Using Special Relativity (SR) as a starting point, then noting a few empirical 4-Vector facts, e.g. classically measurable physical constants, one can instead *derive* the Principles that are normally considered to be the Axioms of Quantum Mechanics (QM).

Hence, [SR→QM]

Since many of the QM Axioms are rather obscure, this seems a far more logical and understandable paradigm than QM as a separate theory from SR, and sheds light on the origin, meaning, and physics of the QM Principles.

For instance, the properties of SR <Events>, encoded as components of 4-Vectors, i.e. [position,momentum], [time,energy], etc., can be “quantized by the Metric”, while SpaceTime & the GR Metric are not themselves “quantized”, in agreement with all known experiments and observations to-date.

The SRQM or [SR→QM] Interpretation of Quantum Mechanics
A Tensor Study of Physical 4-Vectors

or: Why General Relativity (GR) is *NOT* wrong
or: Don’t bet against Einstein ;)
or: QM, the easy way...

SRQM: A treatise of SR→QM by John B. Wilson (SciRealm@aol.com) version 2021-Nov-17 .1
4-Vectors = 4D (1,0)-Tensors are a fantastic language/tool for describing the physics of both relativistic and quantum phenomena. They easily show many interesting properties and relations of our Universe, and do so in a simple and concise mathematical way. Due to their tensorial nature, 4-Vectors are automatically 4D SpaceTime coordinate-frame invariant, their components obey relativistic covariance, and these 4-Vectors can be used to generate *ALL* of the physical Lorentz Scalar 4D (0,0)-Tensors and higher-rank Tensors of Special Relativity (SR).

Let me repeat: You can mathematically build *ALL* of the SR Lorentz Scalars and "larger" SR Tensors from empirically discovered SR 4-Vectors.

(Time·Space) SR 4-Vectors are likewise easily shown to be related to the standard physics 3D vectors { 3-vectors = 3D (1,0)-tensors } that are used in Newtonian Classical Mechanics (CM), Maxwellian Classical ElectroMagnetism (EM), and standard Quantum Mechanics (QM).

In addition, each and every physical SR 4-Vector also fundamentally connects a special-relativistically-related temporal scalar to a spatial 3-vector:

\[
\text{Temporal time (t)} \quad \& \quad \text{Spatial 3-position (r)} \rightarrow (x, y, z) \quad \text{as SR 4-Position } R = R^\mu = (ct, r)
\]

\[
\text{Temporal energy (E)} \quad \& \quad \text{Spatial 3-momentum (p)} \rightarrow (p_x, p_y, p_z) \quad \text{as SR 4-Momentum } P = P^\mu = (E/c, p)
\]

\[
\text{Temporal charge-density (p)} \quad \& \quad \text{Spatial 3-current-density (j)} \rightarrow (j_x, j_y, j_z) \quad \text{as SR 4-CurrentDensity } J = J^\mu = (\rho c, j)
\]

Why 4-Vectors and Tensors as opposed to some of the more abstract mathematical approaches to Quantum Mechanics?

Experiment is the ultimate arbiter of which theories actually correspond to reality/nature. If your quantum-logics and string-theories give no testable/measurable predictions, and if your invented/hypothetical/wished-for particles can't be detected, then they are basically useless for real, actual, empirical physics. The components of 4-Vectors are the physical properties of real particles that *CAN* actually be empirically detected/measured/tested, and Tensors are the well-known and well-established mathematical/physical objects which describe these concepts in an invariant, coordinate-independent way.

In this treatise, I will first **extensively** demonstrate how 4-Vectors are used in the context of Special Relativity (SR), and then show that their use in Relativistic Quantum Mechanics (RQM) is really not fundamentally different. Quantum Principles, without need of QM Axioms, then emerge in a natural and elegant way.

SR is a theory of Measurement, even in QM.

I also introduce the **SRQM (Physical) Diagramming Method**: a highly instructive, graphical charting-method, which visually shows how the SRQM 4-Vectors, Lorentz 4-Scalars, and higher rank 4-Tensors are all related to each other. This symbolic representation clarifies a lot of physics and is a great tool for teaching and understanding.
A Tensor Study of Physical 4-Vectors

Chapter Sections:
- Introduction
- Notation
- Physics Diagramming Method

Mostly SR
- 4-Vectors
- 4-Tensors
- Basis of Classical SR
- Lorentz Transforms

Mostly QM
- QM Connection
- Canonical QM Commutation
- Heisenberg Uncertainty
- QM → CM Correspondence
- SR → QM RoadMap

Credits

SR 4-Tensor
- \( T_{\mu
\nu}^{\sigma} \)
- \( T_{\mu}^{\nu} \) or \( T_{\nu}^{\mu} \)
- \( T_{\mu\nu} \)

SR 4-Vector
- \( V_{\mu} = (v_{0}, v_{\nu}) \)

SR 4-CoVector: One Form
- \( V_{\mu} = (v_{0}, -v_{\nu}) \)

SR 4-Scalar
- \( S \) or \( S_{0} \), Lorentz Scalar

SRQM: A treatise of SR → QM by John B. Wilson (SciRealm@aol.com)
SRQM = The [SR→QM] Interpretation of Quantum Mechanics, by John B. Wilson

In full, with each a subset of the former: [GR→SR→RQM→QM→(EM & CM)]
### SRQM

#### Some Physics: Mathematics

<table>
<thead>
<tr>
<th>Conventions &amp; Notation</th>
<th>SRQM Tensor Symbolic Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time-Space 4-Vector Name matches its spatial 3-vector component name</td>
<td>4-Vector (0 index): Ellipse</td>
</tr>
<tr>
<td>SR 4-Vector = (temporal 3-scalar, spatial 3-vector)</td>
<td>4-Vector (1 index): Rectangle</td>
</tr>
<tr>
<td>4-Vectors (4D) in <strong>bold</strong> UPPERCASE: ex. A</td>
<td>4-Tensor (2 index): Octagon</td>
</tr>
<tr>
<td>3-vectors (3D) in <strong>bold</strong> lowercase: ex. a : sometimes with vec=over-arrow ( \vec{a} )</td>
<td>Technically All Tensors</td>
</tr>
<tr>
<td>Temporal scalars (1D) non-bold, usually lowercase, 0\textsuperscript{th} component: ex. ( a^0, a_0 )</td>
<td></td>
</tr>
<tr>
<td>Individual non-vec scalar components non-bold: ex. ( A = (a^0, a^1, a^2, a^3) = (a^0, a) )</td>
<td>Temporal object (+): blue, Spatial object (-): red</td>
</tr>
<tr>
<td>Rest scalars (invariants) normal non-bold, denoted with naught (( a )): ex. ( a_0 )</td>
<td>Mixed Time-Space object (generic ( &lt;\text{event}&gt; )): purple</td>
</tr>
<tr>
<td>Tensor-index-notation normal non-bold: ex. ( A^\mu = (a^\mu, a) = (a^0, a^1, a^2, a^3) )</td>
<td>The mnemonic being blue and red mixed makes purple</td>
</tr>
<tr>
<td>4D Tensors use Greek indices: ex. ( { \mu, \nu, \sigma, \rho, \ldots } ): ex. 4-Position ( R^\mu = (r^\mu) )</td>
<td></td>
</tr>
<tr>
<td>3D tensors use Latin indices: ex. ( i, j, k, \ldots ) : ex. 3-position ( r^i = (r^i) ) = ( (r^1, r^2, r^3) )</td>
<td>Null: Photonic: Light-like object (0): white</td>
</tr>
<tr>
<td>Upper indices ( A^\mu = (a^\mu) = (a^\mu, a) ) : Lower indices ( B_\mu = (b_\mu) = (b_0, b) )</td>
<td>SpaceTime: I often write it as &quot;Time-Space&quot;</td>
</tr>
<tr>
<td>LightSpeed Factor (c) in temporal component to match dimensional units</td>
<td>just to match this ordering convention of 4-Vector (temporal, spatial) components</td>
</tr>
<tr>
<td>4-Vector: ( A = \vec{A}^\mu ) : ex. 4-Momentum ( P = P^\mu = (E/c, p) = (mc^2, \mathbf{p}) )</td>
<td></td>
</tr>
<tr>
<td>4-CoVector = OneForm: ( A = A_\mu ) : ex. 4D Gradient OneForm ( \partial_\mu = \partial_\mu = (\partial_0/c, \nabla) )</td>
<td></td>
</tr>
<tr>
<td>Null 4-Vector ( N^\mu = (\mathbf{n}<em>0, \mathbf{n}) ) = Lorentz Scalar Invariant ( N\cdot N = N^\mu N</em>\mu = 0 )</td>
<td></td>
</tr>
<tr>
<td>SR: Metric Convention: Particle-Physics, Temporal-0\textsuperscript{th}-Positive (+, -, -, -)</td>
<td></td>
</tr>
<tr>
<td>that is used herein: West-Coast, Time-Like, Mostly-Minuses</td>
<td></td>
</tr>
<tr>
<td>RQM &amp; QM are derivable from principles of SR</td>
<td></td>
</tr>
<tr>
<td>Let that sink in...</td>
<td></td>
</tr>
<tr>
<td>Quantum Mechanics is derivable from Special Relativity</td>
<td></td>
</tr>
</tbody>
</table>

#### Existing Quantum Rules

- 4-Position \( R = (ct, r) = (c\text{\ when, where}) \in \langle \text{What Event} \rangle \)
- 4-Position \( R = (ct, r) = (c\text{\ then, there}) \in \langle \text{That Event} \rangle \)
- 4-Origin \( R_0 = (0, 0) = (c\text{\ now, here}) \in \langle \text{This Event} \rangle \)

#### SRQM Interpretation of QM

- **SR 4-Tensor** (2,0)-Tensor \( T^{\mu\nu} \) or \( T_{\mu\nu} \) and \( T_{\mu\nu} \)
- **SR 4-Vector** \( V^\mu = (\nu, v) \) and \( V_\mu = (v, \nu) \)
- **SR 4-CoVector** OneForm: \( \mathbf{V}^\mu = (\nu, v) \) and \( \mathbf{V}_\mu = (v, \nu) \)
- **SR 4-Scalar** (0,0)-Tensor \( S_0 \) or \( S_0 \) Lorentz Scalar
- **3-Tensor** 3D (2,0)-Tensor \( T^k \) of \( T_k \)
- **Classical (Scalar)**
  - 3D Galilean Invariant
  - 3-vector not Lorentz Invariant
- **3-Scalar** 3D (0,0)-Tensor \( S \)
- **Trace** \( \text{Trace}[T^\mu] = \eta_\mu = T^\mu \) = \( T^\mu = T \)
- **V\cdot V = V^\mu V_\mu = (V^\mu) \cdot (V_\mu) = (V^\mu V_\mu) = (V^\lambda V_\lambda) \)

#### Idea

- Both are the SpaceTime-reversed situations of the other... equivalent under CPT Symmetry

#### Idea

- Normal Matter ↔ Anti Matter (Black Holes ↔ White Holes)

#### Idea

- Both are the SpaceTime-reversed situations of the other... equivalent under CPT Symmetry

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See also:
http://scirealm.org/SRQM.html (alt discussion)
http://scirealm.org/SRQM-RoadMap.html (main SRQM website)
http://scirealm.org/SRQM-Summary.pdf (SRQM Summary .pdf)
http://scirealm.org/4Vectors.html (4-Vector study)
http://scirealm.org/SRQM-Tensors.html (Tensor & 4-Vector Calculator)
http://scirealm.org/SciCalculator.html (Complex-capable RPN Calculator)

or Google “SRQM”

http://scirealm.org/SRQM.pdf (this .pdf document: most current version at SciRealm.org)
A Tensor Study of Physical 4-Vectors

SRQM Study: Physical / Mathematical Tensors

4D Tensor Types: 4-Scalar, 4-Vector, 4-Tensor

Component Types: Temporal, Spatial, Mixed

Matrix Format

SRQM Diagram Format

SR 4-Scalar
4D (0,0)-Tensor S often as $S_0$
Lorentz Scalar

SR 4-Vector
4D (1,0)-Tensor $V = \vec{V}$
uses a single upper index: bar, contravariant

SR 4-Tensor
4D (0,n)-Tensor $T^{\mu_1 \mu_2 \ldots \mu_n}$
uses multiple upper indices: (1,2) or (1,2,3)

SRQM Diagram Ellipse:
4-Vectors, 1 index = rank 1
4*1 = 4 corners in diagram

SRQM Diagram Octagon:
4-Tensors, 2 index = rank 2
4*2 = 8 corners in diagram

SRQM Diagram Rectangular:
4-Tensors, 3 index = rank 3
4*3 = 12 corners in diagram

SRQM Diagram Octahedron:
4-Tensors, 4 index = rank 4
4*4 = 16 corners in diagram

SR Mixed 4-Tensor
4D (1,1)-Tensor $T_{\mu \nu} = \eta_{\mu \nu} T^{\rho \sigma}$

SR Lowered 4-Tensor
4D (0,2)-Tensor $T_{\mu \sigma} = \nabla_{\mu} T^{\nu \sigma}$

Each 4D index = {0,1,2,3} → Tensor Dimension = 4

SRQM Diagram Format

SR 4-Scalar
4D (0,0)-Tensor $S$ often as $S_0$
Lorentz Scalar

SR 4-Vector $V^\mu$
an "arrow": magnitude and 1 direction

SR 4-Tensor $T^{\mu \nu}$
a "matrix or dyadic": magnitude and 2 directions

Tensor Property:
Rank = # of indices
{0 = Scalar}
{1 = Vector}
{2 = Dyadic: Matrix}
etc...

Dimension = # of values a tensor index can take
(SR Tensors = 4D)
(4-Scalar = 1D), (4-Vector = 2D), etc...

Trace[T] = $\sum_{\mu=0}^{3} T^{\mu \mu}$

4D SRQM Interpretation of Quantum Mechanics

SR: Minkowski Metric
$\partial[R] = \partial[R^\mu] = \eta^{\mu \nu} = V^{\mu \nu} + H^{\mu \nu} \rightarrow$

Diag[0,1,2,3] = Diag[1,1,1,1] = Diag[1,-1,-1,-1]
{in Cartesian form} "Particle Physics" Convention

$\{n_{\mu \nu} = 1/\eta^{\mu \nu} : n_{\mu \nu} = \delta_{\mu \nu} \}$

$Tr[T]^{\mu \nu} = T^{\mu \nu}$

4-Gradient $\partial$ in SpaceTime
$\partial = \partial[R] = \partial x$ / SpaceTime Dimension

SRQM Diagram Octagon:
4D Tensor Types: 4-Scalar, 4-Vector, 4-Tensor

Component Types: Temporal, Spatial, Mixed

Technically, all these objects are "SR 4-Tensors"; but we usually reserve the name "4-Tensor" for 4D objects with 2 (or more) indices, and use the "(m,n)-Tensor" notation to specify all the objects more precisely.
### Euclidean 3-vectors & SR:Minkowski 4-Vectors

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Regime</th>
<th>Metric</th>
<th>Transform:Invariance:Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D</td>
<td>Classical:QM</td>
<td>Euclidean</td>
<td>Galilean</td>
</tr>
<tr>
<td>4D</td>
<td>Relativistic:RQM</td>
<td>Minkowski</td>
<td>Lorentz Poincaré</td>
</tr>
</tbody>
</table>

**Classical & QM**

- **3D**
  - approximation only
  - valid regime: limited to $|v| < c$
  - only weak gravity

**Relativistic & RQM**

- **4D**
  - valid regime: all cases except strong gravity GR
  - still good "locally" even then: limited to valid regime: approximation only

---

**Galilean Transformations $G$**

- $\Lambda_i^j = \Lambda_A^j = \Lambda_i = \Lambda_{ij}^j = \Lambda^i_j = \Lambda^i_{kj} = E_i = \delta_i^j = \text{Diag}[1,1,1]$ spaatial-velocity-shifts, spatial-rotations, (P)arity, (T)ime-reversal
- $|\Lambda| = \pm 1$

**Lorentz Transformations $\Lambda$**

- $\Lambda_{\mu\nu}(\Lambda^{-1})_{\nu\lambda} = \Lambda_{\mu\lambda}$ spatial components only, 3x3 matrix
- $\Lambda_{\mu\nu} = \Lambda_{\nu\mu}$ temporal: mixed components, 4x4 matrix
- $\Lambda_{\mu\nu} = \text{Diag}[1,1,1,1]$ 4D Lorentz Transformation

**Lorentz Invariant**

- $\Lambda_{\mu\nu} = \text{Det}[\Lambda_{\mu\nu}] = \pm 1$

**4D SR:Minkowski Metric $\eta$**

- $\eta_{\mu\nu} = \text{Diag}[+1,-1,-1,-1]$ in Cartesian form
- $\eta_{\mu\nu} = \text{Diag}[1,1,1,1]$ 4D Lorentz Invariant
- $\eta_{\mu\nu} = \text{Diag}[1,1,1,1]$, others are zero

**4D SpaceTime**

- $\partial[R] = \partial[R'] = \text{Det}[\eta_{\mu\nu}] = \eta_{\mu\nu} = \eta_{\mu\nu} = \eta_{\mu\nu}$
- $\partial[R] = \eta_{\mu\nu} = \text{Diag}[+1,-1,-1,-1]$ in Cartesian form, "Particle Physics" Convention

---

**3D Classical:Euclidean Metric $E$**

- $\nabla[r] = \sqrt{|E_{ij}|} = E_{ij} = \text{Det}[E_{ij}] = \text{Det}[\Lambda_{ij}^j]$
- $\{E_{ik}\} = \{\delta_{ik}\}$ 3D Space

**4D SR-Vector $V_4$**

- $V_4 = (v_0, v_1, v_2, v_3)$ 4D SpaceTime
- $V_4 = (ct, r, \theta, \phi)$ 4D SR-Vector

---

**Tensor & Vector Interpretation**

- 3D & 4D vector internal components labeled with superscript index, not an exponent
- Only scalars outside of a vector will have exponents, such as in the Lorentz Scalar Product $A = A_{\mu} = (a_0, a_1, a_2, a_3)$
- 4-Vector Position $R = R^\mu = (ct, r, \theta, \phi)$
- Interval $ct = |R|$
Euclidean 3-Vectors & SR:Minkowski 4-Vectors

Dot Product, Lorentz Scalar Product

Einstein Summation Convention

In Classical Mechanics (CM), an ex. of the magnitude of a 3-vector is the length |Ar| of a 3-displacement Ar = (r₁, r₂, r₃).
Examine 3-position r, which is a 3-displacement Δr → r, with the base at the origin r₀ → 0 = (0,0,0) & r₁ → r = (x,y,z).

The 3D Dot Product: \( \text{r} \cdot \text{r} = r₀^2 + r₁^2 + r₂^2 = (x² + y² + z²) = (r²) \) is the Pythagorean Theorem. It uses the Euclidean Metric E, which in Cartesian form is equivalent to the 3D Kronecker delta \( δ_{\text{i} \text{j}} \). \( \text{Diag} \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \) = Identity I₀.
The 3D magnitude² is r₀r₁. The |magnitude| is \( \sqrt{r₀r₁} = \sqrt{r²} = |r| = r. \) 3D magnitudes are always positive (+) spatial.

t & |r| are scalar invariants only in Euclidean 3D space. However, our universe is locally Minkowski 4D, not 3D.

3-vector 1-time t = (t₀) = (t) 3-position r = r₁ = (rᵣ) = (r₀, r₁, r₂, r₃) = (x,y,z) Galilean Invariant r-r = r₀δᵢᵣ = (x²) + (y²) + (z²) = (r²)

The magnitude of an SR 4-Vector is very similar to the magnitude of a 3-vector, but there are some interesting differences. One uses the Lorentz Scalar Product, a 4D Dot Product, which includes time & space components, and is based on the SR:Minkowski Metric Tensor. "Particle Physics' sign-convention \{temporal,0,+,+\}=(0,0,0,0) of the Minkowski Metric gives \( η_{\mu \nu} \rightarrow \text{Diag}[1,-1,-1,-1] \) (Cartesian form), with the other entries zero. Note the 3D (spatial,1²+2²+3²) part is 4D negative(-).

Only the mixed \( (0,1) \)-Tensor form of Minkowski Metric \( η_{\mu \nu} \) is equivalent to the 4D Kronecker Delta \( δ_{\text{μ} \text{ν}} \). \( \text{Diag}[1,-1,-1,-1] \) = (1,4).

A²⁻¹⁻⁻⁻ = (A⁺⁺⁻⁻⁻⁻⁻⁻) = (a⁺⁺⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻˓→ \( \text{Diag}[1,-1,-1,-1,1] \) \{Cartesian form\}. \( \text{Particle Physics' Convention} \)

using Einstein Summation Convention which has upper- lower paired-indices summed over.

R⁴⁺⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻⁻˓→ \( \text{Diag}[1,-1,-1,-1,1] \) \{Cartesian form\}, with the other entries zero. Note the 3D (spatial,1²+2²+3²) part is 4D negative(-).

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The 4-Vector version has the Pythagorean elements in the spatial components, the temporal component is of opposite sign. This gives a "causality condition", with Space-Time intervals in the \(+ \rightarrow - \) SR: Minkowski Metric that can be: $$\Delta R = \Delta t \text{Time-like:Temporal}$$ $$\Delta R = \Delta r \text{Space-like:Spacial}$$ $$\Delta R = (\Delta t, \Delta r)$$ $$\Delta t \geq \text{causal = 1D temporally-ordered, spatially relative}$$ $$\Delta r \geq \text{causal & topological, maximum signal speed}$$ $$\text{Spatial}$$ and \( \text{causal} \) time {} Time (t) and length |r| are NOT 4D SR invariant scalars. They are 4-vector components. ProperTime (r) & Interval |r| are 4D SR invariants.

3D Classical: Euclidean Metric E

\[ \nabla[r] = \nabla[r] = E² = -H^r → \text{Kronecker delta} \delta_{r}^\text{r} \text{ Diag}[+1,+1,+1] = \text{Diag}[1,1,1,1] \]

3D Space

\[ \nabla r = \nabla r = (\delta_{r}^\text{r}) = \nabla r = (r²) \]

4D SR:Minkowski Metric \( \eta \)

\[ \eta_{\mu \nu} = \eta_{\mu \nu} = \nabla_{\mu} \nabla_{\nu} = \nabla_{\mu} \nabla_{\nu} + H_{\mu \nu} \approx \text{Diag}[1,-1,-1,-1,1] \]

\[ \eta_{\mu \nu} = \delta_{\mu \nu} = \text{Diag}[1,1,1,1] \]

Time-like: Temporal & Spatial

\[ \nabla r = \nabla r = (\delta_{r}^\text{r}) = \nabla r = (r²) \]

Interval ct = |r|

4D SpaceTime

\[ \delta R = \delta \eta_{\mu \nu} R^\nu = \delta R^\nu = 4 \]

\[ = (\partial_0 [ct] - \partial_0 [x] = -\partial_0 [y] + \partial_0 [z]) \]

\[ = (((\partial_0 [ct] - \partial_0 [x] = -\partial_0 [y] + \partial_0 [z]) \]

\[ = (1+1+1+1=4) \]

\[ \eta_{\mu \nu} = \delta_{\mu \nu} = \text{Diag}[1,1,1,1] \]

\[ \nabla r = \nabla r = (\delta_{r}^\text{r}) = \nabla r = (r²) \]

4D SpaceTime

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4D SpaceTime

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\[ = ((\partial_0 [ct] - \partial_0 [x] = -\partial_0 [y] + \partial_0 [z]) \]

\[ = (1+1+1+1=4) \]

\[ \eta_{\mu \nu} = \delta_{\mu \nu} = \text{Diag}[1,1,1,1] \]

\[ \nabla r = \nabla r = (\delta_{r}^\text{r}) = \nabla r = (r²) \]
SR: Minkowski Metric [η] Operations

Invarian Lorentz Scalar Product

Tensor Index {Raising, Lowering, Gymnastics}

Both GR and SR use a metric tensor (gμν) to describe measurements in 4D SpaceTime (Time:Space).

SR uses the “flat” Minkowski Metric g0μ → ημν → diag[1,-1,-1,-1] = diag[1,-1,-1,-1] in Cartesian form, which is the (curvature ~ 0 limit = low-mass limit) of the GR metric gμν.

SR is valid everywhere except extreme gravity, ex. near BH’s.

4-Vectors are tensorial entities of Minkowski SpaceTime which maintain covariance for inertial observers, meaning that they may have different relativistic components for different observers, but describe the same physical object.

(like viewing a sculpture from different angles – snapshot pictures “look” different, but it's actually the same object)

There are (4-CoVectors = One-Forms = 4D (0,1)-Tensors) which are dual to the (4-Tensors = 4D (1,0)-Tensors).

4-Vectors = 4D (1,0)-Tensors

A = Aμ = ημνAν = (aμ) = (a0,a1,a2,a3) → (aμ,aν,aα,aβ)

B = Bμ = ημνBν = (bμ) = (b0,b1,b2,b3) → (bμ,bν,bα,bβ)

4-CoVectors = 4D (0,1)-Tensors = 4D Dual-Tensors = 4D One-Forms

A = Aν = ηνμAμ = (aν) = (a0,a1,a2,a3) = (aν,aμ,aα,aβ)

B = Bν = ηνμBμ = (bν) = (b0,b1,b2,b3) = (bν,bμ,bα,bβ)

A·B = Σμν[aμbν ] = Σνμ[aνbν ] = (a0b0 - a1b1 - a2b2 - a3b3) = (a0,b0)

Lorentz Scalar Product using the Einstein Summation Convention where upper-lower-paired indices are summed over

Proof of invariance (using Tensor gymnastics and the properties of the Minkowski Metric η & Lorentz Transforms Λ):

A·B = ΛμνAμBν

Lorentz Scalar Product of 4-Vectors (A·B) → Lorentz Invariant Scalars = 4D (0,0)-Tensors.

They have the same measured value for all inertial observers, i.e. the same value in all 4D inertial reference-frames.

SR 4-Vector

(1,0)-Tensor V = Vμ = (V0,V1,V2,V3)

SR 4-Scalar

(0,0)-Tensor S or S0 Lorentz Scalar

SR 4-Scalar

(0,1)-Tensor Vμ = (V0,V1,V2,V3)

SR 4-Vector

(2,0)-Tensor Tμν = (T00,T01,T02,T03,T10,T11,T12,T13,T20,T21,T22,T23,T30,T31,T32,T33)

SR 4-Vector

(0,1)-Tensor V = Vμ = (V0,V1,V2,V3)

SR 4-Scalar

(0,0)-Tensor S or S0 Lorentz Scalar

Einstein & Lorentz “saw” the physics of SR,
Minkowski & Poincaré “saw” the mathematics of SR.

We are indebted to all of them for the simplicity, beauty, and power of how SR and 4-vectors work...
The **SRQM (Physics) Diagramming Method** shows the properties & relationships of various physical objects/tensors in a graphical way. This “flowchart” method aids understanding.

**Representation:** 4-Scalars by ellipses, 4-Vectors by rectangles, 4-Tensors by octagons. Physical/mathematical equations and descriptions inside each shape/object. Sometimes there will be additional clarifying descriptions around a shape/object.

**Relationships:** Lorentz Scalar Products or tensor compositions of different 4-Vectors are on simple lines(→) between related 4-Vectors. Lorentz Scalar Products of a single 4-Vector, or Invariants of Tensors, are next to that object and often highlighted in a different color.

**Flow:** Objects that are some function of a Lorentz 4-Scalar with another 4-Vector or 4-Tensor are on lines with arrows(→) indicating the direction of flow. (ex. multiplication by SR 4-Scalar, or 4-Scalar function of SR 4-Vector indicated by [..])

**Properties:** Some objects will also have a symbol representing its properties nearby, and sometimes there will be color highlighting within the object to emphasize temporal:spatial properties. I use blue=Temporal ⊹ red=Spatial → purple=mixed Time-Space.

**Alternate ways of writing 4-Vector expressions in physics:**

\((\mathbf{A} \cdot \mathbf{B})\) is a 4-Vector style, which uses vector-notation (ex. inner product "dot= \cdot" or exterior product "wedge=\wedge"), and is typically more compact, always using bold **UPPERCASE** to represent the 4-Vector, ex. \((\mathbf{A} \cdot \mathbf{B}) = (A^\mu \eta_{\mu\nu} B^\nu)\), and **bold lowercase** to represent 3-vectors, ex. \((\mathbf{a} \cdot \mathbf{b}) = (a^i b^i)\). Most 3-vector rules have analogues in 4-Vector mathematics.

