation:

Relational QM treats the state of a quantum system as being observer-dependent, that is, the QM State is the relation between the observer and the system. This is inspired by the key idea behind Special Relativity, that the details of an observation depend on the reference frame of the observer.

All systems are quantum systems: no artificial Copenhagen dichotomy between classical/macroscopic/conscious objects and quantum objects.

The QM States reflect the observers' information about a quantum system. Wave function "collapse" is informational – not physical. A particle always knows it's complete properties. An observer has at best only partial information about the particle's properties.

No Spooky Action at a Distance. When a measurement is done locally on an entangled system, it is only the partial information about the distant entangled state that "changes/becomes-available-instantaneously". There is no superluminal signal. Measuring/physically-changing the local particle does not physically change the distant particle.

ex. Place two identical-except-for-color marbles into a box, close lid, and shake. Without looking, pick one marble at random and place it into another box. Send that box very far away. After receiving signal of the far box arrival at a distant point, open the near box and look at the marble. You now instantaneously know the far marble's color as well. The information did not come by signal. You already had the possibilities (partial knowledge). Looking at the near marble color simply reduced the partial knowledge of both marble's color to complete knowledge of both marbles' color. No signal was required, superluminal or otherwise.

ex. The quantum version of the same experiment uses the spin of entangled particles. When measured on the same axis, one will always be spin-up, the other will be spin-down. It is conceptually analogous. Entanglement is only about correlations of system that interacted in the past and are determined by conservation laws.
SRQM Interpretation:

Interpretation of EPR-Bell Experiment

Einstein and Bohr can both be “right” about EPR:
Per Einstein: The QM State measured is not a “complete” description, just one observer’s point-of-view.
Per Bohr: The QM State measured is a “complete” description, it’s all that a single observer can get.

The point is that many observers can all see the “same” system, but see different facets of it. But a single measurement is the maximal information that a single observer can get without re-interacting with the system, which of course changes the system in general. Remember, the Heisenberg Uncertainty comes from non-zero commutation properties which require separate measurement arrangements*. The properties of a particle are always there. Properties define particles. We as observers simply have only partial information about them.

Relativistic QM, being derived from SR, should be local – The low-velocity limit to QM may give unexpected anomalous results if taken out of context, or out of the applicable validity range, such as with velocity addition $v_{12} = v_1 + v_2$, where the correct formula should be the relativistic velocity composition $v_{12} = (v_1 + v_2)/(1 + v_1 v_2/c^2)$

These ideas lead to the conclusion that the wavefunction is just one observer’s state of information about a physical system, not the state of the physical system itself. The “collapse” of the wavefunction is simply the change in an observer’s information about a system brought about by a measurement or, in the case of EPR, an inference about the physical state.

EPR doesn’t break Heisenberg because measurements are made on different particles. The happy fact is that those particles interacted and became correlated in the causal past. The EPR-Bell experiments prove that it is possible to maintain those correlations over long distances. It does not prove superluminal (FTL) signaling.
SRQM Interpretation:
Range-of-Validity Facts & Fallacies

We should not be surprised by the "quantum" probabilities being correct instead of "classical" in the EPR and Bell Inequalities experiments. Classical thinking (in both CM and QM) has a number of fallacies when it is mistakenly applied outside of its range-of-validity.

Examples

* Classical Physics as the limit of $\hbar \to 0$ {Fallacy}:
  $\hbar$ is a Lorentz Scalar Invariant and Fundamental Physical Constant. It never becomes 0. {Fact}

* The classical commutator being zero $[p^i, x^j] = 0$ {Fallacy}:
  $[\mathbf{P}^\mu, \mathbf{X}^\nu] = i\hbar \eta^{\mu\nu}$; $[\mathbf{p}^i, \mathbf{x}^j] = -i\hbar \delta^{ij}$; $[\mathbf{p}^0, \mathbf{x}^0] = [\mathbf{E}/c, \mathbf{ct}] = [\mathbf{E}, \mathbf{t}] = i\hbar(1)$; Again, it never becomes 0 {Fact}

* Using Maxwell-Boltzmann (distinguishable) statistics for counting probabilities of (indistinguishable) quantum states {Fallacy}:
  Must use Fermi-Dirac statistics for Fermions: Spin=$n+1/2$; Bose-Einstein statistics for Bosons: Spin=$n$ {Fact}