\((\mathbf{A}^\mu \eta_{\mu\nu} B^\nu)\) is a Ricci Calculus style, which uses tensor-index-notation and is useful for more complicated expressions, especially to clarify those expressions involving tensors with more than one index, such as the Faraday EM Tensor \(F^\mu_\nu = (\partial^\mu A^\nu - \partial^\nu A^\mu) = (\partial^\mu \mathbf{A})\)

**Examples:**

**SR 4-Tensor**

\((2,0)-Tensor T^{0\nu}\) or \(T^{0\nu}\)

**SR 4-Scalar**

\((0,0)-Tensor S)\) or \(S\)

**SR 4-Vector**

\((1,0)-Tensor V^\mu = (v^\mu)\) or \(V\)

\((1,1)-Tensor T^\mu_\nu\) or \(T^\mu_\nu\)

**SR 4-CoVector:**

OneForm \(V\) or \(V\)

**Lorentz Factor = Relativistic Gamma**

\(\gamma = 1/\sqrt{1 - \beta^2}\) : \(\beta = u/c\)
### Special Relativity → Quantum Mechanics

#### SRQM Tensor Invariants

**Inherent 4D SpaceTime Properties**

One of the extremely important properties of Tensor Mathematics is the fact that there are numerous ways to generate **Tensor Invariants**. These Invariants lead to Physical Properties that are fundamental in our Universe, and are totally independent of any coordinate-systems used to measure them. Thus, they represent symmetry properties that are inherent in the fabric of 4D SpaceTime (Time-Space). See the Cauchy-Hamilton Theorem, esp. for the Anti-Symmetric Tensor Products.

---

<table>
<thead>
<tr>
<th><strong>Tensor Invariants</strong></th>
<th><strong>Description</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Trace Tensor Invariant:</td>
<td>$\text{Tr}[T^\mu] = \eta_{\mu\nu}T^{\nu} = T^\nu = T_{\mu} = \lambda_i [\text{EigenValues } \lambda_i]$ for $T^\nu$.</td>
</tr>
<tr>
<td>Determinant Tensor Invariant:</td>
<td>$\text{Det}[T^\mu] = \Pi_i [\text{EigenValues } \lambda_i]$ for $T^\nu$</td>
</tr>
<tr>
<td>Inner Product Tensor Invariant:</td>
<td>$\text{IP}[T^\mu] = T^{\mu}T_{\nu} = T_{\mu} : IP[T] = T^\mu T_\nu = T^\nu T_\mu = T \cdot T$</td>
</tr>
<tr>
<td>4-Divergence Tensor Invariant:</td>
<td>$\text{div}[T^\mu] = \partial_\mu T^\nu = \partial_\nu T^\mu = \partial_\nu T^\mu = S^\mu$</td>
</tr>
<tr>
<td>Lorentz Scalar Product Tensor Invariant:</td>
<td>$\text{LSP}[T^\mu,S^\nu] = T^{\mu}S_{\mu} = T_{\mu}S^\mu = T \cdot S = t^2s^0 - t^0s^2$</td>
</tr>
<tr>
<td>Phase Space Tensor Invariant:</td>
<td>$\text{PS}[T^\mu] = (dt^2 + t^0)^2 = (dt^1dt^2dt^3t^0)^2$ for $(T \cdot T) = T^\nu T_\nu = \text{constant}$</td>
</tr>
<tr>
<td>The Ratio of 4-Vector Magnitudes (Ratio of Rest Value 4-Scalars):</td>
<td>$T \cdot V / S \cdot V = (t^2 / s^2)$</td>
</tr>
<tr>
<td>Tensor EigenValues $\lambda_i = { \lambda_0, \lambda_1, \lambda_2, \lambda_3 }$: indexed as $0..3$, counted as $1..4$</td>
<td></td>
</tr>
</tbody>
</table>

#### The 4D Anti-Symmetric Tensor Products: index bracket 1] notation indicates anti-symmetric indices:

$S^\mu = \text{Trace} = \sum_i [\text{EigenValues } \lambda_i]$ for (1,1)-Tensors

$T^{\mu\nu}_{\rho\sigma} = \text{AntiSymm Bi-Product} \rightarrow \text{Inner Product}$

$T^{\mu\nu}_{\rho\sigma} T_{\sigma\tau} = \text{AntiSymm Tri-Product} \rightarrow \text{?Name}$

$S^\mu T^{\nu,\rho\sigma}_{\tau} = \text{AntiSymm Quad-Product} \rightarrow \text{4D Determinant} = \Pi_i [\text{EigenValues } \lambda_i]$ for 4D (1,1)-Tensors

These invariants are not all always independent, some invariants are functions of other invariants.

---

The **SRQM (Physics) Diagramming Method**

1. **4-Gradient** $\partial_\nu = \partial / \partial R_\mu$
2. **4-Position** $R^\mu = (ct, r) \in \langle \text{Event} \rangle$
3. **4-Scalar** $\text{R} = \langle \text{r} \rangle$
4. **4-Vector** $\text{U} = \gamma(c, u)$
5. **4-Divergence** $\partial_\rho R^\rho = \Lambda^\nu_\rho$ for 4D SpaceTime (Time-Space)

#### Lorentz Transformation

$L_\nu = \Lambda^\nu_\rho \text{U}^\rho$

$L_\nu \text{U}^\nu = \text{Vel}^4 = (c^4, 0)$

$L_0 = \gamma(c, 0)$ = Rest Mass $m$; $L_1/L_2/L_3 = \Lambda^\nu_\rho \text{U}^\rho = \text{Rest Energetics}$

#### Einstein's Equations

$E = mc^2 = \gamma m_0 c^2 = \gamma E_0$

### 4-Vector SRQM Interpretation of QM

- **SR 4-Tensor** $(2,0)$-Tensor $T^{\mu\nu}_{\rho\sigma}$
- **SR 4-Vector** $(1,0)$-Tensor $V^\nu = V / \text{c}$
- **SR 4-CoVector** OneForm $(0,1)$-Tensor $V_\mu = (\nu, -\nu)$
- **SR 4-Scalar** $(0,0)$-Tensor $S$ or $\text{SRQM}$

Lorentz Factor = $\gamma = 1 / \sqrt{1 - (c^2)} : \beta = u / c$
**SRQM Study: Physical/Mathematical Tensors**

### Tensor Types: 4-Scalar, 4-Vector, 4-Tensor

**SR 4-Vector**
- 4D (1,0)-Tensors
- Covariant: \( \mathbf{V} = (v^0, v^1, v^2, v^3) \)
- Contravariant: \( \mathbf{V} = (v_0, v_1, v_2, v_3) \)

**SR 4-Scalar**
- 0-index count Tensors
- \( \mathbf{S} \)

**SR 4-CoVector**
- 0-index count Tensors
- \( \mathbf{C} = (c_0, c_1, c_2, c_3) = (v_0^*, v_1^*, v_2^*, v_3^*) \)

**SR 4-Tensor**
- 1-index count Tensors
- \( \mathbf{T} = (T^0_0, T^0_1, ..., T^3_3) \)

**SR Mixed 4-Tensor**
- \( \omega = \epsilon_{\mu\nu\alpha\beta} T^\mu_\nu \)

**SR Lowered 4-Tensor**
- \( \mathbf{P} = (P^0_0, P^0_1, ..., P^3_3) \)

**SR Ricci Decomposition of Kronecker Tensor**
- \( \mathbf{R} = (R^0_0, R^0_1, ..., R^3_3) \)

### Physical 4-Tensors: Objects of Reality which have Invariant 4D SpaceTime (Time:Space) properties

<table>
<thead>
<tr>
<th>0-index count Tensors</th>
<th>1-index count Tensors</th>
<th>2-index count Tensors</th>
<th>3-index count Tensors</th>
</tr>
</thead>
<tbody>
<tr>
<td>EM Charge (Q) =</td>
<td>-speed of light (c) =</td>
<td>Proper Time</td>
<td>SpaceTime</td>
</tr>
<tr>
<td>Lorentz Scalar S</td>
<td>( \eta_{\mu\nu} = \delta_{\mu\nu} + \frac{x^\mu x^\nu}{c^2} )</td>
<td>Derivative</td>
<td>Dimension</td>
</tr>
<tr>
<td>Planck's Constant ( (h) )</td>
<td>d(^{\mu\nu} / \partial x^\mu )</td>
<td>V =</td>
<td>( \Lambda_{\mu\nu} \Lambda^\mu\nu = 4 )</td>
</tr>
<tr>
<td>Lorentz 4-Vector</td>
<td>SR Mixed 4-Tensor</td>
<td>Lorentz Tensor</td>
<td>Tr[( \eta_{\mu\nu} )] = ±1</td>
</tr>
<tr>
<td>Lorentz 4-CoVector</td>
<td>Lorentz 4-Scalar</td>
<td>Lorentz Tensor</td>
<td>Determinant ( \eta_{\mu\nu} )</td>
</tr>
<tr>
<td>Lorentz 4-Tensor</td>
<td>Lorentz 4-Scalar</td>
<td>Lorentz Tensor</td>
<td>Lorentz Tensor</td>
</tr>
<tr>
<td>Lorentz 4-CoVector</td>
<td>Lorentz 4-Scalar</td>
<td>Lorentz Tensor</td>
<td>Lorentz Tensor</td>
</tr>
</tbody>
</table>

### Physical Examples – Venn Diagram

- **SR 4-Vector**
- **SR 4-Scalar**
- **SR 4-CoVector**
- **SR 4-Tensor**

---

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http://scirealm.org/SRQM.pdf

---

Ricci Decomposition of Riemann Tensor

**Trace** of Riemann Tensor

\[ \text{Trace}[T_{\mu\nu}] = \eta_{\mu\nu} T_{\mu\nu} = T_{\mu\mu} = T \]

**V-V**

\[ V \cdot V = V^\mu \eta_{\mu\nu} V^\nu = (v^0)^2 + \sum_{i=1}^{3} (v^i)^2 \]

---

**Riemann Curvature Tensor**

\[ R_{\mu\nu\rho\sigma} = \partial_{\rho} \Gamma_{\mu\nu}^{\lambda} - \partial_{\nu} \Gamma_{\mu\rho}^{\lambda} + \Gamma_{\rho\sigma}^{\lambda} \Gamma_{\mu\nu}^{\lambda} - \Gamma_{\rho\nu}^{\lambda} \Gamma_{\mu\sigma}^{\lambda} \]

**Schouten Tensor**

\[ S_{\mu\nu} = \partial_{\mu} \Gamma_{\nu\rho}^{\lambda} - \partial_{\nu} \Gamma_{\mu\rho}^{\lambda} + \Gamma_{\rho\sigma}^{\lambda} \Gamma_{\mu\nu}^{\lambda} - \Gamma_{\rho\nu}^{\lambda} \Gamma_{\mu\sigma}^{\lambda} \]

**Bach Tensor**

\[ \mathbf{B}_{\mu\nu} = \partial_{\mu} \mathbf{R}_{\nu\rho\sigma} - \partial_{\nu} \mathbf{R}_{\mu\rho\sigma} + \mathbf{R}_{\rho\sigma} \mathbf{R}_{\mu\nu} - \mathbf{R}_{\rho\nu} \mathbf{R}_{\mu\sigma} \]

**Traceless part of Riemann Tensor**

\[ R_{\mu\nu} = \mathbf{R}_{\mu\nu}^{(\text{traceless part})} \]

---

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**SRQM Study:**

**SRQM 4-Vectors = 4D (1,0)-Tensors**

Measurable objects of Reality

---

**4-Vector = 4D (1,0)-Tensor**

- 4-Position \( \mathbf{R} = (ct, \mathbf{x}) = \mathbf{X} = X^\mu \) (alt notation)
- 4-Differential \( \mathbf{dR} = \mathbf{dR}^\mu = (c dt, \mathbf{dx}) \)
- 4-Displacement \( \Delta \mathbf{R} = \Delta \mathbf{R}^\mu = (c \Delta t, \Delta \mathbf{x}) \)
- 4-Velocity \( \mathbf{U} = (\gamma_c, \mathbf{u}) = (c, \gamma_c \mathbf{u}) \)
- UnitTimeSpatial \( \mathbf{T} = \mathbf{T}_\mu = (\gamma_0, \mathbf{0}) = (1, \mathbf{0}) \)
- UnitSpatial \( \mathbf{S} = \mathbf{S}^\mu = \gamma (\mathbf{0}, \mathbf{1}) = (\gamma_0, \mathbf{1}) \)
- UnitSR \( \mathbf{T} \perp \mathbf{S} = \gamma (\mathbf{T} \perp \mathbf{S}) \)
- UnitSpatialN \( \mathbf{N} = \mathbf{N}^\mu = \gamma (\mathbf{N}) = (\gamma N, \mathbf{N}) = (\gamma_0, \mathbf{N}) \)
- 4-Momentum \( \mathbf{P} = \mathbf{P}^\mu = \left( \frac{E}{c}, \mathbf{p} \right) = (\gamma m, \mathbf{p}) \)
- 4-SpaceDifferential \( \mathbf{dx} = \gamma (\mathbf{dx}) = (\gamma_0, \mathbf{dx}) \)
- 4-Space \( \mathbf{C} = \mathbf{C}^\mu = \gamma (\mathbf{C}) = (\gamma_0, \mathbf{C}) \)
- 4-Density \( \rho = \rho^\mu = \gamma \rho \)
- 4-Price \( \mathbf{P} = \mathbf{P}^\mu = \gamma \mathbf{P} \)
- 4-Force \( \mathbf{F} = \mathbf{F}^\mu = \gamma \mathbf{F} \)
- 4-Torque \( \mathbf{M} = \mathbf{M}^\mu = \gamma \mathbf{M} \)
- 4-Vectors can be constructed from the Tensor Products of 4-Vectors. Technically, 4-Tensors

---

**SI Dimensional Units**

<table>
<thead>
<tr>
<th>Temporal : Spatial ] components</th>
<th>(Relativistic Gamma ( \gamma = 1/\sqrt{1 - \mathbf{\beta} \cdot \mathbf{\beta}} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time ( t ) : Space/length/extent ( r )</td>
<td>TimeDifferential ( dt ) : SpaceDifferential ( dr ) <strong>Infinitesimal</strong></td>
</tr>
<tr>
<td>TemporalDisplacement ( \Delta t ) : SpatialDisplacement ( \Delta r ) <strong>Finite</strong></td>
<td></td>
</tr>
<tr>
<td>Temporal &quot;velocity&quot; factor ( \gamma_c ) : Spatial &quot;velocity&quot; factor ( \mathbf{u} ), Spatial 3-velocity ( \mathbf{u} )</td>
<td></td>
</tr>
<tr>
<td>Temporal &quot;acceleration&quot; factor ( \gamma_0 ) : Spatial normalized &quot;velocity&quot; factor ( \mathbf{\beta} ), Spatial 3-beta ( \mathbf{\beta} )</td>
<td></td>
</tr>
<tr>
<td>Temporal &quot;force&quot; factor ( \gamma_0 \mathbf{\beta} ) : Spatial normalized &quot;force&quot; factor ( \gamma_0 \mathbf{\beta} \cdot \mathbf{\mathbf{n}} ), Spatial 3-beta ( \mathbf{\beta} \cdot \mathbf{\mathbf{n}} )</td>
<td></td>
</tr>
<tr>
<td>Temporal factor ( \gamma_0 \mathbf{\beta} \cdot \mathbf{\mathbf{n}} ) : Spatial factor ( \gamma_0 \mathbf{\beta} \cdot \mathbf{\mathbf{n}} ) with ( \mathbf{N} = N = 0 ) : (\mathbf{n} : unit-direction 3-vector</td>
<td></td>
</tr>
<tr>
<td>mass ( m ) : energy ( E ) : 3-momentum ( \mathbf{p} = m \mathbf{u} ) : { ( E = mc^2 = \gamma m c^2 = \gamma E_0 ) &amp; ( \mathbf{p} = m \mathbf{u} )</td>
<td></td>
</tr>
<tr>
<td>total-energy ( E_0 ) : Hamiltonian ( H ) : 3-total-momentum ( \mathbf{p} )</td>
<td></td>
</tr>
<tr>
<td>mass density ( \rho ) : 3-mass-flux : 3-momentum-density ( \mathbf{g} )</td>
<td></td>
</tr>
</tbody>
</table>

---

**Lorentz Factor:**

Relativistic Gamma \( \gamma = 1/\sqrt{1 - \mathbf{\beta} \cdot \mathbf{\beta}} \) : \( \mathbf{\beta} = \mathbf{u}/c \)

\[ \gamma' = \frac{dy}{dt} = \gamma^2 \mathbf{\beta} \cdot \mathbf{a} = \gamma^2 \mathbf{u} \cdot a/c^2 \]

---

**Notational Clash Warnings:**

- 4-Acceleration A vs. 4-VectorPotential A
- 4-Energy/Momentum EnergyQuotient vs. 4-PotentialMomentum
- 4-UnitSpatialS vs. 4-Spin S
- 4-SurfaceNormal N vs 4-NumberDensity Flux

---

**SRQM 4-Tensor**

- (2,0)-Tensor \( T_{\mu
\nu} \)
- (1,1)-Tensor \( T^\mathbf{\mu} \)
- (0,2)-Tensor \( T_{\mathbf{\nu}} \)

**SRQM 4-Vector**

- (1,0)-Vector \( V = (\mathbf{v}, V) \)
- OneForm \( 0,1 \)-Tensor \( V_{\mu} = (v_{\mu}, -v) \)

**SRQM 4-Scalar**

- (0,0)-Tensor or \( S_{\mathbf{\mu}} \)

---

4-Tensors can be constructed from the Tensor Products of 4-Vectors. Technically, 4-Tensors refer to all SR objects (4-Scalars, 4-Vectors, etc.) but typically reserve the name 4-Tensor for SR Tensors of 2 or more indices. Use #D (m,n)-Tensor notation to specify types more precisely.
**SRQM Study:**

SRQM 4-Tensors = 4D (2,0)-Tensors {or higher index #}

*Made from 4-Vector relations*

4-Tensors can be constructed from the Tensor Products of 4-Vectors. Technically, 4-Tensors refer to all SR objects (4-Scalars, 4-Vectors, etc), but typically reserve the name 4-Tensor for SR Tensors of 2 or more indices. Use #D (m,n)-Tensor notation to specify types more precisely.
SRQM Study:

4-Scalars = 4D (0,0)-Tensors = Lorentz Scalars

= 4D SR Invariants ↔ Physical Constants

*Made from 4-Vector relations*

4-Scalar = 4D (0,0)-Tensor (generally composed of 4-Vector combinations using LSP)

Faraday EM Determinant Invariant 2(b⋅b-e⋅e/c²)²

Faraday EM InnerProduct Invariant 2(b⋅b·e·e/c²)²

SI Dimensional Units

4-Scalar = 4D (0,0)-Tensor = SR Invariant

RestTime:ProperTime (t = τ) [s]

RestTime:ProperTime Differential (dt = dτ) [s]

ProperTimeDerivative (d/dτ) [1/s]

MaxSignalSpeed = LightSpeed (c) [m/s]

RestMass (m₀ = E₀/c²) = hω₀/c² [kg]

RestEnergy (E₀ = m₀c²) = hω₀ [J = kg·m²/s²]

RestThermodynamicBeta (β₀ = 1/k₀T₀) [1/(1/s²·kg·m²)]

RestAngFrequency (ω₀ = E₀/c) [rad/s]

RestChargeDensity (ρ₀) [C/m³]

RestScalarePotential (φ₀) [V = J/C = kg·m²/C·s²]

restNumberDensity (n₀) [kg/m³]

SR Phase (Φₚh = Sₚh(φ/h))

SR Action (S₀ = hΦ₀) [J·s]

Planck Constant (h = 6.626×10⁻³⁴J·s)

Dirac:Planck-Reduced Constant (h = h/2π)

SpaceTime Dimension (4) → [4D]

Electric Constant (ε₀) [C/m³]

Magnetic Constant (μ₀) [H/m]

EM Charge (q₀) [C]

EM Charge (Q) *alt method*

Particle # (N) [#]

Rest Volume (V₀) [m³]

Rest(MCRF) EnergyDensity (ρ₀ = n₀E₀) [J/m³]

Rest(MCRF) Pressure (ρ₀) [J/m³]

Faraday EM InnerProduct Invariant 2(b⋅b·e·e/c²)

Faraday EM Determinant Invariant (b⋅b·e·e/c²)

4-Vector SRQM Interpretation of QM

Lorentz Scalars = 4D (0,0)-Tensors can be constructed from the Lorentz Scalar Products (LSP) of 4-Vectors: ex. “multiplication” (A⋅B) & “division” (A/V)/(B/V). Both of these give invariant scalar products. A “mediating” 4-Vector V is required for division, since vectors by themselves don’t divide, they must be made into scalars 1st.
SRQM Study: Physical 4-Vectors

Some SR 4-Vectors and Symbols

- **4-Gradient**
  \[ \nabla = \frac{\partial}{\partial x_i} = (\frac{\partial}{\partial x_0}, \frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}) \]

- **4-Momentum**
  \[ p = (mc, \mathbf{p}) = (m\gamma(c, u), \mathbf{p}) = m\gamma(c, u) \]

- **4-Wavevector**
  \[ k = (\omega/c, \mathbf{k}) = (\omega /c, \mathbf{k}) \]

- **4-ChargeFlux**
  \[ j = (\mu, \mathbf{j}) = (\rho, \mathbf{c}, \mathbf{u}) = \rho \mathbf{c} + \mathbf{u} \]

- **4-ThermalVector**
  \[ \Theta = (\Theta^\mu) = (\Theta^0, \mathbf{\Theta}) = (\Theta_0, \mathbf{\Theta}) T = (1/k_B T)(c, u) = (1/k_B T)(c, u) \]

4-Vector SRQM Interpretation of QM

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http://scirealm.org/SRQM.pdf
SRQM Study:

Primary/Primitive/Elemental 4-Vectors: 4-UnitTemporal \( T \) & 4-UnitSpatial \( S \)

- **4-UnitTemporal**, \( \text{[dimensionless=1]} \)
  - Magnitude\(^2\) = \((+1)\)
  - "Magnitude" = \((+1)\)
  - |Magnitude| = \((1)\)

- **4-UnitSpatial**, \( \text{[dimensionless]} \)
  - Magnitude\(^2\) = \((0)\)
  - "Magnitude" = \((0)\)
  - |Magnitude| = \((0)\)

### 4-UnitTemporal, \( T \)
- \( T = T^\mu = \gamma(1, \beta) \)
- \( \gamma = \gamma(1, \beta) = \gamma(1, \frac{u}{c}) = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \)
- \( T \cdot T = \gamma(1, \beta) \cdot \gamma(1, \beta) = \gamma^2(1 - \beta \cdot \beta) = 1 \)
- LightSpeed Invariant \( c \) = \( \frac{1}{\sqrt{1 - \beta^2}} \)

### 4-UnitSpatial, \( S \)
- \( S = S^\mu = \gamma(\beta \cdot \nabla, \hat{n}) \)
- (depends on direction \( \hat{n} \))
- \( S \cdot S = \gamma(\beta \cdot \nabla, \hat{n}) \cdot \gamma(\beta \cdot \nabla, \hat{n}) = \gamma^2(\beta \cdot \nabla \cdot \beta \cdot \nabla - \beta \cdot \hat{n}) = -\gamma^2(1 - (\beta \cdot \hat{n})^2) = -1 \)

### 4-Null, \( N \)
- \( N = a_{\text{zero}} N = \{a, a\} = a(1, \hat{n}) = a \gamma_{a_{\text{zero}}}(1, \hat{n}) = a^* \gamma_{a_{\text{zero}}}(1, \hat{n}) = 0 \leftrightarrow (N \perp \nabla) \)
- \( 4-\text{Scalar} (a) \) Invariant \( \{a_{\text{zero}}\} \)
- Magnitude\(^2\) = \((0)\)
- "Magnitude" = \((0)\)
- |Magnitude| = \((0)\)

### 4-Vector SRQM Interpretation of QM

- **Relativistic Gamma**
  - \( \gamma = \frac{1}{\sqrt{1 - \beta^2}} = \frac{1}{\sqrt{1 - \beta_0^2}} \)

### 4-Vector Spins
- \( S_{\text{spin}} = S_{\text{spin}}^\mu = \gamma_0 S = (s^0, s) = (\beta \cdot s, s) = -s^0 (s) = -(s_0) \)
- \( 4-\text{Scalar} (s_{\text{spin}}) \) Invariant \( \{s_{\text{spin}}\} \)
- Magnitude\(^2\) = \((s_{\text{spin}})\)²
- "Magnitude" = \((s_{\text{spin}})\)
- |Magnitude| = \((s_{\text{spin}})\)

### 4-Nulls
- \( N = a_{\text{zero}} N = \{a, a\} = a(1, \hat{n}) = \gamma_{a_{\text{zero}}}(1, \hat{n}) = a^* \gamma_{a_{\text{zero}}}(1, \hat{n}) = 0 \leftrightarrow (N \perp \nabla) \)
- \( 4-\text{Scalar} (a) \) Invariant \( \{a_{\text{zero}}\} \)
- Magnitude\(^2\) = \((a)\)²
- "Magnitude" = \((a)\)
- |Magnitude| = \((a)\)

### Lorentz Scalar
- \( \gamma = \frac{1}{\sqrt{1 - \beta^2}} = \frac{1}{\sqrt{1 - \beta_0^2}} \)
- \( \beta = u/c \)
- \( \gamma \) is Lorentzian scalar, \( \beta \) is the component of a vector \( \beta \) along the \( \hat{n} \)-direction, \( (|\hat{n}|=1) \)
- In the RestFrame \( (\beta = 0) \) of a massive particle \( (m_b > 0) \):
  - 4-Velocity appears totally Temporal \( U_\gamma = c(1, 0) \)
  - 4-Spin appears totally Spatial \( S_\beta = s_\beta(0, \hat{n}) \)
- \( \{ 4-\text{UnitTemporal} T : 4-\text{UnitSpatial} S : 4-\text{UnitNull} N \} \) are dimensionless, which allows them to make dimensional 4-Vectors via multiplication by dimensional 4-Scalars, as shown here.
SRQM Study:
Primary/Primitive/Elemental 4-Vectors:
4-"Unit" Null $\bar{N}$

4-Vector SRQM Interpretation of QM

SR Light Cone

SR 4-Tensor
$(2,0)$-Tensor $\bar{T}^{\mu\nu}$
$(1,0)$-Tensor $V^\mu = V = (\gamma v)^\mu$
$(0,2)$-Tensor $\bar{T}_{\mu\nu}$

SR 4-Vector
$(1,0)$-Tensor $V^\mu = V = (\gamma v)^\mu$
$(0,1)$-Tensor $V_\nu = (\gamma v)_\nu$ Lorentz Scalar

SR 4-Scalar
$(0,0)$-Tensor $S$ or $S_0$ Lorentz Scalar

Primary/Primitive/Elemental 4-Vectors:

- $(0,0)$-Tensor $S$ or $S_0$
- $(1,0)$-Tensor $V^\mu = V = (\gamma v)^\mu$
- $(0,1)$-Tensor $V_\nu = (\gamma v)_\nu$
- $(2,0)$-Tensor $\bar{T}^{\mu\nu}$
- $(1,0)$-Tensor $V^\mu = V = (\gamma v)^\mu$
- $(0,2)$-Tensor $\bar{T}_{\mu\nu}$

4-UnitSpatial $S = S^\mu = \gamma_{\beta\hat{\nu}}(\beta \cdot \hat{n}, \hat{n})$
(doesn’t depend on direction $\hat{n}$)

3 independent components
[dimensionless = 1]

4-UnitTemporal $\bar{T} = T^\mu = \gamma(1,\hat{n})$
$= \gamma(1, u/c) = U/c$

T $\cdot$ T = +1

0 independent components – because universal constant $[m/s]$ components

- 3 independent components – because limiting process
- Only 3D direction remains
- Components $[dimensionless = 1]$

N $\cdot$ N = 0/0 = undefined

SR is one physical case which has $(+^\infty 0 = finite)$ $p = \gamma \cdot \! p_{\text{zero}}$
see Dirac Delta Function

Trace[$T_{\mu\nu}$] $= \eta_{\mu\nu}T^{\mu\nu} = T^{\mu\nu} = T$
$V \cdot V = V^\mu \eta_{\mu\nu}V^\nu = (v^\mu)^2 - V\cdot V = (v^\mu)^2$

= Lorentz Scalar Invariant
SR Lorentz Factor = Relativistic Gamma
\[ \gamma = 1/\sqrt{1 - \beta^2} = dt/d\tau = \partial \tau / \partial t \]

The Lorentz Gamma Factor (\( \gamma = 1/\sqrt{1 - \beta^2} \)) defines how relativistic time (t) is related to proper rest time (\( \tau \)).

Many relativistic relations use \( \gamma \): P = (E/c, p) = mγU, Uγ(c, u) = m(c, u)

**Time-Space** factors to Einstein’s \( E = mc^2 \) & \( p = mc \).

The SR Lorentz factor also plays a role in the total differential vs. the partial, and in defining the Hamiltonian (H) and Lagrangian (L) Energies.

The 4-Gradient
\[ \partial = \partial R = \partial x = \partial y = \partial z = (\partial / \partial c, -\nabla) \]
\[ = (\partial / \partial c, \nabla) = (\partial / \partial c, -\nabla) \]

Invarient Proper Time Derivative
\[ U\cdot \partial = \gamma(c,u)(\partial / \partial c, -\nabla) = \gamma(\partial + u\cdot\nabla) \]

Rest Hamiltonian
\[ P_T = (H/c, p_T) = \gamma(c,u) \]
\[ = \gamma(H - p_T \cdot u) = H_0 = -L_0 \]

4-Velocity
\[ \gamma(c,u) : \text{most known 4-Vector with } \gamma \text{ exposed} \]

SR 4-Vector
\[ \{ H \} \rightarrow \{ L \} = (p_T, u) \]

SR 4-Vector: OneForm
\[ T_{\mu\nu} = \sqrt{-\gamma} = 1/\gamma \]

Lorentz Factor (\( \gamma \))
\[ \gamma = 1/\sqrt{1 - \beta^2} \]
\[ (\gamma - 1/\gamma) = (\gamma \beta \cdot \beta) \]
\[ (\gamma - 1/\gamma) (P_T \cdot U) = (\gamma \beta \cdot \beta) (P_T \cdot U) \]
\[ (\gamma P_T \cdot U) + (P_T \cdot U) / \gamma = (p_T \cdot u) \]

The Hamiltonian: Lagrangian Connection

\[ \gamma \]
SRQM: Some Basic 4-Vectors

4-Position, 4-Velocity, & their 4-Gradients

SR SpaceTime Calculus & Invariants

4-Vector SRQM Interpretation of QM

SRQM John B. Wilson SciRealm.org


A Tensor Study of Physical 4-Vectors
SRQM: Some SR Calculus

ProperTime Derivative \((U \cdot \partial_R) = d[..]/d\tau = \gamma d[..]/dt\)

ProperTime Integral \((R \cdot \partial_U) = \int[..]d\tau = \int[..](1/\gamma)dt\)

\(\text{Writing the ProperTime Derivative } = (U \cdot \partial_R) = d[..]/d\tau \text{ and the ProperTime Integral } = (R \cdot \partial_U) = \int[..]d\tau\)

In theory, an integration constant can be added to any of these inputs.

Lorentz Factor:
Relativistic Gamma \(\gamma = 1/\sqrt{1 - \beta \cdot \beta} \quad \beta = u/c\)
\(\gamma' = d\tau/dt = \gamma' \beta \cdot a/c = \gamma' u \cdot a/c^2\)

\(\text{SR 4-Tensor } (2,0)-\text{Tensor } T^{\mu}_{\nu}\)
\(\text{SR 4-Vector } (1,0)-\text{Tensor } V^\nu = (\gamma \cdot v)^\nu\)
\(\text{SR 4-CoVector:OneForm } (0,1)-\text{Tensor } V_\nu = (\gamma \cdot v)_\nu\)
\(\text{SR 4-Scalar } (0,0)-\text{Tensor } S_0 \text{ or } S_0 \text{ Lorentz Scalar}\)

\(\text{4-Vector SRQM Interpretation of QM}\)

\(\text{http://scirealm.org/SRQM.pdf}\)
SRQM Study: Physical 4-Vectors
Some 4-Position $R^\mu$ Relations
SRQM Study: Physical 4-Vectors

Some 4-Velocity \( U^\mu \) Relations

4-Gradient
\[
\vartheta = \partial = \partial_x = \partial_t = \partial_x + \partial_t \cdot \mathbf{v} = \partial \mathbf{R}_u
\]

Lorentz Scalar Invariant
\[
\partial \mathbf{R}_u = \gamma (\partial \mathbf{v} + \mathbf{u} \cdot \nabla) - \gamma (\partial \mathbf{v} + \mathbf{u} \cdot \nabla)
\]

Relativistic Gamma \( \gamma \) combines with Invariant Rest-Scalars to create Relativistic components. This allows the 4-Velocity to create many other standard SR 4-Vectors. For example, with 4-Momentum \( P = (mc,p) = (mc,\mathbf{u}) = m(c,\mathbf{u}) = m_\mathbf{U} \), the relativistic mass \( m \) is \( m = \gamma m_0 \). The invariant rest mass \( m_0 \) is invariant under spacetime transformations.

4-Acceleration
\[
\mathbf{A} = \mathbf{a} = \gamma (c \mathbf{v}, \gamma \mathbf{U} + \gamma \mathbf{a}) = d\mathbf{U}/d\tau = d^2\mathbf{R}/d\tau^2
\]

4-Position
\[
\mathbf{R} = \mathbf{R} = (ct, \mathbf{x}, \mathbf{y}, \mathbf{z}) \quad \text{alt. notation} \quad X^\mu = \mathbf{x}^\mu
\]

4-Position
\[
\gamma d/dt \quad \mathbf{U} \cdot \partial = \gamma (c, \mathbf{u}) \quad \mathbf{U}^\mu = \gamma (c, \mathbf{u}) = \mathbf{c}^\mu \partial t - \mathbf{d}_{x,y,z}
\]

4-Velocity
\[
\mathbf{U} = \mathbf{U} = \gamma (c, \mathbf{u}) = \mathbf{c}^\mu \partial t - \mathbf{d}_{x,y,z}
\]

4-Velocity
\[
\mathbf{U} = \mathbf{U} = \gamma (c, \mathbf{u}) = \mathbf{c}^\mu \partial t - \mathbf{d}_{x,y,z}
\]

4-Velocity
\[
\mathbf{U} = \mathbf{U} = \gamma (c, \mathbf{u}) = \mathbf{c}^\mu \partial t - \mathbf{d}_{x,y,z}
\]

4-Acceleration
\[
\mathbf{A} = \mathbf{a} = \gamma (c \mathbf{v}, \gamma \mathbf{U} + \gamma \mathbf{a}) = d\mathbf{U}/d\tau = d^2\mathbf{R}/d\tau^2
\]

4-Gradient
\[
\vartheta = \partial = \partial_x = \partial_t = \partial_x + \partial_t \cdot \mathbf{v} = \partial \mathbf{R}_u
\]

Lorentz Scalar Invariant
\[
\partial \mathbf{R}_u = \gamma (\partial \mathbf{v} + \mathbf{u} \cdot \nabla) - \gamma (\partial \mathbf{v} + \mathbf{u} \cdot \nabla)
\]

Relativistic Gamma \( \gamma \) combines with Invariant Rest-Scalars to create Relativistic components. This allows the 4-Velocity to create many other standard SR 4-Vectors. For example, with 4-Momentum \( P = (mc,p) = (mc,\mathbf{u}) = m(c,\mathbf{u}) = m_\mathbf{U} \), the relativistic mass \( m \) is \( m = \gamma m_0 \). The invariant rest mass \( m_0 \) is invariant under spacetime transformations.

4-Acceleration
\[
\mathbf{A} = \mathbf{a} = \gamma (c \mathbf{v}, \gamma \mathbf{U} + \gamma \mathbf{a}) = d\mathbf{U}/d\tau = d^2\mathbf{R}/d\tau^2
\]

4-Position
\[
\mathbf{R} = \mathbf{R} = (ct, \mathbf{x}, \mathbf{y}, \mathbf{z}) \quad \text{alt. notation} \quad X^\mu = \mathbf{x}^\mu
\]

4-Position
\[
\gamma d/dt \quad \mathbf{U} \cdot \partial = \gamma (c, \mathbf{u}) \quad \mathbf{U}^\mu = \gamma (c, \mathbf{u}) = \mathbf{c}^\mu \partial t - \mathbf{d}_{x,y,z}
\]

4-Velocity
\[
\mathbf{U} = \mathbf{U} = \gamma (c, \mathbf{u}) = \mathbf{c}^\mu \partial t - \mathbf{d}_{x,y,z}
\]

4-Velocity
\[
\mathbf{U} = \mathbf{U} = \gamma (c, \mathbf{u}) = \mathbf{c}^\mu \partial t - \mathbf{d}_{x,y,z}
\]

4-Acceleration
\[
\mathbf{A} = \mathbf{a} = \gamma (c \mathbf{v}, \gamma \mathbf{U} + \gamma \mathbf{a}) = d\mathbf{U}/d\tau = d^2\mathbf{R}/d\tau^2
\]

SR 4-Vector
\[
\mathbf{V} = \mathbf{V} = (\mathbf{v}^\mu) = (\gamma \mathbf{v}^\mu) = (\gamma \mathbf{v}^\mu)
\]

SR 4-Vector
\[
\mathbf{V} = \mathbf{V} = (\mathbf{v}^\mu) = (\gamma \mathbf{v}^\mu) = (\gamma \mathbf{v}^\mu)
\]

SR 4-Vector
\[
\mathbf{V} = \mathbf{V} = (\mathbf{v}^\mu) = (\gamma \mathbf{v}^\mu) = (\gamma \mathbf{v}^\mu)
\]

SR 4-Vector
\[
\mathbf{V} = \mathbf{V} = (\mathbf{v}^\mu) = (\gamma \mathbf{v}^\mu) = (\gamma \mathbf{v}^\mu)
\]
SRQM Study: Physical 4-Vectors
Some 4-Acceleration Aμ Relations

4-Velocity
U = γ(c, u) = γ(c² - u²)³/² = dR/dτ = cT

Invariant ProperTime
R-U = c²τ from d/dτ[R-R] = d/dτ(c²τ) = 2c²τ
= (R-U) + (U-R) = 2(R-U)

Invariant LightSpeed
U-U = γ(c²-u²) = c²

ProperTime
γd/dτ[...]
= γd/dτ[...]
= γd/dτ[...]
= γd/dτ[...]
= γd/dτ[...]

A Tensor Study of Physical 4-Vectors

A Tensor, A = γ(c²-u²)³/²
= γ²[(γ(u-a)c)/c² - γ²(u-a)u/c²+a]
= γ²[(u-a)c/(u+x(a))c²+a]
=dU/dτ = dR/dτ² : {γdγ/γdt}

4-Acceleration
A = Aμγ = γ(c² - u²)³/²
= γ²[(γ(u-a)c)/c² - γ²(u-a)u/c²+a]
= γ²[(u-a)c/(u+x(a))c²+a]
=dU/dτ = dR/dτ² : {γdγ/γdt}

Invarient Acceleration A-A
= A₀A₀ = (0,a₀)γ(0,a₀) = -a₀ = |a₀| = |a₀|²
= γ²[(c²-u²)³/² - γ²(u-a)u/c²+a]}
= γ²[(u-a)c/(u+x(a))c²+a]}
= γ²[(u-a)c/(u+x(a))c²+a]}
=dU/dτ = dR/dτ² : {γdγ/γdt}

The tedious algebra
Invarient Acceleration A-A
= A₀A₀ = (0,a₀)γ(0,a₀) = -a₀ = |a₀| = |a₀|²
= γ²[(c²-u²)³/² - γ²(u-a)u/c²+a]}
= γ²[(u-a)c/(u+x(a))c²+a]}
= γ²[(u-a)c/(u+x(a))c²+a]}
=dU/dτ = dR/dτ² : {γdγ/γdt}

SR 4-Tensor
(2,0)-Tensor T'ix
(1,1)-Tensor T'v, or T'v, or T'v

SR 4-Vector
(1,0)-Tensor V = (v₀, v'v)

SR 4-Scalar
(0,0)-Tensor 5 or 5, or Lorentz Scalar

MCRF
Momentarily Co-Moving Reference Frame
u → 0
γ → 1 : γ → 0

V-V = V·V = V² = (v₀·v)² = v²

Trace[T'] = ηₜₗ[T'] = T'ₜₗ = T

Lorentz Scalar Invariant

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http://scirealm.org/SRQM.pdf
SRQM Study: Physical 4-Vectors

Some 4-Gradient $\partial^\mu$ Relations

The relations below are for the 4-(Position)Gradient $\partial_R$, 4-Gradients wrt. other 4-Vector variables exist also... ex. 4-WaveGradient $\partial_k$

4-Gradient: 4-(Position)Gradient
\[
\partial = \partial_x = \partial_y = \partial_z = \frac{\partial}{\partial/c} = \nabla \cdot \mathbf{V}
\]

Minkowski
\[
\nabla^\mu = \partial^\mu[\mathbf{V}] = \mathbf{V}^\mu = \mathbf{V} \cdot \mathbf{N}
\]

Lorentz
\[
\nabla^\mu = \partial^\mu[\mathbf{V}] = \mathbf{V}^\mu = \mathbf{V} \cdot \mathbf{N}
\]

Transform
\[
\nabla^\mu = \partial^\mu[\mathbf{V}] = \mathbf{V}^\mu = \mathbf{V} \cdot \mathbf{N}
\]

SpaceTime
\[
\nabla^\mu = \partial^\mu[\mathbf{V}] = \mathbf{V}^\mu = \mathbf{V} \cdot \mathbf{N}
\]

SRQM Non-Zero Commutation
\[
\nabla^\mu = \partial^\mu[\mathbf{V}] = \mathbf{V}^\mu = \mathbf{V} \cdot \mathbf{N}
\]

Total Derivative Chain Rule
\[
\nabla^\mu = \partial^\mu[\mathbf{V}] = \mathbf{V}^\mu = \mathbf{V} \cdot \mathbf{N}
\]

4-Momentum
\[
P = p^\nu = (mc, \mathbf{p}) = m_0 \mathbf{u} = -\nabla[S] = (E/c, \mathbf{p}) = E/c^2 \mathbf{u} = (E/c^2) U = (E/c) T
\]

4-WaveVector
\[
K = k^\nu = (\omega/c^2, \mathbf{k}) = (\omega^2/c^2) \mathbf{u} = -\nabla[\Phi] = (\omega/c^2 \mathbf{u} \mathbf{n} = \mathbf{v}) \mathbf{n} \mathbf{v}_{phase} = (1/c^2 T, \mathbf{n} \cdot \mathbf{A}) = (\omega/c) T
\]

4-EM Vector Potential
\[
A^\nu = (\mathbf{A} \cdot \mathbf{V} = (\mathbf{A} \cdot \mathbf{V}) U = (\mathbf{A} \cdot \mathbf{V}) U = (\mathbf{A} \cdot \mathbf{V}) U
\]

4-Particle: Dust: Number: Flux
\[
N = N^\nu = (n_c, n) = n_c \mathbf{u} = n_c \mathbf{u} = n_c \mathbf{u}
\]

Minkowski Region, $\partial^\mu = \mathbf{V}^\mu$ is it's 3D boundary
\[
d^\mu = 4D \text{Minkowski} \ 3D \text{Surface Element}
\]

Math/Phys
\[
\nabla^\mu = \partial^\mu[\mathbf{V}] = \mathbf{V}^\mu = \mathbf{V} \cdot \mathbf{N}
\]

Faraday EM Tensor
\[
\nabla^\mu = \partial^\mu[\mathbf{V}] = \mathbf{V}^\mu = \mathbf{V} \cdot \mathbf{N}
\]

Conservation of EM Potential
\[
\nabla^\mu = \partial^\mu[\mathbf{V}] = \mathbf{V}^\mu = \mathbf{V} \cdot \mathbf{N}
\]

Conservation of Charge
\[
\nabla^\mu = \partial^\mu[\mathbf{V}] = \mathbf{V}^\mu = \mathbf{V} \cdot \mathbf{N}
\]

Conservation of Particle #
\[
\nabla^\mu = \partial^\mu[\mathbf{V}] = \mathbf{V}^\mu = \mathbf{V} \cdot \mathbf{N}
\]

SR 4-Tensor
\[
\nabla^\mu = \partial^\mu[\mathbf{V}] = \mathbf{V}^\mu = \mathbf{V} \cdot \mathbf{N}
\]

SR 4-Vector
\[
\nabla^\mu = \partial^\mu[\mathbf{V}] = \mathbf{V}^\mu = \mathbf{V} \cdot \mathbf{N}
\]

SR 4-Scalar
\[
\nabla^\mu = \partial^\mu[\mathbf{V}] = \mathbf{V}^\mu = \mathbf{V} \cdot \mathbf{N}
\]

4-Vector $V = V^\mu = (V^0, V)$ = (scalar $\cdot c^2$, 3-vector)
SRQM Study: Physical 4-Tensors

Some SR 4-Tensors and Symbols

- Lorentz Identity Transform \( \Lambda^\mu_\nu \rightarrow \delta^\mu_\nu = I_{(4)} \)
- Lorentz Time-Reverse (Parity Inverse) Transform \( \Lambda^\mu_\nu \rightarrow T^\mu_\nu = -I_{(4)} \)
- Lorentz ComboPT Transform \( \Lambda^\mu_\nu \rightarrow (PT)^\mu_\nu = -I_{(4)} \)

4-Tensor \( T^\mu_\nu \) in Minkowski Metric:
\[ \delta[R] = \partial^2 R = \eta^{\mu\nu} = V^{\mu\nu} + H^{\mu\nu} \]
\[ = \text{Diag}[1,1,1,1] = \text{Diag}[1,\delta] \]
\[ = \text{(Cartesian/rectangular basis)} \]

SR: Minkowski Metric
\[ \Lambda^\mu_\nu \rightarrow B^\mu_\nu \]
\[ = \text{Lorentz Transforms} \]

Lorentz x-Boost Transform
\[ \Lambda^\mu_\nu \rightarrow B^\mu_\nu \]
\[ = \text{Lorentz Transforms} \]

Lorentz z-Rotation Transform
\[ \Lambda^\mu_\nu \rightarrow R^\mu_\nu \]
\[ = \text{Lorentz Transforms} \]

Perfect Fluid
\[ T^\mu_\nu = (\rho_0 + p_0) V^\mu V^\nu + (p_0) \eta^{\mu\nu} \]
\[ = \text{Diag}[\rho_0, p_0, 0, 0] \]
\[ \rightarrow \text{MCFR} \]

Faraday EM
\[ F_\phi = \partial^0 A^j - \partial^j A^0 = \partial^\mu A^\nu - \partial^\nu A^\mu \]

4-Angular Momentum
\[ M^a_\mu = \epsilon^a_\mu V^\nu \epsilon^\nu_\mu \]

4-Tensor
- Unit Dimensionless
- Symmetric, Spatial Isotropic
- Anti-symmetric (skew)

SR 4-Vector
\[ (1,0)-Tensor V^\mu = V^1 \]
\[ (0,1)-Tensor V_\mu = V_1 \]
\[ \text{Note that all Lorentz Transforms and the Minkowski Metric are [unit dimensionless=1]. The Perfect Fluid has units of energy density = pressure = Pa = J/m^2 = kg/m/s^2} \]
SRQM Study: Physical 4-Tensors

**Projection 4-Tensors** \( \{ P^{\mu \nu}, P^{\mu}_{\nu}, P^{\mu}_{\nu} \} \)

---

**SR 4-Tensor** 
(0,0)-Tensor \( V \equiv \{ 0, 0 \} \) from \( \mathbf{MCRF} \)

**SR 4-Vector** 
(1,0)-Tensor \( V = (V^x, V^y, V^z) \)

**SR 4-Scalor** 
(0,0)-Tensor \( S \) or \( 0 \)

---

**SR Perfect Fluid** 
\( \rho_0 = \rho_0 c^2, P^e = \rho_0 \) from \( T \rightarrow S = 0 \)

**4-Unit Temporal** 
\( T = T + \gamma(1,0) \) \( \rightarrow (1,0) \) \( \mathbf{RestFrame} \)

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**SR 4-Tensor** 
Spatial \( "(H)orizontal" \) \( \mu \nu \)

**SR 4-Vector** 
Temporal \( "(V)ertical" \) \( \nu \)

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**Note:** That the Projection Tensors are \( \{ \text{Unit Dimensionless} = 1 \} \), the object projected retains its own dimensional measurement units. Note that the (2,0) \& (0,2) - Spatial Projectors have opposite signs from the mixed (1,1) - Spatial due to minuses in the Minkowski Metric.
SRQM Study: SR 4-Vector Properties

General $T^\mu \rightarrow$ Temporal:$\text{Spatial}$ ($1+3$) Decomposition via Projection

Any SR (1-index) Tensor $\{T^\mu, T_\mu\}$ can be decomposed into $\{\text{Temporal}, \text{Spatial}\}$ parts, by using combinations of (V)ertical & (H)orizontal Projection Tensors, with ($\eta^\alpha_\mu = V^\alpha_\mu + H^\alpha_\mu$) and contraction orthogonality ($V^\mu_\alpha H^\nu_\alpha = (V^\mu_\alpha \eta^\alpha_\nu - V^\nu_\alpha \eta^\alpha_\mu) = (V^\mu_\alpha - V^\nu_\alpha - 0)_\mu$)

Temporal$^\mu = V^\mu_\alpha (T^\alpha)$
Spatial$^\mu = H^\mu_\alpha (T^\alpha)$
FullSpaceTime$^\mu = \eta^\mu_\alpha (T^\alpha) = T^\alpha$

$T^\alpha = (\text{Temporal}^\mu + \text{Spatial}^\mu) = (\text{FullSpaceTime}^\mu)$

SR: Minkowski Metric ($\lambda+3$) Splitting
$
\eta^\mu_\alpha = V^\mu_\alpha + H^\mu_\alpha$

Temporal: Spatial Contraction Orthogonality
$V^\mu_\alpha H^\mu_\alpha = (V^\mu_\alpha H^\mu_\alpha) = 0^\mu$

$T^\alpha = (T^0, T^1, T^2, T^3)$

max 1 component

$\text{Spatial}: 4$-Vector
$T^\mu = (T^0, 0, 0, 0) = V^\mu_0 T^\alpha$

max 3 components

$\text{Temporal}: 4$-Vector
$T^\mu = (0, T^1, T^2, T^3)$

max 4$^1 = 4$ components

Note that in this example:
$T = T^\alpha = (1, \beta)$ is the 4-UnitTemporal
$T^\alpha = (T^0, T^1, T^2, T^3)$ is a general 4-Vector = 4D (1,0)-Tensor $T^\mu$

It can be made into the 4-UnitTemporal via Temporal Projection, followed by a Lorentz Boost
SRQM Study: SR 4-Tensor Properties

General $T^\mu\nu \rightarrow$ Temporal:Mixed: Spatial $(1+3)$ Decomposition via Projection

Any SR (2-index) Tensor $\{T^\mu\nu, T^\nu\mu, T^\mu\nu\}$ can be decomposed into $\{\text{Temporal}, \text{Mixed}, \text{Spatial}\}$ parts, by using combinations of (V)ertical & (H)orizontal Projection Tensors, with $(\eta^\mu_{\nu} = V^\mu_{\nu} + H^\mu_{\nu})$

and contraction orthogonality $(V^\mu_{\nu}H^\mu_{\nu} = (V^\mu_{\nu}\eta^\nu_{\mu} - V^\nu_{\mu})) = (V^\mu_{\nu} - V^\nu_{\mu}) = 0_{\mu\nu}$

Temporal$^{\mu\nu} = V^\mu_{\nu}V^\mu_{\nu} (T^{\mu\nu})$
Mixed$^{\mu\nu} = V^\mu_{\nu}H^\mu_{\nu} (T^{\mu\nu})$
Spatial$^{\mu\nu} = H^\mu_{\nu}H^\mu_{\nu} (T^{\mu\nu})$

$T^{\mu\nu} = (\text{Temporal}^{\mu\nu} + \text{Mixed}^{\mu\nu} + \text{Spatial}^{\mu\nu}) = (\text{FullSpaceTime}^{\mu\nu})$

$T^{\mu\nu} = V^\mu_{\nu}V^\mu_{\nu} (T^{\mu\nu}) + [V^\mu_{\nu}H^\mu_{\nu} (T^{\mu\nu}) + H^\mu_{\nu}V^\mu_{\nu} (T^{\mu\nu})] + H^\mu_{\nu}H^\mu_{\nu} (T^{\mu\nu})$

$T^{\mu\nu} = V^\mu_{\nu}V^\mu_{\nu} (T^{\mu\nu}) + [V^\mu_{\nu}V^\mu_{\nu} + V^\mu_{\nu}H^\mu_{\nu} + H^\mu_{\nu}V^\mu_{\nu} + H^\mu_{\nu}H^\mu_{\nu}] (T^{\mu\nu})$

$T^{\mu\nu} = (\eta^\mu_{\nu} (\eta^\nu_{\mu}) (T^{\mu\nu}) = (\delta^\mu_{\nu} (\delta^\nu_{\mu}) (T^{\mu\nu}) = 0_{\mu\nu}$

$\text{True by definition}$

Temporal $T^{\mu\nu} = V^\mu_{\nu}V^\mu_{\nu} (T^{\mu\nu})$
Mixed $T^{\mu\nu} = V^\mu_{\nu}H^\mu_{\nu} (T^{\mu\nu})$
Spatial $T^{\mu\nu} = H^\mu_{\nu}H^\mu_{\nu} (T^{\mu\nu})$

$\text{SR:Minkowski Metric } (1+3) \text{ Splitting } \eta^\mu_{\nu} = V^\mu_{\nu} + H^\mu_{\nu}$

$\text{Temporal } 4\text{-Tensor} \delta[R] = \delta^R = \eta^\mu_{\nu} = V^\mu_{\nu} + H^\mu_{\nu}$

$\rightarrow \text{Diag} [1, -1, 0, 0] = \text{Diag} [1, \delta^\mu_{\nu}]$ (Cartesian/rectangular basis)

$\text{Spatial } 4\text{-Tensor} \delta[R] = \delta^R = \eta^\mu_{\nu} = V^\mu_{\nu} + H^\mu_{\nu}$

$\rightarrow \text{Diag} [0, 0, 0, 0] = \text{Diag} [0, 0, 0, 0]$ (Cartesian/rectangular basis)

4-UnitTemporal $\mathbf{V} \cdot \mathbf{V} = \mathbf{1}$

4-Vector $\mathbf{V} \cdot \mathbf{V} = \mathbf{V}^\mu V^\mu = V^\mu V^\mu + H^\mu H^\mu$

4-Scalar $\mathbf{V} \cdot \mathbf{V} = \mathbf{V}^\mu V^\mu = V^\mu V^\mu + H^\mu H^\mu$

$\text{Trace} [T^{\mu\nu}] = \eta^\mu_{\nu} T^{\mu\nu} = T^{\mu\nu} \rightarrow V \cdot V = V^\mu V^\mu = (V^\mu)^2 - V^\mu V^\mu = (V^\mu)^2 = \text{Lorentz Scalar Invariant}$
SRQM Study: Physical 4-Tensors

**Relativisitic Fluid 4-Tensor**

**SRQM Study: Physical 4-Tensors**

**4-Vector SRQM Interpretation of QM**

http://scirealm.org/SRQM.pdf

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[Image of a detailed diagram related to SRQM study, showing various 4 tensors and their properties, such as energy density, pressure, heat flux, and stress components.]