* Using sums of classical probabilities on quantum states {Fallacy}:
  Must use sums of quantum probability-amplitudes {Fact}

* Ignoring phase cross-terms and interference effects in calculations {Fallacy}:
  Quantum systems and entanglement require phase cross-terms {Fact}

* Assuming that one can simultaneously "measure" non-commuting properties at a single spacetime event {Fallacy}:
  Particle properties always exist. However, non-commuting ones require separate measurement arrangements to get information about the properties. The required measurement arrangements on a single particle/worldline are at best sequential events, where the temporal order plays a role; {Fact}
  However, EPR allows one to "infer (not measure)" the other property of a particle by the separate measurement of an entangled partner. {Fact}
  This does not break Heisenberg Uncertainty, which is about the order of operations (measurement events) on a single worldline. {Fact}
  In the entangled case, both/all of the entangled partners share common past-causal entanglement events, typically due to a conservation law. {Fact}
  Information is not transmitted at FTL. The particles simply carried their normal respective "correlated" properties (no hidden variables) with them. {Fact}

* Assuming that QM is a generalization of CM, or that classical probabilities apply to QM {Fallacy}:
  CM is a limiting-case of QM for when changes in a system by a few quanta have a negligible effect on the whole/overall system. {Fact}
SRQM Interpretation: Quantum Information

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{from Wikipedia}

No-Communication Theorem/No-Signaling:
A no-go theorem from quantum information theory which states that, during measurement of an entangled quantum state, it is not possible for one observer, by making a measurement of a subsystem of the total state, to communicate information to another observer. The theorem shows that quantum correlations do not lead to what could be referred to as "spooky communication at a distance". SRQM: There is no FTL signaling/communication.

No-Teleportation Theorem:
The no-teleportation theorem stems from the Heisenberg uncertainty principle and the EPR paradox: although a qubit $|\psi> \rangle$ can be imagined to be a specific direction on the Bloch sphere, that direction cannot be measured precisely, for the general case $|\psi> \rangle$. The no-teleportation theorem is implied by the no-cloning theorem.

SRQM: Ket states are informational, not physical.

No-Cloning Theorem:
In physics, the no-cloning theorem states that it is impossible to create an identical copy of an arbitrary unknown quantum state. This no-go theorem of quantum mechanics proves the impossibility of a simple perfect non-disturbing measurement scheme. The no-cloning theorem is normally stated and proven for pure states; the no-broadcast theorem generalizes this result to mixed states. SRQM: Measurements are arrangements of particles that interact with a subject particle.

No-Broadcast Theorem:
Since quantum states cannot be copied in general, they cannot be broadcast. Here, the word "broadcast" is used in the sense of conveying the state to two or more recipients. For multiple recipients to each receive the state, there must be, in some sense, a way of duplicating the state. The no-broadcast theorem generalizes the no-cloning theorem for mixed states. The no-cloning theorem says that it is impossible to create two copies of an unknown state given a single copy of the state. SRQM: Conservation of worldlines.

No-Deleting Theorem:
In physics, the no-deleting theorem of quantum information theory is a no-go theorem which states that, in general, given two copies of some arbitrary quantum state, it is impossible to delete one of the copies. It is a time-reversed dual to the no-cloning theorem, which states that arbitrary states cannot be copied. SRQM: Conservation of worldlines.

No-Hiding Theorem:
the no-hiding theorem is the ultimate proof of the conservation of quantum information. The importance of the no-hiding theorem is that it proves the conservation of wave function in quantum theory.
SRQM: Conservation of worldlines. RQM wavefunctions are Lorentz 4-Scalars (spin=0), 4-Spinors (spin=1/2), 4-Vectors (spin=1), all of which are Lorentz Invariant.
SRQM Interpretation:
Quantum Information

We should not be surprised by the "quantum" probabilities being correct instead of "classical" probabilities in the EPR/Bell-Inequalities experiments. Classical thinking (in both CM and QM) has a number of fallacies when it is mistakenly applied outside of its range-of-validity.

{from Wikipedia}

Quantum information (qubits) differs strongly from classical information, epitomized by the bit, in many striking and unfamiliar ways. Among these are the following:

A unit of quantum information is the qubit. Unlike classical digital states (which are discrete), a qubit is continuous-valued, describable by a direction on the Bloch sphere. Despite being continuously valued in this way, a qubit is the smallest possible unit of quantum information, as despite the qubit state being continuously-valued, it is impossible to measure the value precisely.