**Equation of State**

\( EoS[T^\mu] = \rho \cdot c^2 \cdot Q^{0}/c + \Pi^{\mu} \)

**Trace of T**

\( T^\mu_\mu = \sum_{\mu} T^\mu_\mu = \rho \cdot c^2 + P \)

**SR 4-Tensor (2,0)-Tensor**

\( T^{(2,0)} \rightarrow T^{(2,0)} \rightarrow V^{\mu} = (v^{0}, v^{i}) \)

**SR 4-Vector**

\( V^{\mu} = V^{(0,1)} \rightarrow (v^{0}, v^{i}) \)

**SR 4-Scalar (0,0)-Tensor**

\( S^{(0,0)} \rightarrow (\rho, p, \Pi^{\mu}) \)

**Lorentz Scalar**

\( \rho = \frac{\text{dimensional measurement units}}{c} \)

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**SRQM Study: Physical 4-Tensors**

**Relativisitic Fluid 4-Tensor**

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\( \rho = \frac{\text{dimensional measurement units}}{c} \)
SRQM Study: Physical 4-Tensors

Relativisitc Fluid 4-Tensor

Stress-Energy(Density)-Tensor $T^{\mu\nu}$ (Symmetric)
Covariant Decomposition

Tensor Invariants: { Symmetry+AntiSymmetry, Trace→Isotropy+Anisotropy, 4D $(1+3)$ Splitting }

$T^{\mu\nu} \rightarrow (\rho_0)V^{\mu\nu} + (-p_0)H^{\mu\nu} + (T^\mu H^\nu_o + Q^\mu_o H^\nu_o + \Pi^\mu_o)/c + H^\mu_o H^\nu_o \Pi^\mu_o$

$V_{\mu\nu}T^{\mu\nu} \rightarrow (\rho_0)V_{\mu\nu}V^{\mu\nu} + (-p_0)V_{\mu\nu}H^{\mu\nu} + (T^\mu V^\nu_o + Q^\mu_o V^\nu_o + \Pi^\mu_o)/c + V_{\mu\nu}H^\mu_o H^\nu_o \Pi^\mu_o$

$V_{\mu\nu}T^{\mu\nu} \rightarrow (\rho_0)(1) + (-p_0)(0) + (T^\mu_0(0) + Q^\mu_0(0))/c + (0)$

$V_{\mu\nu}T^{\mu\nu} \rightarrow (\rho_0)(1) + (-p_0)(3) + (T^\mu_0 + Q^\mu_0 + \Pi^\mu_0)/c + (0)$

$H^\mu_o T^{\mu\nu} \rightarrow (\rho_0)H^\mu_o H^{\mu\nu} + (T^\mu_0 H^\nu_i + Q^\mu_o H^\nu_i + \Pi^\mu_o)/c + H^\mu_o H^\nu_o \Pi^\mu_o$

$H^\mu_o T^{\mu\nu} \rightarrow (\rho_0)(0) + (-p_0)(3) + (T^\mu_0 + Q^\mu_0 + \Pi^\mu_0)/c + (0)$

$H^\mu_o T^{\mu\nu} \rightarrow (\rho_0)(0) + (-p_0)(3) + (Q^\mu_0 + Q^\nu_0 + \Pi^\mu_0)/c + (0)$

$H^\mu_o T^{\mu\nu} \rightarrow (\rho_0)(3)$ (Isotropic) Pressure $(p_0)=(-1/3)H^\mu_o T^{\mu\nu}$

$T^\mu_o T^{\mu\nu} \rightarrow (\rho_0)T^\mu_o T^{\mu\nu} + (-p_0)T^\mu_o H^{\mu\nu} + (T^\mu_0 H^\nu_i + Q^\mu_o H^\nu_i + \Pi^\mu_o)/c + T^\mu_o H^\nu_o \Pi^\mu_o$

$T^\mu_o T^{\mu\nu} \rightarrow (\rho_0)T^\mu_o T^{\mu\nu} + (-p_0)(0) + ((1)H^\mu_o + Q^\mu_0(0))/c + (0)$

$T^\mu_o T^{\mu\nu} \rightarrow (\rho_0)T^\mu_o T^{\mu\nu} + (Q^\mu_0 + Q^\nu_0 + \Pi^\mu_0)/c + (0)$

$cT^\mu_o T^{\mu\nu} \rightarrow (\rho_0)T^\mu_o T^{\mu\nu} + (Q^\mu_0 + Q^\nu_0 + \Pi^\mu_0)/c + (0)$

$H^\mu_o H^\nu_o T^{\mu\nu} \rightarrow (\rho_0)H^\mu_o H^\nu_o H^{\mu\nu} + (p_0)H^\mu_o H^\nu_o (T^\mu_0 H^\nu_i + Q^\mu_o H^\nu_i + \Pi^\mu_o)/c + H^\mu_o H^\nu_o \Pi^\mu_o$

$H^\mu_o H^\nu_o T^{\mu\nu} \rightarrow (\rho_0)(0) + (p_0)H^\mu_o H^\nu_o + ((0)\mu_o + (0)\nu_o)/c + (0)$

$H^\mu_o H^\nu_o T^{\mu\nu} \rightarrow (\rho_0)(-1/3)H^\mu_o H^\nu_o H^\nu_o T^{\mu\nu}$ (Anisotropic) ViscousShear $\Pi^\mu_o = H^\mu_o H^\nu_o T^{\mu\nu} + (p_0)H^\mu_o H^\nu_o$
A Tensor Study of Physical 4-Vectors

SRQM Study: Physical 4-Tensors

Special-Cases based on “velocity” (β)

4-ForceDensity [N/m² = Pa/m]
F_{den}= F_{den}^\mu \nu = \partial_\mu T^\nu \rightarrow \partial_\mu T^\nu
= 0 \text{ if conserved}

Full Relativistic Fluid Stress-Energy(Density)
T^{\mu \nu} \rightarrow \rho (0^{-}) \nu^\nu + (-p_0) H^\nu \rightarrow \rho_0 (\rho_0) H^\nu

Perfect Fluid Stress-Energy
in terms of velocity (\beta) instead of pressure (p)
T^{\mu \nu} \rightarrow \rho_0 (\rho_0) H^\nu + ((-\rho_0) \beta H^\nu) \rightarrow \rho_0 (\rho_0) H^\nu

Perfect Gas (|v| < c) = Warm Dust
v = v_{thermal} = \sqrt{3RT/\mu } = \sqrt{3k_B T/m_0} = (0..c)

= characteristic rms 3D thermal speed of molecules

\text{essentially this smoothly varies from Matter-Dust (v~0) to Radiation-Fluid (v~c)}

\text{v}^2 = [3RT/\mu ] = [3k_B T/m_0]

\beta^2 = (v/c)^2 = [3RT/c^2] = [3k_B T/m_0 c^2] = [3k_B T/E_v]

\beta^2 p_0/\rho_0 = \beta^2 n_0 E_v/3 = [(3k_B T/E_v)p_0/3] = n_0 k_B T

= EnergyDensity based on temperature

SR 4-Vector
(0,0)-Tensor S or S_0
SR 4-Scalar
Lorentz Scalar

SR 4-CoVector:OneForm
(1,0)-Tensor V = (v_\nu)

Technically, all these are 4D Stress-EnergyDensity Tensors
That they are usually called “Stress-Energy” Tensors is an
	

4-Vector SRQM Interpretation

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SR Dust 4-Tensor
Special-Cases

In a frame with 4-Velocity \( U \)

\[
\begin{array}{ccc}
\gamma^2 n_0 m_0 c^2 & \gamma^2 n_0 m_0 c u_i & \gamma^2 n_0 m_0 c dx/dt \\
\gamma^2 n_0 m_0 c u_i & \gamma^2 n_0 m_0 u_i u_i & \gamma^2 n_0 m_0 dx/dt \\
\gamma^2 n_0 m_0 c & \gamma^2 n_0 m_0 dx/dt & \gamma^2 n_0 m_0 dx/dt dx/dt \\
\end{array}
\]

\( \mu_\nu \) = Lorentz Scalar Invariant

Equation of State
\( EoS[T^{\mu\nu}] = w = \frac{\rho}{\rho_0} \)

Technically, all these are 4D Stress-EnergyDensity Tensors
That they are usually called “Stress-Energy” Tensors is a lazy abbreviation which causes confusion

\( v \cdot v = v^\mu v_\mu = \frac{\rho}{\rho_0} \)

\( \gamma = \frac{1}{\sqrt{1 - v \cdot v/c^2}} \)

\( \rho \) = MassDensity * c^2

HeatFlux / c
Pressure & Viscous Shear

\( \gamma n_0 E \) = \( \gamma n_0 cp_i \) = \( \gamma n_0 mu'u_i \)

\( nE \) = \( ncp_i \) = \( nmu'u_i \)

\( T^{\mu\nu} \to P^{\mu N}=m_0 U^{\nu} N^{\rho}=\rho_0 (v_0)^{\nu} \to \) (MCFR)

\( \rho_\nu = \rho_\nu c^2 \)

\( [\text{t}] \quad [\text{x}] \quad [\text{y}] \quad [\text{z}] \)

\( \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \)

\( nE = \gamma n_0 E \)

\( ncp_i = \gamma n_0 cp_i \)

\( nmu'u_i = \gamma n_0 mu'u_i \)

\( \text{Symmetric, Spatial Isotropic, Pressureless} \)

\( \text{Time-Space} \) Stress-EnergyDensity Tensor:
(temporal) EnergyDensity = MassDensity * c^2
(mixed) HeatFlux / c
(spatial) Pressure & Viscous Shear
all have the same dimensional measurement units:
[Pa = J/m^3 = N/m^2 = kg/m-s^2]

\( \text{SR 4-Vector} \)
\( (1,0)-Tensor V^\nu = V = (v^\mu, v^\nu) \)
\( (0,1)-Tensor V_\nu = \) Lorentz Scalar

\( \text{SR 4-Scalar} \)
\( (0,0)-Tensor S \text{ or S}_\rho \text{ or S}_o \text{ Scalar} \)

\( \text{SR 4-Tensor} \)
\( (2,0)-Tensor T^{\mu\nu} \)
\( (0,2)-Tensor T_{\mu\nu} \)

\( \text{SR 4-CoVector: OneForm} \)
(0,1)-Tensor \( V_\nu = (v_\nu, v_\rho) \)

\( \text{SR 4-Vector SRQM} \)
\( \text{Interpretation} \)

\( \text{SRQM} \to \text{QM} \)
\( \text{4-Vector} \)

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SRQM Study: Physical 4-Tensors

SR Fluid: Dust: Vacuum 4-Tensors

Also known as GR “Solutions”

Maxwell 4D EM Stress-Energy Tensor
\[ T^{\mu \nu} \rightarrow \left( -\mu_0 / c \right) \left( \mathbf{E} \times \mathbf{B} \right) = \left( \mathbf{H} \times \mathbf{E} \right) / c \]

4-Vector SRQM Interpretation of QM

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Some SR 4-Tensors and Symbols

4-Unit Temporal Tensor
\[ T = T_{\mu \nu \rho \sigma} = (1,0,0,0) \quad \text{and} \quad U = T / c \]

Temporal \( (V) \) Vertical Projection Tensor
\[ P_{\mu \nu} \rightarrow T_{\mu \nu} = \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \]

Faraday EM Tensor
\[ F_{\mu \nu} = \varepsilon_{\mu \nu \alpha \beta} A^\alpha / c = \partial A^\mu / \partial t - \partial A^\nu / \partial x^\mu \]

Maxwell 4D EM Stress-Energy Tensor
\[ T^{\mu \nu} = -\left( 1 / \mu_0 \right) F^{\mu \nu} F_{\mu \nu} - (1 / 4 \mu_0) F^{\mu \nu} F_{\mu \nu} \]

\( 4 \)-Force Density
\[ F_{\text{den}} = F_{\text{den}}^\gamma = -\partial \gamma^\nu T^{\mu \nu} \]

Perfect Fluid Stress-Energy
\[ T^{\mu \nu} \rightarrow (p_o) \gamma^\nu \rightarrow (p_o) \gamma^\nu \quad \text{(MCF)} \]

\( (C) \) (Cold) Matter-Dust
\[ T^{\mu \nu} \rightarrow \rho \gamma^\nu \rightarrow \rho \gamma^\nu \quad \text{(MCF)} \]

Stress Energy 4-Tensor
\[ \text{Symmetric, Spatial Isotropic, Pressureless} \]

Lambda Vacuum
\[ T^{\mu \nu} \rightarrow (\Lambda) \gamma^\nu \rightarrow (\Lambda) \gamma^\nu \quad \text{(MCF)} \]

Zero: Nothing Vacuum
\[ T^{\mu \nu} \rightarrow 0 \quad \text{(MCF)} \]

\( \epsilon \) Energy density
\[ \rho_o = \rho_m c^2 0^i \]

Unit Dimensionless
\[ \dot{x}, \dot{y}, \dot{z} \]

Minkowski Metric
\[ \delta[R] = \partial^2 \gamma = \eta^\mu \gamma^\mu \]

Perfect Fluid Energy Density
\[ E_o S[T^\mu] = w = p_o / \rho_o \]

\( \text{Perfect Fluid} \)
\[ T^{\mu \nu} \rightarrow (p_o) \gamma^\nu \rightarrow (p_o) \gamma^\nu \]

\( 4 \)-Force Density
\[ F_{\text{den}} = F_{\text{den}}^\gamma = -\partial \gamma^\nu T^{\mu \nu} \]

\( \epsilon \) Energy density
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\[ \rho_o = \rho_m c^2 0^i \]

\( \epsilon \) Energy density
\[ \rho_o = \rho_m c^2 0^i \]
SRQM Study: Physical 4-Tensors

Metric Sign-Convention: Signature

Perfect Fluid 4D Stress-Energy Tensor (technically an Energy Density)

\[
T^{\mu\nu} = \left( \rho_0 + p_0c^2 \right) U^\mu U^\nu + \{\{sc\}\} p_0 \eta_{\mu\nu}
\]

for \((+,+,+\)\) Spacelike+, Relativity, EastCoast convention

\[
\text{Tr}[T^{\mu\nu}] = \left( p_0 - (p) - (p) - (p) \right) = \eta_{\mu\nu} T^{\mu\nu} = T^0 = \rho_0 - 3p_0
\]

Maxwell 4D EM Stress-Energy Tensor (technically an Energy Density)

\[
T^{\mu\nu} = \left( \rho + \varepsilon_0 \right) F^{\mu\nu} - \frac{1}{4} \varepsilon_0 \eta^{\mu\nu} F_{\alpha\beta} F_{\alpha\beta}
\]

for \((+,+,+\)\) Spacelike+, Relativity, EastCoast conv.

\[
\text{Tr}[T^{\mu\nu}] = 0
\]

Stress-Energy Tensors are technically Energy Densities, not Energies: Energy Density (temporal) & Pressure/Stress (spatial) have the same dimensional measurement units. [Pa = J/m² = N/m² = kg/m³ s²]

\[
\text{Trace}[T^{\mu\nu}] = \eta_{\mu\nu} T^{\mu\nu} = T^0 = T
\]

\[
V^2 = \left( \varepsilon_0 \right)^2 - \left( \varepsilon_0 \right) V^\mu V_\mu = \left( \varepsilon_0 \right)^2 = \text{Lorentz Scalar Invariant}
\]
SR Invariant Waves

SR Action → 4-Momentum
SR Lorentz Transforms
SR SpaceTime Dimension=4
SR SpaceTime "Flat" Minkowski 4D Metric
4-Gradient=

A Tensor Study

Physics

(1,1)-Tensor T
(2,0)-Tensor T
(2,1)-Tensor T
of Physical 4-Vectors

SR & QM Invariant Waves
SRQM Diagram:

Special Relativity → Quantum Mechanics
RoadMap of SR → QM

4-Gradient=Alteration of SR <Events>
SR SpaceTime "Flat" Minkowski 4D Metric
SR SpaceTime Dimension=4
SR Lorentz Transforms
SR Action → 4-Momentum
SR Phase → 4-WaveVector
SR ProperTime Derivative
SR & QM Invariant Waves

SR d’Alembertian & Klein-Gordon Relativistic Quantum Wave Relation
Schrödinger QWE is |v|<<c limit of KG QWE

**[SR → QM]**

4-WaveVector=Substantiation of SR Wave <Events>
oscillations proportional to mass:energy & 3-momentum

K=⟨ω,κ⟩=(ω/c,ωh/Vphase)

4-Position R=⟨ct,r⟩<Event>

4-Vector SRQM Interpretation of QM

http://scirealm.org/SRQM.pdf

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SRQM: The [ SR→QM ] Interpretation of Quantum Mechanics

Special Relativity (SR) Axioms: Invariant Interval + LightSpeed (c) as Universal Physical Constant lead to SR, although technically SR is itself the Minkowski-SpaceTime low-curvature:"flat" limiting-case of GR.

\{ c, \tau, m_0, \hbar, i \} = \{ c: \text{SpeedOfLight}, \tau: \text{ProperTime}, m_0: \text{RestMass}, \hbar: \text{Dirac/PlanckReducedConstant}(h=\hbar/2\pi), i: \text{ImaginaryNumber}\}:

are all Empirically-Measured SR Lorentz-Invariant Physical and/or Mathematical Constants

\[ i = +\sqrt{-1} = (0,1) \text{ complex} # \]

Standard SR 4-Vectors:

- 4-Position \[ R = (ct, r) \in <\text{Event}> \in <\text{Time}\cdot\text{Space}> \]
- 4-Velocity \[ U = (\gamma(c, u)) = (\gamma(c, u)) \]
- 4-Momentum \[ P = (E/c, p) = m_0 U \]
- 4-WaveVector \[ K = (\omega/c, k) = P/\hbar \]
- 4-Gradient \[ \partial = (\partial/c, \nabla) \]

SR + Empirically Measured Physical Constants lead to RQM via the (KG) Klein-Gordon Quantum Eqn, and thence to QM via the low-velocity limit \{ |v| \ll c \}, giving the Schrödinger Eqn. This fundamental KG Relation also leads to the other Quantum Wave Equations:

**Spin=0 boson field = 4-Scalar:**
- Free Scalar Wave (Higgs)

**Spin=1/2 fermion field = 4-Spinor:**
- Weyl
- Dirac (RQM w/ EM charge)

**Spin=1 boson field = 4-Vector:**
- Maxwell (EM photonic)
- Proca

SRQM Chart:

Special Relativity → Quantum Mechanics

SR→QM Interpretation Simplified

SRQM: A treatise of SR→QM by John B. Wilson (SciRealm@aol.com)
SRQM 4-Vector Topic Index

SRQM 4-Vector Topic Index
SR & QM via 4-Vector Diagrams

Mostly SR Stuff
4-Vector Basics, SR 4-Vectors = Physical 4D (1,0)-Tensors, Coordinate-Independent Objects
Paradigm Assumptions: Right & Wrong
Minkowski:SR SpaceTime, <Events> ∈ <TimeSpace>, WorldLines, 4D Minkowski Metric
SR (4-Scalars, 4-Vectors, 4-Tensors) & Tensor Invariants, Cayley-Hamilton Theorem
SR Lorentz Transforms, CPT Symmetry, Trace Identification, Antimatter, Feynman-Stueckelberg
Fundamental Physical Constants: Lorentz Scalar Invariants = SR 4-Vectors = 4D (0,0)-Tensors
Projection Tensors: Temporal ("vertical") & Spatial ("horizontal"): (V), (h) refer to Light-Cone
Stress-Energy Tensors: Relativistic Fluids → (PerfectFluid, Dust, Radiation, EM, DarkEnergy, etc)
Invariant Intervals, Measurement, Metrics, Metric Signature
SpaceTime Kinematics & Dynamics, ProperTime Derivative
Einstein's E = mc^2 = γm_0c^2 = γE_0, Rest Mass (m_0), Rest Energy (E_0), Scalar Invariants
SpaceTime Orthogonality: Time-like 4-Velocity, Space-like 4-Acceleration
Relativity of Simultaneity: Stationarity; Invariance/Absolutes of Causality: Topology
Relativity: Time Dilation (→ clock moving [→]), Length Contraction (→ ruler moving← )
Invariants: Proper Time (clock at rest [→]), Proper Length (ruler at rest [→])
Temporal Ordering: (Time-like) Causality is Absolute; (Space-like) Simultaneity is Relative
Spatial Ordering: (Time-like) Stationarity is Relative; (Space-like) Topology is Absolute
SR Motion * Lorentz Scalar = Interesting Physical 4-Vector
SR Conservation Laws & Local Continuity Equations, Symmetries
SR Wave-Particle Relation, Invariant d'Alembertian Wave Eqn, SR Waves, 4-WaveVector
Relativistic Doppler Effect, Relativistic Aberration Effect
SpaceTime is 4D = (1+3)D: d=R=c_0R, d_0=c_0R_0, A=A_0=γ(α^0,α^1,α^2,α^3) = 4 comps
Minimal Coupling = Interaction with (a Vector)Potential, usually the EM 4-VectorPotential
Conservation of 4-TotalMomentum (TotalEnergy=Hamiltonian & 3-total-momentum)
SR Hamiltonian/Lagrangian Connection
Lagrangian, Lagrangian Density
Hamilton-Jacobi Equation (differential d), Relativistic Action (integral I)
4-Position:4-Velocity Relation, Euler-Lagrangian Equations
Noether's Theorem, Continuous Symmetries, Conservation Laws, Continuity Equations
Relativistic Equations of Motion, Lorentz Force Equation
\gamma^2 Invariant Relations, The Speed-of-Light (c)
Thermodynamic 4-Vectors, Unruh-Hawking Radiation, Particle Distributions

Mostly QM & SRQM Stuff
Advanced SRQm 4-Vectors
Where is Quantum Gravity?
Relativistic Quantum Wave Equations (RWE : QWE)
Klein-Gordon Equation (KG) / Fundamental Quantum Relation (d=∂/∂τ → (mc/h)^2
Roadmap from SR to QM: SR→Qm, SRQM 4-Vector Connections
QM Schrödinger Relation
QM Axioms? - No, (QM Principles derived from SR) = SRQM
Relativistic Wave Equations: based on mass & spin & relative velocity:energy
RWE's: Klein-Gordon, Dirac, Proca, Maxwell, Weyl, Pauli, Schrödinger, etc
Canonical QM Commutation Relations ↔ derived from SR
Heisenberg Uncertainty Principle (due to non-zero commutation)
Pauli Exclusion Principle (Fermion spin n/2), Bose Aggregation Principle (Boson spin n)
Complex 4-Vectors, Quantum Probability, Imaginary values
CPT Theorem, Lorentz Invariance, Poincaré Invariance, Isometry
Hamiltonian Generators, Unitarity:Anti-Unitarity
QM → Classical Correspondence Principle, similar to SR → Classical Low Velocity
The Compton Effect = Photon:Electron Interaction (neglecting Spin Effects)
Photon Diffraction, Crystal-Electron Diffraction, The Kapitza-Dirac Effect
The (h) Relation, Einstein-de Broglie, Planck:Dirac, Wave-Particle
The Aharonov-Bohm Effect (integral I), The Josephson Junction Effect (differential d)
Dimensionless Quantities
SRQM Symmetries:
Hamilton-Jacobi Equation (differential d), Relativistic Action (integral I)
Differential (d) on (4-Vector) vs. Integral (I) on (4-Scalar)
Schrödinger Relations vs. Cyclic Imaginary Time ↔ Inverse Temperature
4-Velocity:4-Position vs. Euler-Lagrangian Equations
Matter-AntiMatter: Trace Identification of Lorentz Transforms, CPT
Quantum Relativity: GR is NOT wrong, "Never bet against Einstein" :)
There are some paradigm assumptions that need to be cleared up:

**The real, physical world *IS NOT* Euclidean 3-dimensional (3D) with absolute background time.**

Classical and quantum 3D physics is a great approximation; but only for Galilean, slow-moving objects $|v|<<c$. 3D physics uses {3-vectors = 3D (1,0)-tensors}, has 3D Euclidean invariants like lengths (Pythagorean theorem), has 1D Euclidean scalar invariants like absolute time, but it does not contain or predict many of the physical properties and relationships that we now know to be true from SR & RQM.

Also, these 1D & 3D Euclidean invariants have been empirically-proven to *NOT* be invariant in the real world. This is based on a century+ of physics experiments and observations confirming the fact of 4D Relativity. These 1D & 3D scalar invariants are actually just relativistic components of 4-Vectors in 4D.

The fact that 4D Scalar Invariants don’t display similar relativistic variance is a good indication that our universe is empirically 4D.

**The real, physical world *IS* a locally Minkowskian 4-Dimensional SpaceTime (4D), with relativistically-interconnected (1 time + 3 space) dimensions.**

Time and space are interconnected in a very specific Lorentzian way, via SpaceTime 4D Relativistic Metrics, which give a great many special relationships and invariances that 3D physics misses entirely. These properties are easily explained using SR:Minkowskian Physical {4-Vectors = 4D (1,0)-Tensors}.

3D physics can be obtained as a limiting-case approximation from 4D Physics by using relative speed $|v|<<c$. Classical Mechanics (CM) is just a low-speed limiting-case of Special Relativity (SR). Quantum Mechanics (QM) is just the low-speed limiting-case of Relativistic Quantum Mechanics (RQM). This is related to 1+3 Time·Space Splitting.
There are some paradigm assumptions that need to be cleared up:

Minkowskian:SR 4D Physical 4-Vectors *ARE NOT* generalizations of Classical/Quantum 3D physical 3-vectors.
While a “mathematical” Euclidean (n+1)D-vector is the generalization of a Euclidean (n)D-vector, the “physical/physics” analogy ends there.

Minkowskian:SR 4D Physical 4-Vectors *ARE* the primitive elements of 4D Minkowski:SR SpaceTime.
Classical/Quantum physical 3-vectors are just the spatial components of SR Physical {4-Vectors = 4D (1,0)-Tensors}.
There is also a fundamentally-related classical/quantum physical scalar related to each type of 3-vector, which is just the temporal component scalar of a given SR Physical SpaceTime 4-Vector.

These Classical/Quantum {scalar}+{3-vector} are the dual {temporal}+{spatial} components of a single SR Time-Space 4-Vector = (temporal scalar * c^±1, spatial 3-vector) with SR LightSpeed factor (c^±1) to give correct overall dimensional measurement units.

While different observers may see different relative "values" of the Classical/Quantum components \(v^4, v^1, v^2, v^3\) from their point-of-view:frame-of-reference in SpaceTime, each will see the same actual SR 4-Vector \(V = V^\mu\) and its magnitude squared \(V \cdot V = V^\mu V_\mu = (v^4)^2 - v^i v_i = (v^0)^2\) at a given <Event> in SpaceTime. Magnitudes squared can be (+ = temporal) : 0 = null : - = spatial in (+,-,-,+) Special Relativity, due to the {metric signature = (1,3,0)\(^*\}) Lorentzian metric \(\subset\) pseudo-Riemannian metric (non-positive-definite)
There are some paradigm assumptions that need to be cleared up:

Relativistic Physics **IS NOT** the generalization of Classical or Quantum Physics.

Classical & Quantum Physics **ARE** the low-relative-speed \( |v| \ll c \) limiting-case approximation of Relativistic Physics.

This includes (Newtonian) Classical Mechanics and Classical QM (NRQM: meaning the Non-Relativistic Schrödinger QM Equation – it is not fundamental).

The rules of standard QM are just the low-relative-speed approx. \( |v| \ll c \) of RQM rules. Classical EM is for the most part already compatible with Special Relativity. However, Classical EM doesn't include or take into account intrinsic spin, even though spin is a result of SR Poincaré Invariance, not QM.

So far, in all of my research, if there was a way to get a result classically, then there was usually a much simpler way to get the result using tensorial 4-Vectors and SRQM thinking. Likewise, a lot of QM results make much more sense when approached from SRQM (ex: Temporal vs. Spatial relations).

4-Vector formulations are all extremely easy to derive in SRQM and are all relativistically covariant and give invariant results.

Einstein Energy/Mass Eqn: \( P = m_u U \rightarrow \{ E = mc^2 = \gamma m_o c^2 = \gamma E_o : p = m_u = \gamma p_u \} \)

Hamiltonian: \( H = \gamma (P_t U) \) (Relativistic) \( \rightarrow (T + V) = (E_{\text{kinetic}} + E_{\text{potential}}) \) (Classical-limit only, \( |u| \ll c \))

Lagrangian: \( L = - (P_t U)/\gamma \) (Relativistic) \( \rightarrow (T - V) = (E_{\text{kinetic}} - E_{\text{potential}}) \) (Classical-limit only, \( |u| \ll c \))

4-Vector SRQM Interpretation of QM

http://scirealm.org/SRQM.pdf
There are some paradigm assumptions that need to be cleared up:

We will **NOT** be employing the commonly-(mis)used Newtonian classical limits \(\{c \to \infty\}\) and \(\{\hbar \to 0\}\).

Neither of these is a valid physical assumption, for the following reasons:

1. Both \((c)\) and \((\hbar=h/2\pi)\) are **unchanging** Universal Physical Constants and Lorentz Scalar **Invariants**.

   Taking a limit where these change is non-physical. They are **CONSTANT**. Tensor math shows them Invariant.

   Many, many experiments verify that these physical constants have not changed over the lifetime of the universe.

   This is one reason for the 2019 Redefinition of SI Base Units on Fundamental Constants \([c,h,e,k_B,N_A,K_B,\Delta \nu_{\text{Cs}}]\).

2. Photons/waves have energy \((E)\) via momentum \((E=pc)\) & frequency \((E=\hbar \omega)\) : \((\omega = 2\pi \nu)\) \(\{\text{angular [rad/s], circular[cycle/s] , } 2\pi \text{ rad} = 1 \text{ cycle}\}\).

   Let \(E = pc\). If \(c \to \infty\), then \(E \to \infty\). Then Classical EM light rays/waves have infinite energy.

   Let \(E = \hbar \omega = \hbar \nu\). If \(\hbar \to 0\), then \(E \to 0\). Then Classical EM light rays/waves have zero energy.

   Obviously neither of these energy results is true in the Newtonian/Classical limit.

   In Classical EM and Classical Mechanics, LightSpeed \((c)\) remains a large but finite constant.

   Likewise, Dirac’s (Planck-reduced) Constant \((\hbar=h/2\pi)\) remains very small but never becomes zero.

The **correct way** to take the limits is via:

The low-speed non-relativistic limit \(\{ |v| << c \}\), which is a physically-occurring situation.

The Hamilton-Jacobi non-quantum limit \(\{ \hbar |\nabla \cdot p| < (p \cdot p) \}\) or \(\{ |\nabla \cdot k| << (k \cdot k)\}\), which is a physically-occurring situation.
There are some paradigm assumptions that need to be cleared up:

While we are discussing units, note that the following are *ALL* fundamental Relativistic Invariants, meaning also that they are *ALL* Lorentz Scalar Invariants = 4D (0,0)-Tensors:

**Universal Constants (True for all Time·Space)**
- c: LightSpeed in Vacuum Constant [m/s] (maximum speed of causality)  \( E \sim pc \)
- G: Gravitational Constant \([m^2/kg\cdot s^2 = N\cdot m^2/kg^2]\) (GR curvature)  \( E \sim -GMm/r \)
- \( h, \hbar = \hbar/2\pi \): Planck's Constant, Dirac's Constant (Planck Reduced) \([J\cdot s]\) (QM action, waves)  \( E \sim \hbar \omega = \hbar \nu \)
- \( k_B \): Boltzmann's Constant \([J/°K]\) (Stat Mech, temperature, entropy)  \( E \sim k_B T \)
- \( \varepsilon_0 \): Electric Constant \([F/m = C^2\cdot s^2/kg\cdot m^3]\)  
- \( \mu_0 \): Magnetic Constant \([H/m = kg\cdot m/C^2]\)  

**Particle-Dependent Constants** (interestingly, also the “No Hair” scalar parameters of a BlackHole)
- \( m_0 \): Rest Mass \([kg]\)  \( E \sim m_0c^2 \)
- \( q \): EM Charge \([C]\)  \( E \sim -(1/4\pi\varepsilon_0)Qq/r \) or  \( E \sim q\Phi \): ex.  \( E \sim eV \)
- \( s_0 \): Intrinsic Spin \([J\cdot s]\)  \( E \sim (g/2)\mu\cdot b \), with  \( \mu = qs/2m \)

**Fluid-Dependent Scalars**
- \( \rho_{eo} \): MRCF Fluid EnergyDensity (temporal) \([Pa = J/m^3 = N/m^2 = kg/m^2/s^2]\)  
- \( \rho_0 \): MRCF Fluid Pressure (spatial) \([Pa = J/m^3 = N/m^2 = kg/m^2/s^2]\)

\[ T_{\text{perfectfluid}}^{\mu\nu} = (\rho_{eo})V^{\mu}V^{\nu} + (-\rho_0)\delta^{\mu\nu} \]
There are some paradigm assumptions that need to be cleared up:

We will *NOT* be implementing the common (⇒ lazy and extremely misguided) convention of setting physical constants to the value of (dimensionless) unity, often called “Natural Units”, to hide them from equations; nor using mass (m) in place of RestMass (m₀).

Likewise for other components vs Lorentz Scalars with rest-value-naughts (0), like Energy (E) vs RestEnergy (E₀).

One sees this very often in the literature (ex. LightSpeed c→1). The usual excuse cited is “For the sake of brevity” or “For the sake of simplicity”.

Well, the “sake of brevity” forsakes “clarity”. There is nothing physically “natural” about “natural units”.

The *ONLY* situations in which setting constants to unity (1) is practical or advisable is in numerical simulation or mathematical analysis.

When teaching physics, or trying to understand physics: it helps when equations are dimensionally, unit-wise, correct. In other words, the physics technique of “dimensional analysis” is a powerful tool that should not be disdained.

i.e. Brevity only aids speed of computation, Clarity aids understanding.

The situation of using “naught = 0”, for rest-values, such as (m₀) for RestMass and (E₀) for RestEnergy:

is intrinsic to SR, is a very good idea, absolutely adds clarity, identifies Lorentz Scalar Invariants, and will be explained in more detail later.

Essentially, relativistic gamma (γ) pairs with invariant (Lorentz scalar: rest value 0) to make a relativistic component: { m = γm₀ ; E = γE₀ ; p = γp₀ ; ω = γω₀ } Note the multiple equivalent ways that one can write 4-Vectors of SpaceTime (Time:Space) using these rules:

- (1,0)-Tensor V
- (0,1)-Tensor V
- (2,0)-Tensor T
- (0,2)-Tensor T

4-Momentum P = Pµ = (pµ) = (p⁻¹, p⁺) = (mc/E, cE/c, p⁻¹m, u) = -∂[ L_{action,free} ] = -∂[ L_{free,RestLagrangian} ]

= m, U = m, γ(c, u) = γm, (c, u) = m, (c, u) = (mc, mu) = (mc, p) = mc(1, β) = m, γ(1, β) = (mc)T

= (E/c²)U = (E/c²)γ(c, u) = γ(E/c²)(c, u) = (E/c²)(c, u) = (E/c, Eu/c²) = (E/c, p) = (E/c)(1, β) = (E/c, γ)(1, β) = (E/c)T

This notation makes clear what is { relativistically-varying=(frame-dependent) vs. Invariant=(frame-independent) } and { Temporal vs. Spatial } (part 6)

BTW, I prefer the “Particle Physics” Metric-Signature-Convention (+,−,−,−)={temporal:0⁺:−}. {Makes rest values positive, fewer minus signs to deal with}

Show the physical constants and rest naughts (0) in the work. They deserve the respect and you will benefit.

You can always set constants to unity later, when you are doing your numerical simulations.
There are some paradigm assumptions that need to be cleared up:

Some physics books on Electromagnetism (EM) say that:

- Electric field $\mathbf{E}$ and the Magnetic field $\mathbf{B}$ are the "real" physical objects, and that
- EM scalar-potential $\phi$ and the EM 3-vector-potential $\mathbf{A}$ are just "calculational/mathematical" artifacts.

Neither statement is relativistically correct: See Jefimenko's equations & Liénard-Wiechert Potential.

All of these physical EM properties: $\{\mathbf{E}, \mathbf{B}, \phi, \mathbf{A}\}$ are actually just the components of SR tensors, and as such, their values will relativistically vary in different observers' reference-frames.

Given this SR knowledge, and to match our 4-Vector notation, we demote the physical property symbols, (the tensor "components") to their lower-case equivalents $\{\mathbf{e}, \mathbf{b}, \phi, \mathbf{a}\}$.

The truly SR invariant physical objects are:

The 4-Gradient $\partial$, the 4-VectorPotential $\mathbf{A}$, their combination via the exterior (wedge=$\wedge$) product into the Faraday EM 4-Tensor $F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha = \partial \wedge \mathbf{A}$, and their combination via the inner (dot=) product into the Lorenz Gauge 4-Scalar $(\partial \cdot \mathbf{A}) = 0$. Yes, Lorenz, not Lorentz.

Temporal-spatial components of 4-Tensor $F^{\alpha\beta}$: Electric 3-vector field $\mathbf{e} = e^i = e^0$

Spatial-spatial components of 4-Tensor $F^{\alpha\beta}$: Magnetic 3-vector field $\mathbf{b} = b^i = (\frac{1}{2}) \varepsilon_{ijk} F^{kj}$

Temporal component of 4-Vector $\mathbf{A} = A^0$: EM scalar-potential $\phi$

Spatial components of 4-Vector $\mathbf{A} = A^i$: EM 3-vector-potential $\mathbf{a}$

Notice that the Speed-of-Light ($c$) plays a prominent role in the component definitions. Also, QM requires the 4-VectorPotential $\mathbf{A}$ as explanation of the Aharonov-Bohm Effect. The physical measurability of the AB Effect proves the reality of the 4-VectorPotential $\mathbf{A}$. Again, all the lower and higher-rank SR tensors can be built from fundamental 4-Vectors.
There are some paradigm assumptions that need to be cleared up:

A number of QM philosophies make the assertion that particle “properties” do not “exist” until measured. The assertion is based on the QM Heisenberg Uncertainty Principle, and more specifically on quantum non-zero commutation, in which a measurement on one property of a particle alters a different non-commuting property of the same particle. That is an incorrect analysis. Properties define particles: what they do & how they interact with other particles. Particles and their properties “exist” as <events> independently of human intervention or observation. The correct way to analyze this is to understand what a measurement is: an arrangement of some number of particles in a particular manner as to allow an observer to get information:knowledge about one or more of the “subject particle’s” properties. Typically this involves “counting” spacetime <events> and using SR invariant intervals as a basis-of-measurement.

Some properties are indeed non-commuting. This simply means that it is not possible to arrange a set of particles (the measuring device) in such a way as to measure (ie. obtain “complete” information about) both of the “subject particle’s” non-commuting properties at the same spacetime <event>. The measurement arrangement <events> can be done at best sequentially, and the temporal order of these <events> makes a difference in observed results. EPR-Bell, however, allows one to “infer” (due to conservation:continuity laws) properties on a “distant” subject particle by making a measurement on a different “local” {space-like-separated but entangled} particle. This does *not* imply FTL signaling nor non-locality. The (psi-epistemic) measurement just updates local partial-information one already has about particles that interacted/entangled then separated.

So, a better way to think about it is this: The “Measurement→InformationUpdate” of a property does not “exist” until a physical setup <event> is arranged. Non-commuting properties require different physical arrangements in order for the properties to be measured, and the temporally-first measurement alters that particle’s properties in a minimum sort of way, which affects the latter measurement. All observers agree on Causality, the time-order of temporally-separated spacetime <events>. However, individual observers may have different sets of partial information about the same particle(s).

This objective, realist view makes way more sense than the subjective belief that a particle’s actual properties don’t exist until it is “observed”, which is about as unscientific and laughable a statement as I can imagine.

**Relativity is the System-of-Measurement that QM has been looking for: (Ψ) Psi-Epistemic**
There are some paradigm assumptions that need to be cleared up:

**A well-formulated and correctly-used notation is critical for understanding physics**

Unfortunately, there are a number of “sloppy” notations seen in relativistic and quantum physics.

Incorrect: Using $T^{ii}$ as a Trace of 3D tensor $T^{ij}$, or $T^{\mu\mu}$ as a Trace of 4D Tensor $T^{\mu\nu}$.

The Trace operation requires a paired upper-lower index combination (Einstein Sum), which then gets summed over.

Incorrect: Using $T_{ii}$ as the diagonal part of 3-tensor $T^{ij}$, the components: $T_{ii} = \text{Diag}[T_{11}, T_{22}, T_{33}]$.

The Trace operation requires a paired upper-lower index combination (Einstein Sum), which then gets summed over.

Incorrect: Using $T_{\mu\mu}$ as the diagonal part of 4-Tensor $T^{\mu\nu}$, the components: $T_{\mu\mu} = \text{Diag}[T_{00}, T_{11}, T_{22}, T_{33}]$.

The Trace operation requires a paired upper-lower index combination (Einstein Sum), which then gets summed over.

Incorrect: Hiding factors of LightSpeed ($c$) in relativistic equations, ex. $E = m$.

Wrong: $E = m$: Energy [J = kg m²/s²] is *not* identical to mass [kg], not in dimensional units nor in reality.

Correct: $E = mc^2$: Energy is related to mass via the Speed-of-Light ($c$) [m/s], i.e. mass is a type of concentrated energy.

Incorrect: Using $m$ instead of $m_o$ for rest mass; Using $E$ instead of $E_o$ for rest energy.

Correct: $E = mc^2 = \gamma m_o c^2 = \gamma E_o$.

$E$ & $m$, and $p$ are relativistic internal components of 4-Momentum $P = (E/c, p) = (mc, p)$ which vary in different reference-frames.

$E_o$ & $m_o$ are Lorentz Scalar Invariants, the SR Rest Values, which are the same, even in different reference-frames: $P = m_o U = (E_o/c^2) U$. 

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**SRQM: A treatise of SR→QM by John B. Wilson (SciRealm@aol.com)**
There are some paradigm assumptions that need to be cleared up:

Incorrect: Using the same symbol for a tensor-index and a component
The biggest offender in many books for this one is quantum commutation. Unclear because \((i)\) means two different things in the same equation. Correct way: \((i = \sqrt{-1})\) is the imaginary unit; \(\{j, k\}\) are tensor-indices
In general, any equation which uses complex-number math should reserve \((i)\) for the imaginary, not as a tensor-index.

Incorrect: Using the 4-Gradient: Gradient One-Form notation incorrectly
The 4-Gradient is a 4-Vector, a 4D \((1,0)\)-Tensor, uses an upper index, and has a negative spatial component \((-\nabla)\) in \((+,+,-,-)\) SR.
The Gradient (4D) One-Form, its more natural tensor form, a 4D \((0,1)\)-Tensor, uses a lower index in SR.

Incorrect: Mixing styles in 4-Vector naming conventions
There is pretty much universal agreement on the 4-Momentum \(P^\mu = (p^0, p^i) = (E/c, p)\) = \((mc, p)\) = \((E/c, p)\) = \((mc, p)\)
Do not in the same document use 4-Potential \(A = (\varphi, A):\) This is wrong on many levels, inc. dimensional units.
The correct form is 4-Vector Potential \(A = A^\mu = (a^0, a^i) = (\varphi/c, a)\), with \((\varphi)\) the scalar-potential & \((a)\) = the 3-vector-potential

For all SR 4-Vectors, one should use a consistent notation:
The UPPER-CASE SpaceTime (Time·Space) 4-Vector Names match the lower-case spatial 3-vector names
There is a LightSpeed \((c)\) factor in the temporal component to give overall matching dimensional units for the entire 4-Vector
4-Vector components are typically lower-case with a few exceptions, mainly energy \((E)\) vs. energy-density \(\{(e), (\rho_e), (\rho_E)\}\)

SRQM: A treatise of SR→QM by John B. Wilson (SciRealm@aol.com)
Old Paradigm: QM (as I was taught...)

SR and QM as separate theories

Simple GR Axioms:
- Principle of Equivalence
- Invariant Interval Measure
- Tensors describe Physics
- SpaceTime Metric $g^{\mu\nu}$
- $\{c, G\} = \text{physical constants}$

GR limiting-case: $g^{\mu\nu} \rightarrow \eta^{\mu\nu}$
Minkowski “Flat” SpaceTime Metric = (Curvature ~ 0)

Obscure QM Axioms:
- Wave-Particle Duality
- Unitary Evolution
- Operator Formalism
- Hilbert Space Representation
- Principle of Superposition
- Canonical Commutation Relation
- Heisenberg Uncertainty Principle
- Pauli Exclusion Principle (FD-statistics)
- Bose Aggregation Principle (BE-statistics)
- Hermitian Generators
- Correspondence Principle to CM
- Born Probability Interpretation
- $\{\hbar=\hbar/2\pi\} = \text{physical constant}$

SR limiting-case: $|v| \ll c$

QM limiting-case: # particles $N \gg 1$
but not always…
see Bose-Einstein condensate, superfluidity, superconductivity, etc.

This was the QM paradigm that I was taught while in Grad School: everyone trying for Quantum Gravity
Old Paradigm: QM (years later...)  
SR and QM still as separate theories  
QM limiting-case better defined, still no QG

Simple GR Axioms:  
- Principle of Equivalence  
- Invariant Interval Measure  
- Tensors describe Physics  
- SpaceTime Metric $g^{ij}$  
- $\{c, G\} = \text{physical constants}$

Obscure QM Axioms:  
- Wave-Particle Duality  
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- Correspondence Principle to CM  
- Born Probability Interpretation  
- $\{h = \hbar/2\pi\} = \text{physical constant}$

It is known that QM + SR “join nicely” together to form RQM, but problems with RQM + GR...
SRQM Study:
Physical Theories as Venn Diagram
Which regions are empirically real?

**GR:**
General Relativity

**SR:**
Special Relativity

**QM:**
Quantum Mechanics

**CM:**
Classical Mechanics

**RQM:**
Relativistic Quantum Mechanics

**Reality**

**GR limiting-case:** $g_{\mu\nu}^{\infty} \rightarrow \eta_{\mu\nu}$ Minkowski "Flat" SpaceTime = (Curvature ~ 0)

QM physicists think these areas, anything outside of QM, doesn't exist…

Hence the attempt to Quantize Gravity: Unsuccessful for 50+ years…

A new approach is needed: SR→QM (SRQM) fits the facts…

**QM limiting-case:** $\hbar |\nabla \cdot p| << (p\cdot p)$

**SRQM Interpretation**

- Many-Worlds Interpretations
- Non-local interactions
- Instantaneous QM entangled connections
- Instantaneous Physical Wavefunction Collapse
- Spacetime Dimensions >4
- Hidden: Alternate Dimensions
- Super-Symmetry
- String Theory
- Alternate Gravity Theories
- Slews of hypothetical new particles etc.

**Quantum Mysticism**

Basically, lots of stuff for which there's no empirical evidence...
& loads of hype…

Many QM physicists believe that the regions outside of QM don’t exist…
SRQM Interpretation would say that the regions outside of GR probably don’t exist…
SRQM Study: Regimes of Physics
Physical Limit-Cases as Venn Diagram
Which limit-regions use which physics?

- Quantum Gravity? Actual GR?
  - QM limit-case: ħ|∇∙p| << (p∙p)
  - or ψ→Re[ψ] or |∇∙k| << (k∙k)
  - Change by a few quanta has negligible effect on overall state
- GR limit-case: gμν → ημν
  - Minkowski “Flat” SpaceTime = (Curvature ~ 0)

SR → QM (SRQM)
Special Relativity → Relativistic QM

SR limit-case: |v| << c
Non-relativistic velocities

Large gravity fields typically lead to relativistic speeds |v| ~ c

SRQM: A treatise of SR → QM by John B. Wilson (SciRealm@aol.com)
Special Relativity → Quantum Mechanics

Background: Proven Physics

Both General Relativity (GR) and Special Relativity (SR) have passed very stringent tests of multiple varieties. Likewise, Relativistic Quantum Mechanics (RQM) and standard Quantum Mechanics (QM) have passed all tests within their realms of validity:

- Generally micro-scale systems: ex. Single particles, ions, atoms, molecules, electric circuits, atomic-force microscopes, etc.,
- But also a few special macro-scale systems: ex. Bose-Einstein condensates, super-currents, super-fluids, long-distance entanglement, etc.

To-date, however, there is no observational/experimental indication that quantum effects "alter" the fundamentals of either SR or GR. Likewise, there are no known violations, QM or otherwise, of Local Lorentz Invariance (LLI) nor of Local Position/Poincaré Invariance (LPI).

In fact, in all known experiments where both SR/GR and QM are present, QM respects the principles of SR/GR, whereas SR/GR modify the results of QM. All tested quantum-level particles, atoms, isotopes, super-positions, spin-states, etc. follow GR's Universality of Free-Fall & Equivalence Principle and SR's $E = mc^2$ and speed-of-light (c) communication/signaling limit. Meanwhile, quantum-level atomic clocks are used to measure gravitational frequency-shift (gravitational time-dilation) alters atomic=quantum-level timing. Think about that for a moment...

Some might argue that QM modifies the results of SR, such as via non-commuting measurements. However, that is an alteration of CM expectations, not SR expectations. In fact, there is a basic non-zero commutation relation fully within SR: $(\hat{a}, \hat{X}) = \eta \hbar$, which will be derived from purely SR Principles in this treatise. The actual commutation part (Commutator $[a,b]$) is not about (h) or (i), which are just invariant Lorentz Scalar multipliers.

On the other hand, GR Gravity *does* induce changes in quantum interference patterns and hence modifies QM:

- See the COW gravity-induced neutron QM interference experiments, the LIGO & VIRGO & KAGRA gravitational-wave detections via QM interferometry, and now also QM atomic matter-wave gravimeters via QM interferometry, ie. gravitational potential modifies atomic-level (quantum) timing.
- Likewise, SR induces fine-structure splitting of spectral lines of atoms, "quantum" spin, spin magnetic moments, spin-statistics (fermions & bosons), antimatter, QED, Lamb shift, relativistic heavy-atom effects (liquid mercury, yellowish color of gold, lead batteries having higher voltage than classically predicted, heavy noble-gas interactions, relativistic chemistry...), etc. - essentially requiring QM to be RQM to be valid. QM is instead seen to be the limiting-case of RQM for $|v| \ll c$.

Some QM scientists say that quantum entanglement is "non-local", but you still can't send any real messages/signals/information/particles faster than SR's speed-of-light (c). The only "non-local" aspect is the alteration of probability-distributions based on knowledge-changes obtained via measurement. A local measurement can only alter the "partial information" already-known about the probability-distribution of a distant (entangled) system. Getting a Stern-Gerlach "up" here doesn't cause the distant entangled particle to suddenly start moving "down" there. One only knows "now" that it "would" go down "if" the distant experimenter actually performs the measurement.

QM respects the principles of SR/GR, whereas SR/GR modify the results of QM.
Principles/Axioms and Mathematical Consequences of General Relativity (GR):

**Equivalence Principle:** Inertial Motion = Geodesic Motion, Universality of Free-Fall, Mass Equivalency ($m_{inertial} = m_{gravitational}$)

**Relativity Principle:** SpaceTime ($M$) has a Lorentzian $\subset$ pseudo-Riemannian Metric ($g^{\mu\nu}$) & SR:Minkowski Space rules apply locally ($g^{\mu\nu} \rightarrow \eta^{\mu\nu}$) (Minkowski)

**General Covariance Principle:** Tensors describe Physics, General Laws of Physics are independent of arbitrarily chosen Coordinate-Systems

**Invariance Principle:** Invariant Interval Measure comes from Tensor Invariance Properties, 4D SpaceTime ($Time\cdotSpace$) from Invariant Trace $[g^{\mu\nu}] = 4$ (SR)

**Causality Principle:** Minkowski Diagram/Light-Cone gives {Time-Like (+), Light-Like:Null (0), Space-Like (−)} Measures and Causality Conditions

Einstein:Riemann’s Ideas about Matter & Curvature:
Riemann($g$) has 20 independent components $\rightarrow$ too many
Ricci($g$) has 10 independent components = enough to describe/specify a gravitational field = # of Poincaré Invariances (10)

{$c, G$} are Fundamental Physical Constants; so are {$\hbar = h/2\pi$}, but less well-known that this actually comes from SR

To-date, there are no known violations of any of these GR Principles.
GR has passed EVERY observational test to-date, in both weak and strong field regimes.

It is vitally important to keep the mathematics grounded in known physics. There are too many instances of trying to apply top-down, theoretical-only mathematics to physics. (ex. String Theory, SuperSymmetry: no physical evidence to-date; SuperGravity: physically disproven)
Progress in science doesn’t work that way: Nature itself is the arbiter of what math works with physics. Tensor mathematics applies well to known physics {SR and GR}, which have been empirically extremely well-tested in a huge variety of physical situations. Tensors describe physics.

All known experiments to date comply with all of these Principles, including QM and RQM
Old Paradigm: QM Axioms (for comparison)

SR and QM still as separate theories

QM limiting-case better defined, still no QG

Simple GR Axioms:
- Principle of Equivalence
- Invariant Interval Measure
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- SpaceTime Metric \( g^{\mu\nu} \)
- \( \{c,G\} = \text{physical constants} \)

Obscure QM Axioms:
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- Correspondence Principle to CM
- Born Probability Interpretation
- \( \{\hbar=\hbar/2\pi\} = \text{physical constant} \)

It is known that QM + SR “join nicely” together to form RQM, but problems with RQM + GR...

Yet another “would be” fortuitous merging??

50+ years searching for QG with no success...

A fortuitous merging?

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf
*New Paradigm: SRQM or [SR→QM]*

QM derived from SR + a few empirical facts
Simple and fits the data

This new paradigm explains why RQM “miraculously fits” SR, but not necessarily GR.
New Paradigm: SRQM w/ EM
QM, EM, CM derived from SR + a few empirical facts

Derived RQM **Principles**:
Wave-Particle Duality
Unitary Evolution
Operator Formalism
Hilbert Space Representation
Principle of Superposition
Canonical Commutation Relation
Heisenberg Uncertainty Principle
Pauli Exclusion Principle (FD-statistics)
Bose Aggregation Principle (BE-statistics)
Hermitian Generators
{\hbar = \hbar/2\pi} = physical constant

Derived QM **Principles**:
Correspondence Principle to CM
Born Probability Interpretation

This new paradigm explains why RQM “miraculously fits” SR, but not necessarily GR.
Classical SR w/ EM Paradigm (for comparison)

CM & EM derived from

SR + a few empirical facts

Simple GR Axioms:
- Principle of Equivalence
- Invariant Interval Measure
- Tensors describe Physics
- SpaceTime Metric $g^{\mu\nu}$

{c,G} = physical constants

GR limiting-case: $g^{\mu\nu} \rightarrow \eta^{\mu\nu}$
Minkowski “Flat” SpaceTime Metric = (Curvature ~ 0)

The entire classical SR→{EM,CM} structure is based on the limiting-case of quantum effects being negligible.

Notice that only the SR 4-Vector relation: $K=(1/\hbar)P$

is missing from the Classical Interpretation…

All of the SR 4-Vectors, including ($K$ & $\partial$),
are still present in the Classical setting.

$K$ is used in the Relativistic Doppler Effect and EM waves.
$\partial$ is used in the SR Conservation/Continuity Equations,
Maxwell Equations, Hamilton-Jacobi, Lorenz Gauge, etc.

$\partial=(-i)K$ may be somewhat controversial, but it is the equation for complex plane-waves, which are still used in classical EM.

SR limiting-case: $|v| \ll c$

Background Inherent Assumption

QM limiting-case:

$\{ h|\nabla \cdot p| \ll (p\cdot p) \} \text{ or } \{ \psi \rightarrow \text{Re}[\psi] \} \text{ or }$

$\{ |\nabla \cdot k| \ll (k\cdot k) \}$ (doesn’t depend on $h$

Hamilton-Jacobi non-quantum limit
Change by a few quanta has negligible effect on overall state

This (Classical=non-QM) SR→{EM,CM} approximation-paradigm has been working successfully for decades…
The SRQM view:
Each level (range of validity) is a subset of the larger level.

GR
General Relativity

SRQM
Special Relativity → Relativistic QM
GR limiting-case: \( g^{\mu \nu} \rightarrow \eta^{\mu \nu} \) Minkowski “Flat” SpaceTime = (Curvature \( \sim 0 \))

QM
Non-relativistic Quantum Mechanics
SRQM limiting-case: \( |v| \ll c \)

CM
Classical Mechanics
QM limiting-case: \( \hbar |\nabla \cdot p| \ll (p \cdot p) \)
or \( \psi \rightarrow \text{Re}[\psi] \) or \( |\nabla \cdot k| \ll (k \cdot k) \)

Change by a few quanta has negligible effect on overall state

SRQM = New Paradigm:
SRQM View as Venn Diagram
Ranges of Validity

SRQM = A treatise of SR→QM by John B. Wilson (SciRealm@aol.com)
SRQM = New Paradigm:  
SRQM w/ EM View as Venn Diagram  
Ranges of Validity

The SRQM view:  
Each level (range of validity) is a subset of the larger level.