A qubit cannot be (wholly) converted into classical bits; that is, it cannot be "read". This is the no-teleportation theorem.

Despite the awkwardly-named no-teleportation theorem, qubits can be moved from one physical particle to another, by means of quantum teleportation. That is, qubits can be transported, independently of the underlying physical particle. SRQM: Ket states are informational, not physical.

An arbitrary qubit can neither be copied, nor destroyed. This is the content of the no-cloning theorem and the no-deleting theorem. SRQM: Conservation of worldlines.

Although a single qubit can be transported from place to place (e.g. via quantum teleportation), it cannot be delivered to multiple recipients; this is the no-broadcast theorem, and is essentially implied by the no-cloning theorem. SRQM: Conservation of worldlines.

Qubits can be changed, by applying linear transformations or quantum gates to them, to alter their state. While classical gates correspond to the familiar operations of Boolean logic, quantum gates are physical unitary operators that in the case of qubits correspond to rotations of the Bloch sphere.

Due to the volatility of quantum systems and the impossibility of copying states, the storing of quantum information is much more difficult than storing classical information. Nevertheless, with the use of quantum error correction quantum information can still be reliably stored in principle. The existence of quantum error correcting codes has also led to the possibility of fault tolerant quantum computation.

Classical bits can be encoded into and subsequently retrieved from configurations of qubits, through the use of quantum gates. By itself, a single qubit can convey no more than one bit of accessible classical information about its preparation. This is Holevo's theorem. However, in superdense coding a sender, by acting on one of two entangled qubits, can convey two bits of accessible information about their joint state to a receiver.

Quantum information can be moved about, in a quantum channel, analogous to the concept of a classical communications channel. Quantum messages have a finite size, measured in qubits; quantum channels have a finite channel capacity, measured in qubits per second.
Minkowski still applies in local GR

QM is a local phenomenon

The QM Schrodinger Equation is not fundamental. It is just the low-energy limiting-case of the RQM Klein-Gordon Equation. All of the standard QM Axioms are shown to be empirically measured constants or emergent properties of SR. It is a bad approach to start with NRQM as an axiomatic starting point and try to generalize it to RQM, in the same way that one cannot start with CM and derive SR. Since QM *can* be derived from SR, this partially explains the difficulty of uniting QM with GR:

QM is not a “separate formalism” outside of SR that can be used to “quantize” just anything...

Strictly speaking, the use of the Minkowski space to describe physical systems over finite distances applies only in the SR limit of systems without significant gravitation. In the case of significant gravitation, SpaceTime becomes curved and one must abandon SR in favor of the full theory of GR.

Nevertheless, even in such cases, based on the GR Equivalence Principle, Minkowski space is still a good description in a local region surrounding any point (barring gravitational singularities). More abstractly, we say that in the presence of gravity, SpaceTime is described by a curved 4-dimensional manifold for which the tangent space to any point is a 4-dimensional Minkowski Space. Thus, the structure of Minkowski Space is still essential in the description of GR.

So, even in GR, at the local level things are considered to be Minkowskian:

i.e. SR → QM “lives inside the surface” of this local SpaceTime, GR curves the surface.
SRQM Interpretation: Main Result

QM is derivable from SR!

Hopefully, this interpretation will shed light on why Quantum Gravity has been so elusive. Basically, QM rules of “quantization” don’t apply to GR. They are a manifestation-of/derivation-from SR. Relativity *is* the “Theory of Measurement” that QM has been looking for.

This would explain why no one has been able to produce a successful theory of “Quantum Gravity”, and why there have been no violations of Lorentz Invariance, CPT, or the Equivalence Principle.

If quantum effects “live” in Minkowski SpaceTime with SR, then GR curvature effects are at a level above the RQM description, and two levels above standard QM. SR+QM are “in” SpaceTime, GR is the “shape” of SpaceTime...