- **GR**: General Relativity  
- **SR**: Special Relativity  
- **SRQM**: Special Relativity → Relativistic QM  
- **QM**: Non-relativistic Quantum Mechanics  
- **CM**: Classical Mechanics

**GR Limiting Case**
- \( g_{\mu\nu} \rightarrow \eta_{\mu\nu} \) Minkowski “Flat” SpaceTime = (Curvature ~ 0)

**SRQM Limiting Case**
- \(|v| << c\)

**QM Limiting Case**
- \( \hbar |\nabla \cdot p| << (p \cdot p) \) or \( \psi \rightarrow \text{Re}[\psi] \) or \( |\nabla \cdot k| << (k \cdot k) \)

- Change by a few quanta has negligible effect on overall state

**SRQM: A treatise of SR → QM by John B. Wilson (SciRealm@aol.com)**
SRQMQ: SR language beautifully expressed with Physical 4-Vectors

Newton’s laws of classical physics are greatly simplified by the use of physical 3-vector notation, which converts 3 separate space components, which may be different (relative) in various coordinate systems, into a single invariant object: a 3D vector, with an invariant 3D magnitude (but not 4D invariant). The basis-values of these components can differ in certain ways, via Galilean transforms, yet still refer to the same overall 3-vector object.

SR is able to expand the concept of mathematical vectors into the Physical 4-Vector, which combines both (time) and (space) components into a single (Time:Space) object: These 4-Vectors are elements of Minkowski 4D SR SpaceTime. They have Lorentzian (relative) components but invariant 4D Magnitudes. There is a Speed-of-Light factor (c) in the temporal component to make the dimensional units match. ex. \( R = (ct, \mathbf{r}) \); overall dimensional units of [length] = SI Unit [m]

This also allows the 4-Vector name to match up with the 3-vector name.

In this presentation: {Temporal,0,1,2}(,1,2) metric signature, giving \( \mathbf{A} \cdot \mathbf{A} = A^\mu \eta_{\mu\nu} A^\nu = (a_0)^2 - \mathbf{a} \cdot \mathbf{a} = (a_0)^2 \)

4-Vectors will use Upper-Case Letters, ex. \( \mathbf{A} \); 3-vectors will use lower-case letters, ex. \( \mathbf{a} \); I always put the (c) dimensional factor in the temporal component. Vectors of both types will be in bold font; components and scalars in normal font and usually lower-case. 4-Vector name will match with 3-vector name.

Tensor form will usually be normal font with tensor indicies; \( \mathbf{a} \cdot \mathbf{a} = (a_1)^2 + (a_2)^2 + (a_3)^2 = |\mathbf{a}|^2 \)

4-Vector = 4D (1,0)-Tensor
\( \mathbf{A} = A^\mu = (a^1, a^2, a^3) \) Cartesian/Rectangular 4D basis
\( (a^1, a^2, a^3) \) Polar/Cylindrical 4D basis
\( (a^0, a^1, a^2, a^3) \) Spherical 4D basis

Classical scalar (1D)
\( 1\text{-time } t \)
\( \mathbf{c} \)
\( \mathbf{r} \)
\( \mathbf{r} \rightarrow (x,y,z) \) 3-position

Classical 3-vector (3D)
\( \mathbf{A} = A^\mu = (a, \mathbf{v}) \) 3-Position
Classical 4-Vector (4D)
\( \mathbf{R} \) 4-Position
\( R^\mu = (r^0, \mathbf{r}) \)
\( (r^0, r^1, r^2, r^3) \) 4-Position

\( \mathbf{V} \cdot \mathbf{V} = (V \epsilon \mathbf{v})^2 - \mathbf{v} \cdot \mathbf{v} = (\mathbf{v}^\mu)^2 \)

\( \text{Trace} [\mathbf{T}] = \eta_{\mu\nu} T^\mu_\nu = T^\mu_\mu \)

3-Dimensional Lorentz Scalar
\( \mathbf{V} \cdot \mathbf{V} = V^\mu \eta_{\mu\nu} V^\nu = 0 \)

3-Dimensional Lorentz Scalar
\( \mathbf{V} \cdot \mathbf{V} = 0 \) Lorentz Scalar

The triangle/wedge \( \Delta \) (3 sides) represents splitting the components into a scalar and 3-vector.

I style classical 3D objects this way (by a triangle/wedge \( \Delta \)) to emphasize that they are actually just the separated components of SR 4-Vectors.

SR-4 Vector
\( \mathbf{R} \) 4-Position
\( R^\mu = (r^0, \mathbf{r}) \)
\( (r^0, r^1, r^2, r^3) \) 4-Position

\( \mathbf{V} \cdot \mathbf{V} = (V \epsilon \mathbf{v})^2 - \mathbf{v} \cdot \mathbf{v} = (\mathbf{v}^\mu)^2 \)

\( \text{Trace} [\mathbf{T}] = \eta_{\mu\nu} T^\mu_\nu = T^{\mu}_{\mu} \)

3-Dimensional Lorentz Scalar
\( \mathbf{V} \cdot \mathbf{V} = V^\mu \eta_{\mu\nu} V^\nu = 0 \)

3-Dimensional Lorentz Scalar
\( \mathbf{V} \cdot \mathbf{V} = 0 \) Lorentz Scalar Invariant
SR 4-Vectors are primitive elements of Minkowski SpaceTime 4D → (1+3)D

We want to be clear, however, that SR 4-Vectors are NOT generalizations of Classical or Quantum 3-vectors.

SR 4-Vectors are the primitive elements of Minkowski SpaceTime \((\text{Time} \cdot \text{Space})\) = 4D → (1+3)D, which incorporate both:

- A temporal scalar element
- A spatial 3-vector element

Temporal and Spatial components can intermix via a Lorentz Boost Transform. Temporals and Spatials are metrically distinct, but can mix in SR. 4-Vector \(A = A^\mu = (a^0, a^1, a^2, a^3)\) with component scalar \((a^0) \rightarrow (a')\) & component 3-vector \((a^i = a) \rightarrow (a'^i, a'^2, a'^3)\)

It is the {Classical (Newtonian) or Quantum} 3-vector \((a)\) which is a limiting-case approximation of the spatial part of SR 4-Vector \((A)\) for \(|v| \ll c\). i.e. The energy \((E)\) and 3-momentum \((p)\) as “separate” entities occurs only in the low-velocity limit \(|v| \ll c\) of the Lorentz Boost Transform. They are actually part of a single 4D entity: the 4-Momentum \(P = (E/c, p)\) with the components: \(\{\text{temporal energy} (E), \text{spatial 3-momentum} (p)\}\), dependent on a frame-of-reference, while the overall 4-Vector \(P\) is invariant. Likewise with \(\{\text{time} (t), \text{space 3-position} (r)\}\) in the 4-Position \(R = (ct, r)\).

SR is 4D Minkowskian; obeys Lorentz<–Poincaré Invariance.

CM is 3D Euclidean; obeys Galilean Invariance.

\[
\text{4-Momentum} \quad P_{CM} = (E/c, p) \\
\text{4-Position} \quad R_{CM} = (ct, r)
\]

\(E\) can intermix with \((p)\) via a Lorentz Boost Transformation \(\Lambda^\nu \rightarrow \Lambda'^\nu\).

Spatial components can intermix via a Lorentz Rotation Transform \(\Lambda^\nu \rightarrow R^\nu\).

\((t)\) can intermix with \((r)\) via a Lorentz Boost Transformation \(\Lambda^\nu \rightarrow \Lambda'^\nu\).

\(\eta = (0,1,1,1)\) is the Lorentz Metric, as the square of the 4-Dimensional Minkowskian Spacetime Interval.
SR 4-Vectors & Lorentz Scalars

Rest Values ("naughts") are Lorentz Scalars

\[ \mathbf{A} = (a^0 a^i - \mathbf{a} \cdot \mathbf{a}) = (a^0_o)^2, \] where \( a^0_o \) is the rest-value, the value of the temporal coordinate when the spatial coordinate is zero \( (a \rightarrow 0) \). The "rest-values" of several physical properties are all Lorentz scalars.

\[ \mathbf{P} = (m \mathbf{c}, \mathbf{p}) \]
\[ \mathbf{P} \cdot \mathbf{P} = (mc)^2 - \mathbf{p} \cdot \mathbf{p} \]
\( \mathbf{p} \cdot \mathbf{p} \) and \( \mathbf{K} \cdot \mathbf{K} \) are Lorentz Scalars. We can choose a frame that may simplify the expressions.

Choose a frame in which the spatial component is zero. This is known as the "rest-frame" of the 4-Vector. It is not moving spatially = Stationary.

\[ \mathbf{P} = (m_0 \mathbf{U}) = (E_0/c)^2 \mathbf{U} \]
\[ \mathbf{K} = (\omega/c)^2 \mathbf{U} \]

This leads to simple relations between 4-Vectors.

\[ \mathbf{P} = (m_0 \mathbf{U}) = (E_0/c)^2 \mathbf{U} \]
\[ \mathbf{K} = (\omega/c)^2 \mathbf{U} \]

And gives nice Scalar Product relations between 4-Vectors as well.

\[ \mathbf{P} \cdot \mathbf{K} = (m_0 \omega_o) \rightarrow \mathbf{P} = (m_0 \mathbf{c}/\omega_o) \mathbf{K} = (E_0/\omega_o) \mathbf{K} \rightarrow \mathbf{P} = (\text{scalar invariant}) \mathbf{K} \]

This property of SR equations is a very good reason to use the "naught" convention for specifying the difference between restMass \((m_o)\) and RestAngularFrequency \((\omega)\). They are Invariant Lorentz Scalars by construction.

SR 4-Tensor

\[ (2,0)-\text{Tensor } \mathbf{T}^\mu_\nu \]
\[ (1,1)-\text{Tensor } \mathbf{T}^\nu_\mu \text{ or } \mathbf{T}^0_1 \]
\[ (0,2)-\text{Tensor } \mathbf{T}^\mu_\nu \]

SR 4-Vector

\[ (1,0)-\text{Tensor } \mathbf{V} = (V^\nu) \]
\[ 4-\text{Vector Co-Vector OneForm} \]
\[ (1,0)-\text{Tensor } V_\nu = (\nu^\nu) \]

SR 4-Scalar

\[ (0,0)-\text{Tensor } S \text{ or } S_0 \]

Lorentz Scalar

\[ \text{Trace}[\mathbf{T}^\nu_\nu] = \eta_{\nu\nu} \mathbf{T}^\nu_\nu = \mathbf{T}^\nu_\nu = \mathbf{T} \]
\[ \mathbf{V} \cdot \mathbf{V} = V^\nu V_\nu = (\nu^\nu)^2 - \mathbf{v} \cdot \mathbf{v} = (\nu^\nu)^2 \]

Lorentz Scalar Invariant

The resulting simpler expressions then give the "rest values", indicated by \( (o) \). The resulting properties then give the "rest values", indicated by \( (o) \).

This is known as the "rest-frame" of the 4-Vector. It is not moving spatially = Stationary.
SRQM Study: Manifest Invariance
Invariant SR 4-Vector Relations

Consider a particle at a SpaceTime (Time·Space) <Event> that has properties described by 4-Vectors \( A \) and \( B \):

One possible relationship is that the two 4-Vectors are related by a Lorentz 4-Scalar \( S \): ex. \( B = (S)A \).
How can one determine this? Answer: Make an experiment that empirically measures the tensor invariant \( [ B \cdot C / A \cdot C ] \).
If \( B = (S)A \) then \( B \cdot C = (S)(A \cdot C) \), giving \( S = [ B \cdot C / A \cdot C ] \)

if \( C=A \), then \( S = [ B \cdot A / A \cdot A ] \) This basically a standard vector projection.
if \( C=other \), Invariant result mediated by another 4-Vector \( C \), always possible.

Run the experiment many times. If you always get the same result for \( S \), then it is likely that the relationship is true, and thus invariant.

Example: Measure \( (S_p) = [ P \cdot U / U \cdot U ] \) for a given particle type.
Repeated measurement always give \( (S_p) = m_o \).
This makes sense because we know \( [ P \cdot U ] = \gamma(E - p \cdot u) = E_o \) and \( [ U \cdot U ] = c^2 \).
Thus, 4-Momentum \( P = (E_o/c^2)U = (m_oU) = (m_o)^2 \) 4-Velocity \( U \).

Example: Measure \( (S_\omega) = [ K \cdot U / U \cdot U ] \) for a given particle type.
Repeated measurement always give \( (S_\omega) = (\omega/c^2) \).
This makes sense because we know \( [ K \cdot U ] = \gamma(\omega - K \cdot u) = \omega_o \) and \( [ U \cdot U ] = c^2 \).
Thus, 4-WaveVector \( K = (\omega/c^2)U = (\omega_o/c^2)^2 \) 4-Velocity \( U \).

Since \( P \) and \( K \) are both related to \( U \), this would also mean that the 4-Momentum \( P \) is related to the 4-WaveVector \( K \) in a particular Lorentz Invariant manner for each given particle type... a major hint for later...

SR 4-Tensor
(2,0)-Tensor \( T^{uv} \)
(1,1)-Tensor \( T^{uu} \), or \( T^u \)
(0,2)-Tensor \( T_{uv} \)

SR 4-Vector
(1,0)-Tensor \( V^u = V = (v^0, v) \)
SR 4-CoVector:OneForm
(0,1)-Tensor \( V_\nu = (v^\nu, -) \)

SR 4-Scalar
(0,0)-Tensor \( S_\omega \) or \( \omega_o \)
Lorentz Scalar

Trace\( [T^u] = \eta_{uu}T^u = T \)
\( V \cdot V = V^\nu \eta_{\nu\mu}V^\mu = (v^0)^2 - V^\nu V^\nu = (v^0)^2 \)
Lorentz Scalar Invariant
SR 4-Vectors & Lorentz Scalars

Frame-Invariant Equations

SRQM Diagramming Method

4-Vectors are 4D (1,0)-Tensors, Lorentz 4-Scalars are 4D (0,0)-Tensors, CoVectors are 4D (0,1)-Tensors, (m,n)-Tensors have (m) # upper-indices and (n) # lower-indices

Any equation which employs only Tensors, such as those with only 4-Vectors and Lorentz 4-Scalars, is automatically Frame-Invariant, or coordinate-frame-independent. One’s frame-of-reference plays no role in the form of the overall equations. This is also known as being “Manifestly-Invariant”, not showing inner components. This is exactly what Einstein meant by his postulate: “The laws of physics should have the same form for all inertial observers”. Use of the RestFrame-naught (0) helps show this. It is seen when the spatial part (v) of a magnitude can be set to zero (= at-rest). The temporal part (v0) would then equal the rest value (v0).

The components (v0, v1, v2, v3) of the 4-Vector V can relativistically vary depending on the observer and their choice of coordinate system, but the 4-Vector V = Vμ itself is invariant. Equations using only 4-Tensors, 4-Vectors, and Lorentz 4-Scalars are true for all inertial observers. The SRQM Diagramming Method makes this easy to see in a visual format, and will be used throughout this treatise.

The following examples are SR Time·Space frame-invariant equations:

\[ U \cdot U = (c)^2 \]
\[ U = \gamma(c,u) \]
\[ P = (mc,p) = (E/c,p) = m_o U = (E_o/c^2)U \]
\[ K = (\omega/c,k) = (\omega/c,\omega \hat{n}/\text{phase}) = (\omega/c^2)U \]
\[ P \cdot U = E_o \]

The SRQM Diagram Form, on the right, has all of the info of the Equation Form, but shows overall relationships and symmetries among the 4-Vectors much more clearly.

Blue: Temporal components
Red: Spatial components
Purple: Mixed Time·Space components
Some SR Mathematical Tools
Definitions, Approximations, Misc.

\[ \beta = \frac{v}{c} ; \beta = |\beta|; \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}} = \frac{1}{\sqrt{1 - \beta \cdot \beta}}; \]

- dimensionless Velocity Beta Factor
- dimensionless Lorentz Relativistic Gamma Factor

\[
(1+x)^\gamma \sim (1 + nx + O[x^2]) \text{ for } |x| \ll 1 \text{ Approximation used for } SR\rightarrow \text{Classical limiting-cases, typically used on the Relativistic Gamma } \gamma
\]

Lorentz Transformation \( \Lambda^\mu_\nu = \frac{\partial X^\mu}{\partial X'\nu} = \delta[\nu][X'] - \text{a relativistic frame-shift, such as a Rotation or Velocity-Boost.} \)

It transforms a 4-Vector in the following way:

\[
(\gamma^0, \gamma^1, \gamma^2, \gamma^3) = \Lambda^\mu_\nu \cdot (v_0, v_1, v_2, v_3)
\]

Some SR Mathematical Tools

**SR: Minkowski Metric**

\[
\partial[R] = \frac{\partial[R']}{\partial R} = \eta^{\mu \nu} = V^{\mu \nu} + H^{\mu \nu}
\]

Diag\([+1,-1,-1,-1] = \text{Diag}[1,-1,1,3] = \text{Diag}[1,-\delta^0_1]\]

\{\text{in Cartesian form } \eta = \text{"Particle Physics" Convention}

\[
\{(\eta^{\mu \nu}) = \frac{1}{|\eta^{\nu \nu}|} : \eta^{\mu \nu} = \delta^{\mu \nu} \}
\]

**SR: Lorentz Transform**

\[
\partial[R^\mu_v] = \frac{\partial[R'^\mu_v]}{\partial R^\mu_v} = \Lambda^\mu_\nu
\]

\[
\Lambda^\nu_\nu = (\Lambda^{-1})^\mu_\nu : \Lambda^\mu_\alpha (\Lambda^{-1})^\alpha_v = \eta^{\nu \nu} = \delta^\mu_v
\]

\[
\eta_{\nu \mu} \Lambda^\mu_\alpha \Lambda^\alpha_\beta = \eta_{\nu \beta}
\]

\[
\text{Det}[\Lambda^\mu_\nu] = \pm 1
\]

\[
\Lambda^\mu_\nu \Lambda^\nu_\mu = 4 = \Lambda^\nu_\nu \Lambda^\nu_\nu
\]

**Trace**: \(T^\mu_v = \frac{1}{\eta^{\nu \nu}} |\eta^{\nu \nu}| + \frac{1}{\eta^{\mu \nu}} |\eta^{\mu \nu}| = 4\)

**SR 4-Tensor**

- \((0,0)\)-Tensor \(T^{\mu \nu}_0\)
- \((1,0)\)-Tensor \(V^\mu = (v^\mu_0)\)
- \((0,1)\)-Tensor \(V^\nu = (v^\nu\_0)\)

**SR 4-Vector**

- \((2,0)\)-Tensor \(T^{\mu \nu}_2\)
- \((1,1)\)-Tensor \(\Lambda^{\mu \nu}_0\), or \(T^{\mu \nu}_2\)
- \((0,2)\)-Tensor \(T^{\mu \nu}_0\)

**SR 4-Scalar**

- \((0,0)\)-Tensor \(S\) or \(\Lambda_0\)

**Lorentz Scalar**

\[
V\cdot V = \eta^{\nu \nu} V^\nu = (v^\nu_0)^2
\]

\[
V\cdot V = \eta^{\nu \nu} V^\nu = (v^\mu_0)^2
\]

- Lorentz Scalar Invariant

**Space-Time**

\[
\partial [R] = \partial [R^\mu_v] = 4 \text{ Dimension}
\]

**Definitions, Approximations, Misc.**

- Standard Notation
- Einstein Summation Convention
- Lorentz Relativistic Gamma Factor
- Dimensionless Lorentz Boost Transformation
- Typical Lorentz Boost Transformation
- Lorentz Transformation \( \Lambda^\mu_\nu \rightarrow B^\mu_\nu \) for a linear-velocity frame-shift (x,t)-Boost in the \( \hat{x} \)-direction.

**Symmetric Mixed 4-Tensor**

\[
A^\mu_\nu \cdot A^{\nu \mu} = \Lambda^\mu_\nu \cdot A^{\nu \mu}
\]

**General Time-Space Boost**

\[
\begin{bmatrix}
 t' \\
 x' \\
 y' \\
 z'
\end{bmatrix} =
\begin{bmatrix}
 
 t \\
 x \\
 y \\
 z
\end{bmatrix} \begin{bmatrix}
 1 & 0 & 0 & 0 \\
 -\beta \gamma & 1 & 0 & 0 \\
 -\gamma & 0 & 1 & 0 \\
 0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
 t' \\
 x' \\
 y' \\
 z'
\end{bmatrix} =
\begin{bmatrix}
 
 t \\
 x \\
 y \\
 z
\end{bmatrix} \begin{bmatrix}
 \cosh[w] & -\sinh[w] & 0 & 0 \\
 -\sinh[w] & \cosh[w] & 0 & 0 \\
 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 1
\end{bmatrix}
\]

**Trig Functions**

\[
\tan^{-1} \left( \frac{\sinh[w]}{\cosh[w]} \right)
\]

**Trace**

\[
3 \to \text{Trace} = \eta^{\mu \nu} \Lambda^\mu_\nu = 4
\]

**Dual**

\[
\Lambda^\mu_\nu \Lambda^\nu_\mu = 4 = \eta_{\nu \nu} \eta^{\nu \nu}
\]

**Definitions**

- Lorentz Boost Transformation
- Lorentz Scalar
- Lorentz Transformation
- Lorentz Relativistic Gamma Factor
- Lorentz Transform Type
- Lorentz Relativistic Invariant
SRQM Study: Ordering of Time•Space <Events>

Temporal Causality vs. Spatial Topology
Simultaneity vs. Stationarity

Venn Diagram

Properties of Minkowski: SR SpaceTime <Events>

Time-Like Ordering of...

Time-Like Separated <Events>

Causal: Invariant = Absolute Temporal Order (A→B→C)
{ ProperTime (t = τ) for | clock at-rest | }
{ Time Dilation (t = γL = γτ) for ...⇒ moving clock |→ }
All observers agree on temporal order of time-separated events.
Note: although temporal event separation may be ≠ [Time-Dilated]⇒

Time-Like Invariant Interval
ΔR∙ΔR= (cΔt)² - Δr∙Δr → + (cΔt)²

Time-Like Separated <Events>

Non-Topological: Relative → Relativity of Stationarity (A→?→B)
Stationarity: (only if in reference-frame with Same-Place occurrence)
("no motion" for stationary particle/worldline, "motion" in all other frames)
Any 2 time-separated <events> may occur in any spatial order = frame-dependent

Space-Like Ordering of...

Space-Like Separated <Events>

Non-Causal: Relative → Relativity of Simultaneity (A→/?→B)
Simultaneity: (only if in reference-frame with Same-Time occurrence)
("no wait" for simultaneous events, "wait" in all other reference frames)
Any 2 space-separated <events> may occur in any temporal order = frame-dependent

Space-Like Invariant Interval
ΔR∙ΔR= (cΔt)² - Δr∙Δr → -(|Δr|²)

Light-Like (Null) Separated <Events>

Causal: Invariant = Absolute Temporal Order (A→B→C)
All observers agree on temporal order of light-separated events,
and on the invariant TimeSpace <Event> interval measurement.
All observers measure invariant LightSpeed (c) in their own frames.

U U=c²

Light-Like Invariant Interval
ΔR∙ΔR= (cΔt)² - Δr∙Δr → 0

Light-Like (Null) Separated <Events>

Topological: Invariant = Absolute Spatial Order (A→B→C)
All observers agree on spatial order/topology of light-separated events,
and on the invariant TimeSpace <Event> interval measurement.
All observers measure invariant LightSpeed (c) in their own frames.

Space-Like Separated <Events>

Non-Causal: Relative → Relativity of Simultaneity (A→/?→B)
Simultaneity: (only if in reference-frame with Same-Time occurrence)
("no wait" for simultaneous events, "wait" in all other reference frames)
Any 2 space-separated <events> may occur in any temporal order = frame-dependent

Space-Like Invariant Interval
ΔR∙ΔR= (cΔt)² - Δr∙Δr → -(|Δr|²)

Trace[Tμν]= ημνTμν = Tμν = T
V V= VμVν = [(Vμ)² - VνVν] = (Vν)²
= Lorentz Scalar Invariant

SR 4-Tensor (2,0)-Tensor TⅩⅨ
(1,1)-Tensor TⅩⅨ or TⅨ
SR 4-Vector (0,1)-Tensor VⅨ = (Vμ, Vν)
SR 4-CoVector: OneForm (0,1)-Tensor VⅨ = (Vμ, Vν)
SR 4-Scalar (0,0)-Tensor Scalar S or S0
4-Displacement (between <events>)
ΔR=ΔRμ= (cΔt, Δr) = RμR1 (finite)
dR=dRμ= (cdt, dr)

John B. Wilson
SciRealm.org
http://scirealm.org/SRQM.pdf
SRQM Diagram: 

The Basis of Classical SR Physics
Special Relativity via 4-Vectors

Focus on a few of the main SR Physical 4-Vectors:

4-Position
\[ \mathbf{R} = \mathbf{R}(\mathbf{r}) = (r^0, r^1, r^2, r^3) = (ct, \mathbf{x}) \]

4-Velocity
\[ \mathbf{U} = \mathbf{U}(\mathbf{r}) = (u^0, u^1, u^2, u^3) = \gamma(c, \mathbf{u}) \]

4-Gradient
\[ \nabla = \frac{\partial}{\partial x^\mu}, \quad \frac{\partial}{\partial r^\mu} = \nabla \cdot \mathbf{R}, \quad \frac{\partial}{\partial r^\mu} \mathbf{R} = \nabla \mathbf{R} \]

4-Displacement
\[ \Delta \mathbf{R} = (\Delta t, \Delta \mathbf{r}) \]

4-Position
\[ \mathbf{R} = (ct, \mathbf{x}) \]

Note that these main 4-Vectors are all mathematical functions of the 4-Position \( \mathbf{R} = R^\mu \):

4-Displacement
\[ d\mathbf{R} = d[R^\mu] \]

4-Gradient
\[ \partial = \partial/c, \mathbf{V} = \partial \mathbf{R} \]

4-Velocity
\[ \mathbf{U} = d\mathbf{R}/dt = R^\mu \partial R^\mu \]

These 4-Vectors give some of the main classical results of Special Relativity, including 4D SR Minkowski Space concepts like:

The 'flat' Minkowski Metric, \( \text{SpaceTime} \ (\text{Time} \cdot \text{Space}) \) Dimension = 4, Lorentz Transformations, \( \text{<Events}> \), Invariant Interval Measure, Minkowski Diagrams, The Light-Cone, etc.

Relativity: Time Dilation (\( \leftarrow \) clock moving \( \rightarrow \)), Length Contraction (\( \leftarrow \) ruler moving \( \rightarrow \)), Proper Time (\( \leftarrow \) clock at rest \( \rightarrow \)), Proper Length (\( \leftarrow \) ruler at rest \( \rightarrow \))

Temporal 1D Ordering of: (Time-like \( \text{<event> separations} \)) \( \Rightarrow \) Causality is Absolute, (Space-like \( \text{<event> separations} \)) \( \Rightarrow \) Simultaneity is Relative

Spatial 3D Ordering of: (Time-like \( \text{<event> separations} \)) \( \Rightarrow \) Stationarity is Relative, (Space-like \( \text{<event> separations} \)) \( \Rightarrow \) Topology is Absolute

Use of the Lorentz Scalar Product to make Lorentz Invariants, Continuity Equations, etc.

The Invariant Speed-of-Light (c), Invariant Proper Measurements \( \text{(Time} \cdot \text{Space}) \), Relative Components of 4-Vectors = Lorentz Covariance, Invariant SR Wave Equations, via d'Alembertian (Lorentz Scalar Product of 4-Gradient with itself), leads to a 4-WaveVector \( \mathbf{K} \) solution.
The Basis of Classical SR Physics

The Basis of all Classical SR Physics is in the SR Minkowski Metric of “Flat” SpaceTime $\eta^{\mu\nu} = \delta^{\mu\nu} \left[ R \right] = \eta \left[ R \right]$, which is generated from the 4-Gredient $\delta = \partial$ and 4-Position $R = R^\nu$ and determines the invariant measurement interval $R^\nu R_{\nu} = R^\mu \eta_{\mu\nu} R^\nu$ between $<Events>$. This Minkowski Metric $\eta^{\mu\nu}$ provides the relations between the 4-Vectors of SR: 4-Position $R = R^\nu$, 4-Gredient $\delta = \partial$, 4-Velocity $U = U^\nu$.