Thus, this SRQM Treatise explains the following:

- Why GR works so well in it’s realm of applicability {large scale systems}.
- Why QM works so well in it’s realm of applicability {micro scale systems and certain macroscopic systems}.
  i.e. The tangent space to any point in GR curvature is locally Minkowskian, and thus QM is typically found in small local volumes...
- Why RQM explains more stuff than QM without SR {because QM is just an approximation: the low-velocity limiting-case of RQM}.
- Why all attempts to "quantize gravity" have failed {essentially, everyone has been trying to put the cart (QM) before the horse (GR)}. 
- Why all attempts to modify GR keep conflicting with experimental data {because GR is apparently fundamental – passed all tests to-date}.
- Why QM works perfectly well with SR as RQM but not with GR {because QM is derivable from SR, hence a manifestation of SR rules}.
- How Minkowski Space, 4-Vectors, and Lorentz Invariants play vital roles in RQM, and give the SRQM Interpretation of Quantum Mechanics.

SRQM: Special Relativistic Quantum Measurement, Special Relativistic Quantum Mechanics
SRQM: The [SR→QM] Interpretation of Quantum Mechanics

Special Relativity (SR) Axioms: Invariant Interval + LightSpeed (c) as Universal Physical Constant lead to SR, although technically SR is itself the Minkowski-SpaceTime low-curvature: "flat" limiting-case of GR.

\{c, \tau, m_0, \hbar, i\} = \{c: SpeedOfLight, \tau: ProperTime, m_0: RestMass, \hbar: Dirac/Planck Reduced Constant(h=\hbar/2\pi), i: ImaginaryNumber\sqrt{-1}\}:
are all Empirically-Measured SR Lorentz-Invariant Physical and/or Mathematical Constants

Standard SR 4-Vectors:
- 4-Position: \( R = (ct, r) \)
- 4-Velocity: \( U = \gamma(c, u) \)
- 4-Momentum: \( P = (E/c, p) \)
- 4-WaveVector: \( K = (\omega/c, k) \)
- 4-Gradient: \( \partial = (\partial_t/c, \nabla) \)

Related by these SR Lorentz Invariants:
- \( (R \cdot R) = (ct)^2 \)
- \( (U \cdot U) = (c)^2 \)
- \( (P \cdot P) = (m_0c)^2 \)
- \( (K \cdot K) = (m_0c/\hbar)^2 \)
- KG Equation: \( |v| \ll c \rightarrow \) QM Relation → RQM → QM

SR + Empirically Measured Physical Constants lead to RQM via the (KG) Klein-Gordon Quantum Eqn, and thence to QM via the low-velocity limit \( |v| \ll c \), giving the Schrödinger Eqn. This fundamental KG Relation also leads to the other Quantum Wave Equations:

- \( \text{spin}=0 \) boson field = 4-Scalar: Free Scalar Wave (Higgs)
- \( \text{spin}=1/2 \) fermion field = 4-Spinor: Weyl
- \( \text{spin}=1 \) boson field = 4-Vector: Maxwell (EM photonic)

SRQM Chart:

SR → QM Interpretation Simplified

SRQM: A treatise of SR→QM by John B. Wilson (SciRealm@aol.com)
SR ProperTime Derivative
SR Phase → 4-WaveVector
SR Action → 4-Momentum
SR Lorentz Transforms
SR SpaceTime Dimension=4
SR SpaceTime "Flat" 4D Metric
4-Gradient=

A Tensor Study

Physics

(1,1)-Tensor T

+ {\mid}

Schrödinger Klein-Gordon Relativistic
SR d'Alembertian &

-(0,2)-Tensor

-(2,0)-Tensor T

- mass:energy &

3-momentum

oscillations proportional to

4-WaveVector=

\text{limit of KG QWE}

SR 4-Tensor

\text{of Physical 4-Vectors}

http://scirealm.org/SRQM.pdf

John B. Wilson
SciRealm.org
SciRealm@aol.com
Special Relativity $\rightarrow$ Quantum Mechanics

The SRQM Interpretation: Links

See also:
http://scirealm.org/SRQM.html (alt discussion)
http://scirealm.org/SRQM-RoadMap.html (main SRQM website)
http://scirealm.org/4Vectors.html (4-Vector study)
http://scirealm.org/SRQM-Tensors.html (Tensor & 4-Vector Calculator)
http://scirealm.org/SciCalculator.html (Complex-capable RPN Calculator)

or Google “SRQM”

http://scirealm.org/SRQM.pdf (this document: most current ver. at SciRealm.org)
The 4-Vector SRQM Interpretation

QM is derivable from SR!

The SRQM or [SR→QM] Interpretation of Quantum Mechanics
A Tensor Study of Physical 4-Vectors

quantum
relativity

SRQM = SciRealm QM?
A happy coincidence… :)