The Tensor Invariants of these 4-Vectors give the:

Invariant Interval Measures $\rightarrow$ Causality: Topology, from $R^\nu R_{\nu}$. Invariant (Magnitude) LightSpeed (c), from $U^\nu U_{\nu}$. Invariant d’Alembertian Wave Equation & 4-WaveVector $K$, from $\partial^\nu \partial_{\nu}$. The relation between 4-Gredient $\delta$ and 4-Position $R$ gives the Dimension of SpaceTime $= (4 \, , \, )$, the Minkowski Metric $( \eta^{\mu\nu} )$, and the Lorentz Transformations $( \Lambda^\nu_\mu \, )$.

The relation between 4-Gredient $\delta$ and 4-Velocity $U$ gives the invariant ProperTime Derivative $( d/\!\!\!d\tau )$, Rearranging gives the invariant ProperTime Differential $( d\tau )$, which gives relativistic $\leftrightarrow$ (Time Dilation)$\rightarrow$ (temporal) & $\rightarrow$ (Length Contraction)$\rightarrow$ (spatial). The ProperTime Derivative $d\tau/\!\!\!d\tau$ acting on 4-Position $R$ gives 4-Velocity $U$ acting on the SpaceTime Dimension Lorentz Scalar gives the Continuity of 4-Velocity Flow.

The relation between 4-Displacement $\Delta R$ and 4-Velocity $U$ gives Relativity of Simultaneity: Stationarity.

One of the most important properties is the Tensor Invariant Lorentz Scalar Product $( dot \rightarrow )$, provided by the lowered-index form of the Minkowski Metric $\eta_{\mu\nu}$.

SRQM Diagram: The Basis of Classical SR Physics Special Relativity via 4-Vectors

SR is a theory about the relations between 4D Time-Space $<Events>$, i.e. how their intervals are "measured".

SRQM Diagram

From here, each object will be examined in turn...
The 4-Differential (Displacement) is just the infinitesimal version of the finite 4-Displacement. Lorentz and Poincaré transformations are invariant under both the endpoints of a 4-Displacement.

The 4-Position is a specific type of 4-Displacement, which is used to give consistent dimensional units across all SR 4-Vector components. I emphasize the 4-Position as the most fundamental 4-Vectors of SR.

The 4-Position \( \mathbf{R} = (c, r) \) relates time to space via the fundamental invariant physical constant \((c)\): the Speed-of-Light = “c” (c)eerly; \((c)\)erlits”, which is used to give consistent dimensional units across all SR 4-Vector components.

The 4-Position is a specific type of 4-Displacement, for which one of the endpoints is the origin \(0\).

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The Invariant Interval is the Lorentz Scalar Product of the 4-Position \( R = \langle ct, x, y, z \rangle \) and 4-Displacement \( \Delta R = \langle c\Delta t, \Delta x, \Delta y, \Delta z \rangle \), which is the time-displacement of an event measured by a clock at-rest, and the space-displacement measured by a ruler at-rest.

This leads to the concept of the (Minkowski Diagram) Light-Cone. The differential form regardless of differing reference frames. This leads to the idea meaning that all observers must agree on their magnitudes, regardless of direction. The 4D SpaceTime Intervals are Invariant:

\[ \Delta r^2 = (c\Delta t)^2 - \Delta x^2 - \Delta y^2 - \Delta z^2. \]

SRQM Diagram:

- **Absolute/Invariant (Ordering of Events)**
  - Causality is **temporal**
  - Topology is **spatial**

- **Absolute/Invariant (Ordering of Events)**
  - Causality is **temporal**
  - Topology is **spatial**

The 4D SpaceTime Intervals are Invariant, meaning that all observers must agree on their magnitudes, regardless of differing reference frames. This leads to the concept of ProperTime \( \Delta t = \Delta t_o \), which is the time-displacement measured by a clock at-rest, and ProperLength \( L = |\Delta x| \), which is the space-displacement measured by a ruler at-rest. This also leads to the various Causality Conditions of SR, and the concept of the (Minkowski Diagram) Light-Cone. The differential form \( d\mathbf{R} \cdot d\mathbf{R} \) is apparently also still true in the curved spacetime of GR.
A Tensor Study

The only way there can be more dimensions is if there is another SpaceTime direction. This Tensor Invariant Lorentz Scalar relation gives the dimension of SpaceTime.

\[ \delta \cdot R = 4 \]

: The 4-Divergence SpaceTime Dimension Relation

\[ \delta \cdot R = \delta \cdot R_R = (\partial / \partial c) \cdot (c R) \]

4-Gradient

SpaceTime Dimension

This Tensor Invariant Lorentz Scalar relation gives the dimension of SpaceTime. The only way there can more dimensions is if there is another SpaceTime direction available. 4-Divergence \((\partial J) = 0\)

All empirical evidence to-date indicates that there are only the 4 known dimensions: 1 temporal \((t)\) : measured in SI units \(= [s]\), with \((c)\) : measured in SI units \(= [m]\) 3 spatial \((x, y, z)\) : measured in SI units \(= [m]\)

These are the 4 components that appear in:

4-Position

\[ R = (c, t, r) \rightarrow (c, t, x, y, z) \]

measured in SI units

SR 4-Tensor

\((2,0)\)-Tensor \(T^{iv} = \frac{\partial \delta}{\partial c, \cdot V} = (\partial / \partial c, \cdot V) \]

SR 4-Vector

\((1,0)\)-Tensor \(V = V = (v, \cdot v)\)

SR 4-CoVector: OneForm

\((0,1)\)-Tensor \(V_i = \left( v_i, \cdot v_i \right)\)

SR 4-Scalar

\((0,0)\)-Tensor \(S_0 \) or \( S_0 \) Lorentz Scalar

4D Kronecker Delta = 4D Identity

\[ \delta^{\mu \nu} = \delta^{\nu \mu} = \delta_{\mu \nu} = \delta_{\nu \mu} = \{ 1 \text{ if } \mu = \nu, \text{ else } 0 \} = \text{Diag}[1, 1, 1, 1] \]

4D Invariant Magnitude LightSpeed

\[ U \cdot U = c^2 \]

SRQM Diagram:

The Basis of Classical SR Physics

SpaceTime Dimension = 4D = \((1+3)D\)
A Tensor Study

**Physics**

\[ T^{\mu \nu} = \eta_{\mu \nu} + H^{\mu \nu} \]

**Derivation:**

\[ \frac{\partial}{\partial t} T^{\mu \nu} = \gamma \frac{\partial}{\partial \tau} T^{\mu \nu} \]

\( \eta_{\mu \nu} = \text{Diag}[1, -1, -1, -1] \)

\( \delta \rightarrow \text{SR} 4-\text{Vector} \)

The Minkowski Metric \( \eta^{\mu \nu} \) is the fundamental SR (2,0)-Tensor, which shows how intervals are "measured" in SR Time-Space. It is itself the low-mass = (Curvature = 0) limiting-case of the more general GR Metric \( g^{\mu \nu} \). It can be divided into temporal and spatial parts. The Minkowski Metric can be used to raise/lower indices on other SR tensors, i.e., 4-Vectors. The GR Metric \( g^{\mu \nu} \) is used in strong gravity.

**SR Diagram:**

The Basis of Classical SR Physics

The Minkowski Metric \( \eta^{\mu \nu} \), Measurement

\[ \delta^{\mu \nu} = \delta_{\mu \nu} = \delta_{\mu \nu} = \{ \text{if } \mu = \nu, \text{ else } 0 \} = \text{Diag}[1, 1, 1, 1] \]

4D Kronecker Delta = 4D Identity
The Basis of Classical SR Physics

The Lorentz-Transform \( \partial_v[R^\mu] = \partial R^\mu / \partial R^v = \Lambda^\mu_v \)

SRQM Diagram:

- Invariant Tr\( [\Lambda^\mu_v] \) → \(-∞, -(4), -(2), 0, +2, +4, +∞\)
- Trace identifies CPT Symmetry in the Lorentz Transform

4-Vector SRQM Interpretation of QM

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http://scirealm.org/SRQM.pdf
The Lorentz transformation can also be derived empirically. In order to achieve this, it's necessary to write down coordinate transformations that include experimentally testable parameters. For instance, let there be given a single "preferred" inertial frame \((t,x,y,z)\) in which the speed of light is constant, isotropic, and independent of the velocity of the source.

It is also assumed that Einstein synchronization and synchronization by slow clock transport are equivalent in this frame. Then assume another frame \((t',x',y',z')\) in relative motion, in which clocks and rods have the same internal constitution as in the preferred frame.

The following relations, however, are left undefined:

- \(a(v)\): differences in time measurements,
- \(b(v)\): differences in measured longitudinal lengths,
- \(d(v)\): differences in measured transverse lengths,
- \(\varepsilon(v)\): depends on the clock synchronization procedure in the moving frame.

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The value of LightSpeed \((c)\) was 1st empirically measured by Ole Rømer to be finite using the timing of Jovian moon eclipses. His estimate was 1 Earth-Orbit-Diameter/22 minutes, about 75% of the actual value of \((c)\).
SRQM Diagram:
The Basis of Classical SR Physics

Time • Space

Dimension = 4D = (1+3)D

SR : Minkowski

Time • Space is 4D

(1+3)D → QM

A Tensor Study of Physical 4-Vectors

SR 4-Vector

\[ V = (v_x, v_y, v_z, c) \]

SR 4-Scalar

\[ S = (s_0, s_1, s_2, s_3) \]

SR 4-Position

\[ R = (x, y, z, c) \]

SR 4-Displacement

\[ \Delta R = (\Delta x, \Delta y, \Delta z, \Delta c) \]

SR 4-Divergence

\[ \nabla \cdot R = \frac{\partial c}{\partial t} \]

SR 4-Gradient

\[ \nabla \nabla \cdot R = \nabla \nabla \cdot \nabla R \]

SR 4-Vector

\[ \mathbf{V} = (V_x, V_y, V_z, c) \]

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SR 4-Displacement

\[ \Delta R = (\Delta x, \Delta y, \Delta z, \Delta c) \]

SR 4-Divergence

\[ \nabla \cdot R = \frac{\partial c}{\partial t} \]

SR 4-Gradient

\[ \nabla \nabla \cdot R = \nabla \nabla \cdot \nabla R \]
The Tensor Invariant Lorentz Scalar Product (LSP) is the SR 4D (Dot=·) Product. It is used to make Invariant Lorentz Scalars from two 4-Vectors. A tensor A·B is set to zero, giving the rest-frame invariant value, as well as the Invariant Magnitude of a single 4-Vector. If the 4-Vector is temporal, it is used to make Invariant Lorentz Scalars from two 4-Vectors.

\[
\eta_{\mu\nu} = \begin{cases} 
\delta_{\mu\nu} & \text{in Cartesian basis} \\
\text{as basis vectors} \ A = \delta^\mu_\nu \rightarrow \delta_{\mu\nu} (\text{Cartesian basis}) 
\end{cases}
\]

\(\eta_{\mu\nu}\) is itself just the lowered-index form of the SR Minkowski Metric (\(\eta^{\mu\nu}\)), with individual components \(\eta_{\mu\nu} = 1\) if \(\eta^{\mu\nu}\) else 0. In Cartesian basis, this gives \(\eta_{\mu\nu} = \eta^{\mu\nu}\) (Cartesian basis).

The LSP is used in just about every relation between any two interesting 4-Vectors. It also gives the Minkowski Metric with \(\eta^{\mu\nu}\) or \(T^{\mu\nu}\) as basis vectors.

4-Displacement
\[
\Delta R = (c\Delta t, \Delta r) \rightarrow \text{Diag}[1,-1,-1,-1] = \text{Diag}[1,-1,-1,-1]
\]

4-Position
\[
R = (ct, r) \rightarrow \text{Diag}[1,-1,-1,-1] \quad \text{Minkowski Metric}
\]

Relativity of Simultaneity: Stationarity
\[
U \cdot \Delta R = \gamma(c,u) \cdot (c\Delta t, \Delta r)
\]
\[
= c^2\Delta t - u \cdot \Delta r = c^2\Delta t - c^2\Delta r
\]

Continuity of 4-Velocity Flow
\[
\partial U = 0
\]

ProperTime Derivative
\[
U \cdot \partial = \gamma(c,u) \cdot (\partial/c, -V) = \gamma(c,u) \cdot (\partial/c, -V)
\]
\[
= \gamma(d/dt, \partial/c) + (dy/dt, -d/dz) + (dy/dt, \partial/c) = d/dt, d/dz
\]

ProperTime Differential
\[
dr = (1/\gamma)dt
\]

4-Momentum
\[
P = (mc, p) = (E/c, p)
\]

4-Momentum
\[
P \cdot P = (mc, p) \cdot (mc, p) = (E/c)^2 + p^2
\]

4-Covariant Components
\[
\lambda_{\mu\nu} = 
\]

4-Component As Basis Vectors
\[
\lambda^{\mu\nu} = \begin{cases} 
\delta_{\mu\nu} & \text{in Cartesian basis} \\
\text{as basis vectors} \ A = \delta^\mu_\nu \rightarrow \delta_{\mu\nu} (\text{Cartesian basis}) 
\end{cases}
\]

The Basis of Classical SR Physics
Lorentz Scalar (Dot) Product \((\eta_{\mu\nu} = \cdot)\)

SRQM Diagram
4-Vector SRQM Interpretation

The Basis of Classical SR Physics

4-Velocity U, SpaceTime <Event> Motion

4-UnitTemporal

4-Velocity U is the ProperTime Derivative (dU/dt) of the 4-Position R or of the 4-Displacement ΔR.

It is the SR 4-Vector that describes the motion of <Events> through SpaceTime.

(a) For an un-accelerated observer, the 4-Velocity U is a constant along the WorldLine at all points.

(b) For an accelerated observer, the 4-Velocity U is still tangent to the WorldLine at each point, but changes direction as the WorldLine bends thru SpaceTime.

The 4-UnitTemporal Ṭ & 4-Velocity U are unlike most of the other SR 4-Vectors. They have 3 independent components, whereas the others usually have 4. This is due to the constraints placed by the LSP Tensor Invariants. Ṭ Ṭ = +1 & U-U = c^2 have constant magnitudes, giving the Speed-of-Light (c) in SpaceTime.

Components:
3 independent + 0 independent → 3 independent + 1 independent = 4 independent

They also usually have the Relativistic Gamma factor (γ) exposed in component form, whereas most of the other temporal 4-Vectors have it absorbed into the Lorentz 4-Scalar factor that goes into their components.

4-UnitTemporal Ṭ = γ(1, β)
4-Velocity U = γ(c, u)
m_c E_c /c^2
4-Momentum P = (mc, p) = (E_c, p) = m_c U

4-Displacement ΔR = (cΔt, Δr) = (cΔt)^2-dr = (cdt, dr) = (cΔt)Δr

Invariant Interval
R-R = (ct)^2-r^2=(ct)^2

Relativity of Simultaneity: Stationary
U-ΔR = γ(c, u)-(cΔt, Δr) = γ(c^2Δt - u·Δr) = c^2Δt, c^2Δr

ProperTime Derivative
U-∂A = γ(∂(dx/dt)∂t, + (dy/dt)∂y, + (dz/dt)∂z)
= γ d/dt = d/dτ

ProperTime Differential dt = (1/γ)dt = Time Dilation

SR Diagram

4-Vector SRQM

Relativistic Gamma γ = 1/√[1 - β·β], β = u/c

SR 4-Vector
(1,0)-Tensor V = V = (v_0, v)
4-CoVector:OneForm (0,1)-Tensor V = (v_0, v)
SR 4-Scalar (0,0)-Tensor S or Sc Lorentz Scalar

Trace[V] = γ, V = V = [(v_0)^2] = (v^2)^2 = Lorentz Scalar Invariant
SRQM Diagram: The Basis of Classical SR Physics

4-Velocity U, SpaceTime <Event> Motion

4-Velocity U = γ(c, u) = (γc, γu) = (U ∙ ∂τ/R = γ(∂t U)/R = dr/dτ = (dR/dτ) = γ(dR/dτ) = γ(c, u) = U = c ∙ ΛγV

Components: 0 independent, 4-"Rest" UnitTemporal Tτ = (1, 0)

Components: 3 independent + 0 independent → 3 independent + 1 independent = 4 independent

4-UnitTemporal τ = γ(1, β)

4-Velocity U = γ(c, u)

Components: 3 independent + 0 independent → 3 independent + 1 independent = 4 independent

4-Velocity U = γ(c, u)

Components: 4 independent

4-Momentum P = m0 E/c²

4-Displacement ΔR = (cΔt, Δr) dr = (cdt, dr) = c(Δt² - dr²)

Invariant Interval R ∙ R = (ct)² - r²

ΔR ∙ ΔR = (cΔt)² - Δr²

4-Position R = (ct, r)

4-Momentum SRQM Diagram

γd/dτ [...] = γ(∂t U)/∂τ

4-Gradient δ(Δ, Δ) = ∂/∂τ

Continuity of 4-Velocity Flow δ(Δt, 0)

ProperTime Derivative δ(Δ, Δ) = ∂/∂τ

Relativistic Gamma γ = 1/√(1 - β · β), β = u/c

SR 4-Tensor (2,0)-Tensor τ(0,0)-Tensor V(0,0)

SR 4-Vector (1,0)-Tensor V = V = (v, V)

SR 4-CoVector: OneForm (0,1)-Tensor V₁ = (v₁, V₁)

SR 4-Scalar (0,0)-Tensor S or S₀

Relativistic Gamma γ = 1/√(1 - β · β), β = u/c

SRQM Diagram: The Basis of Classical SR Physics

4-Velocity U, SpaceTime <Event> Motion

SRQM Diagram: The Basis of Classical SR Physics

4-Velocity U, SpaceTime <Event> Motion

SRQM Diagram: The Basis of Classical SR Physics

4-Velocity U, SpaceTime <Event> Motion
The Lorentz Scalar Product of the 4-Velocity leads to the Invariant |Magnitude| Speed-of-Light (c), one of the fundamental SR physical constants of physics.

\[ \text{Invariance: } U \cdot U = (E/c)^2 - (\mathbf{p} \cdot \mathbf{v})^2 = (E/c)^2 - (\mathbf{p} \cdot \mathbf{v}/c)^2 = (E/c)^2 \]

\[ \text{Components: } U = (E/c)^2, \mathbf{p} = (E/c)^2 - (\mathbf{p} \cdot \mathbf{v}/c)^2 = (E/c)^2(1 - v^2) \]

This fundamental constant Lorentz Invariant (c) provides an extra constraint on the components of 4-Velocity U, making it have only 3 independent components (u). This allows one to move new 4-Vectors related to 4-Velocity by multiplying other by Lorentz scalars. (Lorentz Scalar)^4(4-Velocity) = (New 4-Vector)

An interesting thing to note is that all <events> move at the Speed-of-Light (c) in 4D Space-Time. Massive at-rest particles simply travel at (c) temporally as \( U = (c, 0) \), while massless photons move at (c) spatially also (in vacuum) as \( U = (c, -c, -c, -c) \). Magnitude \[ |U| = \sqrt{\eta \cdot U} \]

If (c) was not a constant, but varied somehow, then all 4-Vectors made from the 4-Velocity would have more than 4 independent components, which is not observed. It seems a very strong, compelling argument against variable light-speed theories.

A Tensor Study of Physical 4-Vectors

**SRQM Diagram:**

The Basis of Classical SR Physics

4-Velocity |Magnitude| = Invariant Speed-of-Light (c)
SRQM Diagram: The Basis of Classical SR Physics

**Relativity of Simultaneity:Time-Delay**
(Simultaneity ↔ Same-Time Occurrence ↔ Δt=0)
(Time-Delay ↔ Different-Time Occurrence ↔ Δt≠0)

If Lorentz Scalar (\(U \cdot \Delta X = 0 = c^2 \Delta t\)), then the ProperTime displacement (\(\Delta t\)) is zero, and the \(<\text{Event}>\)'s separation (\(\Delta X = X_2 - X_1\)) is orthogonal to the worldline at \(U\).

\(<\text{Event}>\)'s X₁ and X₂ are therefore simultaneous (\(\Delta t = 0\)) for the observer on this worldline at \(U\).

Examining the equation we get Boost Frame \(\gamma(c^2 \Delta t - u \cdot \Delta x) = 0\).

The coordinate time difference between the events is \((\Delta t = u \cdot \Delta x/c^2)\).

The condition for simultaneity in an alternate reference frame (moving at 3-velocity \(u\) wrt. the worldline \(U\)) is \(\Delta t = 0\), which implies \((u \cdot \Delta x) = 0\).

This condition can be met by:

- \((u) = 0\), the alternate observer is not moving wrt. the events, i.e. is on worldline \(U\) or on a worldline parallel to \(U\).
- \((\Delta x) = 0\), the events are at the same spatial location (co-local).
- \((u \cdot \Delta x = 0 = u \cdot |\Delta x| \cos(\theta))\), the alternate observer's motion is perpendicular (orthogonal, \(\theta=90^\circ\)) to the spatial separation \(\Delta x\) of the events in that frame.

If none of these conditions is met, then the events will not be simultaneous in the alternate reference-frame.

This can be shown on a Minkowski Diagram.

---

**SR 4-Tensor**

\[ (2,0)-T^{\mu \nu} \quad (0,1)-T^{\nu}_{\mu} \quad (1,1)-T^{\nu \nu} \quad (0,2)-T^{\mu \nu} \]

---

**SR 4-Vector**

\[ V = (v^0, v^1, v^2, v^3) \]

---

**SR 4-Scalar**

\[ S \quad S_0 \]

---

**SRQM Diagram**

Realizing that Simultaneity (no-delay) is not an invariant concept was a breakthrough that lead Einstein to Special Relativity (SR).

---

**Temporal Ordering:**

- Simultaneity (=same time occurrence) is Relative
- Stationarity (=same place occurrence) is Relative

**Spatial Ordering:**

- Causality is Absolute ↔ Invariant Proper Time
- Topology is Absolute ↔ Invariant Proper Length
Let $<\text{Event}>$'s $X_1$ and $X_2$ be local ($\Delta x' = 0$) for the observer on worldline at $U$.

This has equation $(\mathbf{U} \cdot \Delta \mathbf{x}) = \gamma (c \Delta t - \mathbf{u} \cdot \Delta \mathbf{x}) = c^2 \Delta \tau \neq 0$.

To be stationary/motionless in the Rest-Frame is $\Delta x' = 0$.

In a Boosted Frame, using $\gamma = \sqrt{1 - v^2}$ and $\Delta t = \gamma \Delta t$:

$\gamma (c^2 \Delta t - \mathbf{u} \cdot \Delta \mathbf{x}) = c^2 \Delta \tau$

$\gamma c^2 \Delta t - c^2 \Delta t = \mathbf{u} \cdot \Delta \mathbf{x}$

$(\gamma^2 - 1)c^2 \Delta t = \mathbf{u} \cdot \Delta \mathbf{x}$

$(\gamma^2 - 1)c^2 \Delta t = \mathbf{u} \cdot \Delta \mathbf{x}$

$\mathbf{u} \cdot \Delta \mathbf{x} = \mathbf{u} \cdot \Delta \mathbf{x}$

$\gamma^2 - 1 = \mathbf{n} \cdot \Delta \mathbf{x}$

If $\mathbf{u} = 0$, then $\gamma = 1$, then $\mathbf{n} \cdot \Delta \mathbf{x} = 0$, which is RestFrame.

If $\mathbf{u} > 0$, then $\gamma > 1$, then $\mathbf{n} \cdot \Delta \mathbf{x} \neq 0$

If $\mathbf{u} = 0$, then $\gamma = 1$, then $\mathbf{n} \cdot \Delta \mathbf{x} = 0$, which is RestFrame.

If $\mathbf{u} > 0$, then $\gamma > 1$, then $\mathbf{n} \cdot \Delta \mathbf{x} \neq 0$

So, in any Boosted Frame, $\Delta x' \neq 0$.

If this condition is met, then the events will not be stationary in the alternate reference-frame.

This can be shown on a Minkowski Diagram.
The derivation shows that the ProperTime Derivative \( (d/d\tau) \) is an Invariant Lorentz Scalar. Therefore, all observers must agree on its magnitude, regardless of their frame-of-reference. \( (d/d\tau) \) is used to derive some of the physical 4-Vectors: 4-Velocity, 4-Acceleration, 4-Force, 4-Torque, etc.

### SRQM Diagram: The Basis of Classical SR Physics

The ProperTime Derivative \( (d/d\tau) = (U \cdot \partial) \)

- **4-Vector SRQM Interpretation of QM**
- **SR 4-Vector** (2,0)-Tensor \( T^{\mu\nu} \) or \( T^{\nu\mu} \) (1,1)-Tensor \( T^\gamma \), or \( T^\gamma \)
- **SR 4-CoVector:** OneForm
- **SR 4-Scalar** (0,0)-Tensor \( S_\alpha \) or \( S_\alpha \) Lorentz Scalar

The ProperTime Derivative can be used to make new tensors from existing tensors, as it is taking the derivative of an existing tensor by a Lorentz Scalar: the ProperTime Derivative of a Lorentz scalar is used to derive some of the physical 4-Vectors: 4-Velocity, 4-Acceleration, 4-Force, 4-Torque, etc.

**SRQM Diagram**

- **SR 4-Vector**: \( \mathbf{V} = (c, u) \)
- **SR 4-Scalar**: Lorentz Scalar
- **SR 4-CoVector**: OneForm
- **SR 4-Tensor**: (2,0)-Tensor \( T^{\mu\nu} \) or \( T^{\nu\mu} \) (1,1)-Tensor \( T^\gamma \)

**Lorentz Factor = Relativistic gamma**

\[ \gamma = \frac{1}{\sqrt{1 - \beta \cdot \beta}} \]

\[ \beta = u/c \]
SRQM Diagram:
The Basis of Classical SR Physics
ProperTime Derivative in SR:
4-Tensors, 4-Vectors, and 4-Scalars

As one can see from the list, the ProperTime Derivative gives the tensors that are the change in status of the tensor that ProperTime Derivative acts on. It can also act on Scalar Values to give deep SR results.
SRQM Diagram: The Basis of Classical SR Physics

ProperTime Differential $(d\tau) \rightarrow$

Time Dilation & Length Contraction

There are several ways to derive Time Dilation.

The ProperTime Derivative

$$U \cdot \partial = \gamma(c, u) \cdot (\delta/c, -\mathbf{V}) = \gamma(\partial + \mathbf{u} \cdot \mathbf{V}) = \frac{\gamma}{d\tau} = \frac{d}{d\tau}$$

ProperTime Differential (Lorentz 4-Scalar): $d\tau = (1/\gamma)d\tau$

$\mathbf{dR} \cdot \mathbf{dR} = (c^2 d\tau^2)$

4-Differential $\mathbf{dR} = (c d\tau, d\mathbf{r})$

4-Position $\mathbf{R} = (c \tau, \mathbf{r})$

4-Displacement $\Delta \mathbf{R} = (c \mathbf{\Delta} \tau, \mathbf{\Delta} \mathbf{r}) = (c^2 \mathbf{\Delta} \tau, \mathbf{\Delta} \mathbf{r})$

$$\mathbf{dR} \cdot \mathbf{dR} = (c^2 d\tau^2 - \mathbf{d} \cdot \mathbf{d} \mathbf{r} = (c^2 d\tau^2)$$

The coordinate time $\Delta \tau$ measured by an observer is "dilated", compared to the ProperTime as measured by a clock moving with the object. This has the effect that moving objects appear to age more slowly than at-rest objects. The effect is reciprocal as well. Since velocity is relative, each observer will see the other as ageing more slowly, similarly to the effect that each will appear smaller to the other when seen at a distance.

Now multiply both sides by the moving-frame speed $v = |\mathbf{v}|$

$\mathbf{v} \Delta \tau = \gamma \mathbf{v} \Delta \mathbf{r}$

$\mathbf{v} \Delta \mathbf{r} = \text{distance } \mathbf{L}_0$ the moving clock travels wrt. frame, which is a proper (fixed-to-frame) displacement length.

$L_0 = \gamma \mathbf{L}$

$L = (1/\gamma) L_0$: →Length Contraction← {in spatial $\mathbf{v}$ direction}

SR 4-Tensor $(1,1)$-Tensors $T^{\mu \nu}$, or $T_\mu$ ⎮ SR 4-Vector $(0,1)$-Tensors $V^{\mu} = (c \tau, \mathbf{v})$, or $S_{\mu \nu} = (0,1)$-Tensor $V_\mu = (\mathbf{v}, 0)$}

SR 4-Scalar $(0,0)$-Tensor $S_\mu$ or $S_{\mu \nu}$ Lorentz Scalar

Relativity: Time Dilation (←) clock moving (→), Length Contraction (→) ruler moving (←), Proper Time (←) clock at rest (→), Proper Length (←) ruler at rest (→)

Invariants: Proper Time (←) Length Contraction (→)}
SR Diagram: 

The Basis of Classical SR Physics

4-Gradient \( \partial, \) SR 4-Vector Function: Operator

4-Gradient

The 4-Gradient plays a major role in advanced physics, showing how SR waves are formed, creating the Hamilton-Jacobi equations, the Euler-Lagrange equations, Conservation Equations (\( \partial \cdot \partial = 0 \)), Maxwell's Equations, the Lorenz Gauge, the d’Alembertian, etc. It gives the Dimension of SpaceTime, the Minkowski Metric, and the Lorenz Transformations.

In QM, it provides the Schrödinger relations.

The 4-Gradient is fundamental in connecting SR to QM.
The Lorentz Scalar Product Invariant of the 4-Gradient gives the d'Alembertian Wave Equation, describing SR wave motion. It is seen, for example, in the SR Maxwell Equation for EM light waves.

SR is the "natural" 4D arena for the description of waves, using the d'Alembertian

\[ \partial \vec{\phi} = (\partial/c)^2 \vec{\nabla} \cdot \vec{\nabla} \vec{\phi} = (\partial/c)^2 \vec{\nabla}^2 \vec{\phi} \]

Trace[\(T^\nu\nu\)] = \(\eta_{\nu\nu}T^\nu\nu = T^\nu\nu = T\)

\(V \cdot V = V^\nu\eta_{\nu\nu}V^\nu = (V^\nu)^2 - (V^\nu c)^2 = \text{Lorentz Scalar} \)
Conservation of Charge, continuity eqn:
These are local continuity equations which basically
Invariant Scalar equation, a continuity equation.
All of the Physical Conservation Laws are in the form of
Conservation of 4-Velocity Flow
\[ \partial \cdot U = 0 \]
This leads to all the SR Conservation Laws.

A Tensor Study
Physics
SR 4-Vector
\( V^\mu \) = \( \gamma (c, u) \cdot \gamma (c, \Delta \tau, \Delta r) \)
Conservation of 4-Velocity Flow
\[ \partial \cdot U = 0 \]

SRQM Diagram:
The Basis of Classical SR Physics
Continuity of 4-Velocity Flow (\( \partial \cdot U = 0 \))

\( \partial \cdot U = \frac{d}{dt}(\partial \cdot U) = \frac{d}{dt}(\partial \cdot U) = \frac{d}{dt}(\partial \cdot U) = \frac{d}{dt}(\partial \cdot U) = \frac{d}{dt}(\partial \cdot U) = 0 \)
Conservation of the 4-Velocity Flow (4-Velocity Flow-Field)

All of the Physical Conservation Laws are in the form of a 4-Divergence (\( \partial \cdot U = 0 \)), which is a Lorentz Invariant Scalar equation, a continuity equation.

These are local continuity equations which basically say that the temporal change of a quantity is balanced by the flow of that quantity in-to or out-of a local region.

Conservation of Charge, continuity eqn:
\[ \rho \cdot \partial \cdot U = \rho \cdot \partial \cdot J = (\partial \cdot \rho + \nabla \cdot J) = 0 \]
A Tensor Study of Physical 4-Vectors

Now focus on a few more of the main SR 4-Vectors.

4-Position \( R = (ct, r) \)

\[ \Rightarrow \text{<Event> Location} \]

4-Velocity \( U^\mu \)

\[ \Rightarrow \text{<Event> Motion} \]

4-Gradient \( \partial^\mu \)

\[ \Rightarrow \text{<Event> Alteration} \]

4-Momentum \( P^\mu \)

\[ \Rightarrow \text{<Event> Substantiation (particle: mass } m_0 \text{)} \]

4-WaveVector \( K^\mu \)

\[ \Rightarrow \text{<Event> Substantiation (wave: phase oscillation } \omega_0 \text{)} \]

4-CurrentDensity: ChargeFlux \( J^\mu \)

\[ \Rightarrow \text{<Event> Substantiation (charge Q or q)} \]

4-(Dust)NumberFlux \( N^\mu \)

\[ \Rightarrow \text{<Event> Substantiation (dust: number N or } n_0 \text{)} \]

These 4-Vectors give more of the main classical results of Special Relativity, including SR concepts like:

- SR Particles and Waves, Matter-Wave Dispersion
- Einstein’s \( E = mc^2 = \gamma m_0 c^2 = \gamma E_0 \), Rest Mass, Rest Energy
- Conservation of Charge (Q), Conservation of Particle Number (N), Continuity Equations

SR 4-Vector \( (2,0) \)-Tensor \( T^{\mu \nu} \)

SR 4-Vector \( (1,1) \)-Tensor \( T^\mu_\nu \) or \( T^\nu_\mu \)

SR 4-Vector \( (0,2) \)-Tensor \( T^{\mu \nu \rho} \)

SR 4-Scalar \( \gamma \)

\[ \Rightarrow \text{4-Position: } R^\mu = (ct, r) \]

\[ \Rightarrow \text{4-CurrentDensity: } J^\mu = \text{ChargeFlux} \]

\[ \Rightarrow \text{4-Gradient: } \partial^\mu = \frac{\partial}{\partial x, y, z} \]

\[ \Rightarrow \text{4-Momentum: } P^\mu = (mc, p) = (mc, mu) \]

\[ \Rightarrow \text{4-WaveVector: } K^\mu = (\omega c, k) = (\omega c, \omega \vec{n}/\text{phase}) \]

\[ \Rightarrow \text{4-CurrentDensity: } J^\mu = (pc, j) = (pc, \gamma (c, u)) \]

\[ \Rightarrow \text{4-Gradient: } \partial^\mu = \frac{\partial}{\partial x, y, z} \]

\[ \Rightarrow \text{4-Momentum: } P^\mu = (mc, p) = (mc, mu) \]

\[ \Rightarrow \text{4-WaveVector: } K^\mu = (\omega c, k) = (\omega c, \omega \vec{n}/\text{phase}) \]

\[ \Rightarrow \text{4-CurrentDensity: } J^\mu = (pc, j) = (pc, \gamma (c, u)) \]

4-Vector SRQM Interpretation of QM

Motion of various Lorentz Scalars leads to the “Substantiation” of the various physical SR 4-Vectors from the 4-Velocity.

Lorentz 4-Scalar \( a_0 \)

4-Vector \( A = A^\mu = (a, a) = a_\mu U = a_\mu (c, u) = a(c, u) = (ac, au) \)

Trace[\( T^\mu_\nu \)] = \( \eta^\mu_\nu T^\mu_\nu = T^\mu_\nu = T \)

\[ V \cdot V = V^\mu \eta^\mu_\nu V^\nu = (\vec{v}^2)^2 = (\vec{v} \cdot \vec{v})^2 = \text{Lorentz Scalar Invariant} \]
SRQM Diagram:
The Basis of Classical SR Physics
4-Momentum, Einstein's $E = mc^2$, $p = mu$

4-Position $R = (ct, r)$
4-Gradient $\partial = (\partial/c, \mathbf{V})$
4-Velocity $\mathbf{U} = \gamma(c, \mathbf{u})$

4-Momentum $\mathbf{P} = (E/c, \mathbf{P}) = m_o \mathbf{U} = \gamma m_o(c, \mathbf{u}) = m(c, \mathbf{u}) = (mc, m\mathbf{u})$

Temporal part: $E = \gamma E_o = \gamma m_o c^2 = mc^2$
{energy}

$p = E\mathbf{u}/c^2 = \gamma E_o \mathbf{u}/c^2 = \gamma m_o \mathbf{u} = \mathbf{mu}$
{3-momentum}

Spatial part: $\mathbf{P} = (E/c, \mathbf{P}) = -\partial [S_{\text{action,free}}] = - (\partial/c, \nabla)[S_{\text{action,free}}]$
4-TotalMomentum $\mathbf{P}_T = (E_T/c = H/c, \mathbf{P}_T) = -\partial [S_{\text{action}}] = - (\partial/c, \nabla)[S_{\text{action}}]$

Temporal part: $E = -\partial [S_{\text{action,free}}] : E_T = H = -\partial [S_{\text{action}}]$
{energy}

Spatial part: $\mathbf{P}_T = +\nabla [S_{\text{action,free}}] : \mathbf{p}_T = +\nabla [S_{\text{action}}]$
{3-momentum}

Einstein's $E = \gamma E_o = \gamma m_o c^2 = mc^2$
Energy/Mass
Equivalence
Rest Mass

Hamilton-Jacobi Equation
$\mathbf{P} = \partial [S_{\text{action,free}}]$ (SR 4-Vector Equation of Motion)

4-Momentum $\mathbf{P} = (E/c, \mathbf{P}) = (mc, \mathbf{P}) = m_o \mathbf{U}$

ProperTime Derivative $\mathbf{U} = \mathbf{U}/d\tau = d\mathbf{V}/d\tau$

Einstein's $E = \gamma E_o = \gamma m_o c^2 = mc^2$
Energy/Mass
Equivalence
Rest Mass

Tracer $[\mathbf{T}^\gamma] = \eta_{\mu\nu} T^\nu = \mathbf{T}^\gamma = \mathbf{T}$
$\mathbf{V} \cdot \mathbf{V} = \mathbf{V}^\gamma \eta_{\nu\nu} V^\nu = (\mathbf{V}^\gamma)^2 - \mathbf{V} \cdot \mathbf{V} = (\mathbf{V}^\gamma)^2$
Lorentz Scalar

Relativistic Energy $E$: Mass($m_o$) vs Invariant Rest Energy ($E_o$): Mass($m_o$)

$E = \gamma E_o = \gamma m_o c^2 = mc^2$
SRQM Diagram: The Basis of Classical SR Physics

4-WaveVector, $|u * v_{\text{phase}}| = |v_{\text{group}} * v_{\text{phase}}| = c^2$

4-Position $R=(ct,r)$

4-Gradient $\partial=(\partial/c,-\nabla)$

4-Velocity $U = \gamma(c,u)$

4-WaveVector $K = (\omega/c, k) = (\omega/c^2)U = \gamma(\omega,c^2)(c,u)$

$= (\omega/c\omega_{\text{phase}}/v_{\text{phase}})$

4-Displacement $\Delta R = (\Delta ct, \Delta r)$

$\Delta R = (ct,dr)$

4-WaveVector $K = (\omega/c, k) = -\partial[\Phi_{\text{phase,free}}] = -(\partial/c, -\nabla)[\Phi_{\text{phase,free}}]$

4-Total WaveVector $K_T = (\omega/c, k_T) = -\partial[\Phi_{\text{phase}}] = -(\partial/c, -\nabla)[\Phi_{\text{phase}}]$

Temporal part: $\omega = \gamma \omega_0$

Spatial part: $k = \gamma(\omega/c^2)U = (\omega/c^2)U = \omega_{\text{phase}}/v_{\text{phase}}$

{3-wavevector}

$|u * v_{\text{phase}}| = c^2 = |v_{\text{group}} * v_{\text{phase}}|$

4-WaveVector $K = (\omega/c, k) = -\partial[\Phi_{\text{phase,free}}] = -(\partial/c, -\nabla)[\Phi_{\text{phase,free}}]$

4-WaveVector $K_T = (\omega/c, k_T) = -\partial[\Phi_{\text{phase}}] = -(\partial/c, -\nabla)[\Phi_{\text{phase}}]$

Temporal part: $\omega = -\partial[\Phi_{\text{phase,free}}]: \omega_T = -\partial[\Phi_{\text{phase}}]$

Spatial part: $k = +\nabla[\Phi_{\text{phase,free}}]: k_T = +\nabla[\Phi_{\text{phase}}]$

{3-wavevector}

Wave Frequency $\omega_{\text{group}} * v_{\text{phase}} = c^2$

4-Velocity $U = \gamma(c,u) = dR/dt$

SRQM Diagram:

Wave Speed $V_{\text{wave}} = \gamma(c,u)$

Point Time Derivative $U \partial = -\gamma d/dt = d/dt$

4-WaveVector $K = (\omega/c, k) = (\omega/c^2)U = (\omega/c^2)\gamma d/dt$

(1,0)-Tensor $V = (v^x, 0)$

Spatial part:

$\omega = \gamma \omega_0$

$\omega^2 = (|k|)^2 + (\omega)^2$ : Matter-Wave Dispersion Relation

Relativistic AngFreq($\omega$) vs Invariant Rest AngFreq($\omega_0$)

$\omega = \gamma \omega_0$

$\omega^2 = |k|^2 + \omega_0^2$

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SRQM Diagram: The Basis of Classical SR Physics

4-CurrentDensity, Charge Conservation

4-Position \( R=(ct,r) \)
4-Gradient \( \partial=(\partial/c,-\nabla) \)
4-Velocity \( U = \gamma(c,u) \)

4-CurrentDensity \( J = (pc,j) = \rho_0 U = \gamma \rho_0 (c,u) = \rho(c,u) \)
4-ChargeFlux \( J \)

Temporal part: \( \rho = \gamma \rho_0 \) {charge-density}

Spatial part: \( \mathbf{j} = \gamma \rho_0 \mathbf{u} = \rho \mathbf{u} \) {3-current-density}

Conservation of Charge (Q)
\[ Q = \int \rho \, d^3x = \int \gamma \rho_0 \, d^3x = \int \gamma \, d\mathbf{r} \, \rho_0 \, d\mathbf{A} \]
\[ \rho \mathbf{V}_o \rightarrow \rho_0 V_o \]
\[ \int \mathbf{dT} \cdot \mathbf{j} = -cQ/V_o \]

\[ \partial \cdot \mathbf{J} = (\partial/\gamma c, -\nabla) \cdot (pc,j) = (\partial \rho + \nabla \cdot j) = 0 \]
Continuity Equation: Noether’s Theorem
The temporal change in charge density is balanced by the spatial change in current density.
Charge is neither created nor destroyed It just moves around as charge currents...

Relativistic ChargeDensity(\( \rho \)) vs Invariant Rest ChargeDensity(\( \rho_0 \))
\[ \rho = \gamma \rho_0 \]
SRQM Diagram: The Basis of Classical SR Physics

4-(Dust)NumberFlux, Particle # Conservation

4-Position \( R = (ct, r) \)

4-Gradient \( \partial = (\partial_t/c, -\nabla) \)

4-Velocity \( U = \gamma(c, u) \)

4-NumberFlux \( N = (nc, n) = n_0U = \gamma n_0(c, u) = n(c, u) \) aka. 4-ParticleFlux: 4-DustFlux

Temporal part: \( n = \gamma n_0 \) {number-density}

Spatial part: \( n = \gamma n_0 u = nu \) {3-number-flux}

Conservation of Particle # (N)

\[ N = \int nd^3x = \int \gamma n_0 d^3x = \int \gamma dr n_0 dA \]

\[ \partial \cdot N = (\partial_t/c - \nabla) \cdot (nc, n) = (\partial_t n + \nabla \cdot n) = 0 \]

Continuity Equation: Noether’s Theorem

The temporal change in number density is balanced by the spatial change in number-flux. Particle # is neither created nor destroyed. It just moves around as number currents...

Relativistic NumberDensity(n) vs Invariant Rest NumberDensity(n₀)

\[ n = \gamma n_0 \]

SR 4-Tensor

(2,0)-Tensor \( T^{\mu\nu} \)

(1,1)-Tensor \( T^{\mu} \), or \( T_{\mu} \)

(0,2)-Tensor \( T_{\mu\nu} \)

SR 4-Vector

\( V = (v^0, v) \)

SR 4-CoVector: OneForm

\( V_\mu = (v_\mu) \)

SR 4-Scalar

\( S_0 \) or \( S_0 \) Lorentz Scalar

\[ V \cdot V = \gamma^2 n_0 \]

Lorentz Scalar Invariant

\[ V \cdot V = (v^0)^2 - (\vec{v})^2 \]

\[ \gamma = n_0 \]

\[ n = \gamma n_0 \]
Lorentz Transforms $\Lambda^\mu_\nu = \partial_\nu [X^\mu]$ (Continuous) vs (Discrete) (Proper $\text{Det}=+1$) vs (Improper $\text{Det}=-1$)

The main idea that makes a generic 4-Vector into an SR 4-Vector is that it must transform correctly according to an SR Lorentz Transformation \( \{ \Lambda^\mu_\nu = \partial_\nu [X^\mu] \} \), which is basically any linear, unitary or antiunitary, transform (Determinant[$\Lambda^\mu_\nu$] = ±1) which leaves the Invariant Interval unchanged.

The SR continuous transforms (variable with some parameter) have (Det = +1, Proper) and include:

- "Rotation" (a mixing of space-space coordinates) and "(Velocity) Boost" (a mixing of time-space coordinates).

The SR discrete transforms can be (Det = +1, Proper) or (Det = -1, Improper) and include:

- "Space Parity-Inversion" (reversal of all space coordinates), “Time-Reversal” (reversal of the temporal coordinate), “Identity” (no change), various single dimension “Flips”, “Fixed Rotations”, and combinations of all of these discrete transforms.

### Typical Lorentz Boost Transformation
for a linear-velocity frame-shift \( \hat{x} \)-Boost:

- \( A^\nu = (a^0, a^1, a^2, a^3) \)
- \( \hat{x}^\nu = (a^1, a^2, a^3, a^0) \)
- \( = B^\nu \Lambda^\nu_\nu \)
- \( = (\gamma a^0 - \gamma \beta a^1, \gamma \beta a^0 + \gamma a^1, a^2, a^3) \)

### Lorentz Parity-Inversion Transformation:

- \( A^\nu = (a^0, a^1, a^2, a^3) \)
- \( \Lambda^\nu_\nu \rightarrow - \Lambda^\nu_\nu \)
- \( = (a^0, -a^1, -a^2, a^3) \)
- \( \rightarrow (a^0, -a^1, -a^2, a^3) \)

---

**Continuous: Boost depends on variable parameter \( \beta \), with \( \gamma^2 = 1/(1-\beta^2) \)**

**Discrete: Parity has no variable parameters**

- **Proper:** preserves orientation of basis
- **Improper:** reverses orientation of basis

---

- **SR 4-Tensor**
  - (2,0)-Tensor \( T^{\mu\nu} \)
  - (1,1)-Tensor \( V^\nu = (v_0, v_1, v_2, v_3) \)

- **SR 4-Vector**
  - \( V^\nu = V^\nu = (v_0, v_1, v_2, v_3) \)

- **SR 4-Scalar**
  - (0,0)-Tensor \( S \)
  - Lorentz Scalar
Proper Lorentz Transforms (Det=+1):
Continuous: (Boost) vs (Rotation)

\[ \Lambda_{\mu '}^{\nu} = \partial_{\nu} [X^{\mu'} ] \]

\[ \Lambda_{\mu '}^{\nu} = \partial_{\nu} [X^{\mu'} ] \]

 Typical Lorentz Boost Transform (symmetric):
for a linear-velocity frame-shift (x,t)-Boost in the \( \hat{x} \)-direction:

\[ \Lambda_{\mu '}^\nu = B_{\mu }^\nu \] \( \equiv e^\left( \gamma \beta \hat{x} \right) = \) 

\[ \gamma - \beta y \]

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Lorentz Transforms $\Lambda^\mu_{\nu'} = \partial_{\nu'}[X^\mu']$

Proper Lorentz Transforms (Det=+1): (Boost) vs (Rotation) vs (Identity)

General Lorentz Boost Transform (symmetric, continuous):
for a linear-velocity frame-shift (Boost) in the $v/c = \beta = (\beta^2, \beta^0)$-direction:

$\Lambda^\mu_{\nu'} \rightarrow B^\mu_{\nu'} = \gamma (I - \beta \gamma \beta')$

$\Lambda_{\mu\nu} = \begin{bmatrix} 1 & \beta^0 \beta^0 \beta^0 \\ \beta^0 & -1 & \beta^0 \\ \beta^0 & \beta^0 & -1 \end{bmatrix}$

Lorentz Identity Transform: $\Lambda^\mu_{\nu'} \rightarrow \eta^\mu_{\nu'} = \delta^\mu_{\nu'} = I_{(4)}$

Lorentz Rotation Transform (non-symmetric, continuous): for an angular-displacement frame-shift (Rotation) angle $\theta$ about the $\hat{\mathbf{n}}=(n^1, n^2, n^3)$-direction:

$\Lambda^\mu_{\nu'} \rightarrow R^\mu_{\nu'} = \cos(\theta) I + \hat{\mathbf{n}} \otimes \hat{\mathbf{n}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Lorentz Boost Transform (symmetric, continuous):
for a linear-velocity frame-shift (Boost) in the $v/c = \beta = (\beta^2, \beta^0)$-direction:

$\Lambda^\mu_{\nu'} \rightarrow B^\mu_{\nu'} = \gamma (I - \beta \gamma \beta')$

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Lorentz Boost Transform (symmetric, continuous):
for a linear-velocity frame-shift (Boost) in the $v/c = \beta = (\beta^2, \beta^0)$-direction:

$\Lambda^\mu_{\nu'} \rightarrow B^\mu_{\nu'} = \gamma (I - \beta \gamma \beta')$
Lorentz Transforms $\Lambda_{\mu'}^\nu = \partial_\nu [X^\mu']$

Discrete (non-continuous) 
(Parity-Inversion) vs (Time-Reversal) vs (Identity)

<table>
<thead>
<tr>
<th>General Lorentz Parity-Inversion (Space-Reversal) Transform:</th>
<th>No mixing</th>
<th>General Lorentz Time-Reversal Transform:</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda_{\nu'}^\mu = P^\nu_{\nu'}$ (Improper,symmetric,discrete)</td>
<td>$\text{Tr}[\Lambda_{\nu'}^\mu] = +4$</td>
<td>$\Lambda_{\nu'}^\mu = {A_0^\nu, A_0^{\nu'}}$</td>
<td>$\text{Tr}[T_{\nu'}^\mu] = +2$</td>
</tr>
<tr>
<td>$\Lambda_{\nu'}^\mu = {A_0^\nu, A_0^{\nu'}}$</td>
<td>$\text{Det}[\Lambda_{\nu'}^\mu] = +1$</td>
<td>$\text{Tr}[P_{\nu'}^\mu] = -2$</td>
<td></td>
</tr>
<tr>
<td>General Lorentz Time-Reversal Transform: $\Lambda_{\nu'}^\mu = T^\mu_{\nu'}$ (Improper,symmetric,discrete)</td>
<td>$\text{Det}[T_{\nu'}^\mu] = -1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{Det}[\Lambda_{\nu'}^\mu] = +1$</td>
<td>$\text{Det}[\Lambda_{\nu'}^\mu] = -1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>General Lorentz Identity Transform: $\Lambda_{\nu'}^\mu = \delta_{\nu'}^\mu = I_{(4)}$ (Proper,symmetric,discrete)</td>
<td>$\text{Det}[\Lambda_{\nu'}^\mu] = +1$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Identical 4-Vector $A' = A_0^\nu, A_0^{\nu'} = (a_0^\nu, a_0^{\nu'}) = (a_0^\nu, a_0^{\nu'}) = A$

Time-Reversed 4-Vector $A' = A_0^\nu, A_0^{\nu'} = (a_0^\nu, a_0^{\nu'}) = (-a_0^\nu, a_0^{\nu'})$

Parity-Inverted 4-Vector $A' = A_0^\nu, A_0^{\nu'} = (a_0^\nu, a_0^{\nu'}) = (a_0^\nu, -a_0^{\nu'})$

Combo PT'd 4-Vector $A' = A_0^\nu, A_0^{\nu'} = (a_0^\nu, a_0^{\nu'}) = (-a_0^\nu, -a_0^{\nu'})$

Original 4-Vector $A = A_0^\nu, A_0^{\nu'} = (a_0^\nu, a_0^{\nu'})$

Both the Parity-Inversion (P) and Time-Reversal (T) have a Determinant of -1, which is an improper transform. However, combinations (PP), (TT), (PT) have overall Determinant of +1, which is proper. Classical SR Time Reversal neglects spin and charge. When included, there is also a Charge-Conjugation(C) transform. Then one gets (CC),(PP),(TT),({PT}{PT}); & permutations of (CPT) transforms all leading back to the Identity ($I_{(4)}$).

Note that the Trace of Discrete Lorentz Transforms goes in steps from (-4,-2,2,4). As we will see in a bit, this is a major hint for SR antimatter and CPT Symmetry.
SRQM Lorentz Transforms $\Lambda^{\mu'}_{\nu} = \partial_{\nu} [X^{\mu}]$

Discrete & Fixed Rotation $\rightarrow$ Particle Exchange

Lorentz Coordinate-Flip Transforms

SR Lorentz Transform

$\partial_{[\nu}] = \partial_{\nu \mu} / \partial_{\nu} = \Lambda^{\nu}_{\mu}$

$\Lambda^{\nu}_{\mu} = (\Lambda^{-1})^{\nu}_{\mu} : \Lambda^{\nu}_{\mu} (\Lambda^{-1})_{\nu} = \eta_{\nu \mu} = \delta^{\nu}_{\mu}$

$\eta_{\nu \mu} \Lambda^{\nu}_{\mu} = \delta_{\nu \mu}$

$\det[\Lambda^{\nu}_{\mu}] = \pm 1$  
$\Lambda^{\nu}_{\mu} \Lambda^{\mu}_{\nu} = 4 = \Lambda^{\nu}_{\mu} \Lambda^{\mu}_{\nu}$

Any single Lorentz Flip Transform is Improper, with a Determinant of -1. However, pairwise combinations are Proper, with a Determinant of +1. All single flips have Trace of 2.

The combination of any two Spatial Flips is the equivalent of a Spatial Rotation by ($\pi$) about the associated rotational axis.

$sin(\pi) = 0, cos(\pi) = -1$  
Since this is a Proper transform, it is also the equivalent of a particle location exchange.

The combination of all three Spatial Flips, Flip-xyz, gives the Lorentz Parity Transform, which is again Improper, with Trace of -2.

The Flip-t is the standard Lorentz Time-Reversal, Improper.

Lorentz Transform $\partial_{[\nu]} = \partial_{\nu \mu} / \partial_{\nu} = \Lambda^{\nu}_{\mu}$

$\Lambda^{\nu}_{\mu} = (\Lambda^{-1})^{\nu}_{\mu} : \Lambda^{\nu}_{\mu} (\Lambda^{-1})_{\nu} = \eta_{\nu \mu} = \delta^{\nu}_{\mu}$

$\eta_{\nu \mu} \Lambda^{\nu}_{\mu} = \delta_{\nu \mu}$

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The Flip-t is the standard Lorentz Time-Reversal, Improper.
SRQM Lorentz Transforms $\Lambda^\nu_\mu = \partial_\nu [X^\mu]$  

Lorentz Transform Connection Map – Discrete Transforms  

CPT, Big-Bang, [Matter↔AntiMatter], Arrow(s)-of-Time  

Examine all possible combinations of **Discrete Lorentz Transformations** which are Linear (Determinant of ±1).  

A lot of the standard SR texts only mention (P)arity-Inverse and (T)ime-Reversal. However, there are many others, including (F)lips and (R)otations of a fixed amount. However, the (T)imeReversal and Combo(P)arity(T)ime take one into a separate section of the chart. Taking into account all possible discrete Lorentz Transformations fills in the rest of the chart. The resulting interpretation is that there is CPT **Symmetry** (Charge:Parity:Time) and Dual **Time-Space** (with reversed timeflow). In other words, one can go from the Identity Transform (all +1) to the Negative Identity Transform (all -1) by doing a Combo PT Lorentz Transform or by Negating the Charge (Matter↔Antimatter). The Feynman-Stueckelberg CPT Interpretation (AntiMatter moving spacetime backward = NormalMatter moving spacetime-forward) aligns with this as a Dual-Universe “AntiMatter” Side.  

This is similar to Dirac’s prediction of AntiMatter, but without the formal need of Quantum Mechanics, or Spin. In fact, it is more general than Dirac’s work, which was about the electron. This is from general Lorentz Transforms for any kind of particle: event.

Tao – I Ching – YinYang  

fantastic metaphors for  

SR SpaceTime...  

Tao: “Flow of the Universe”  

“way, path, route, road”  

I Ching: “Book of Changes”  

Transformations  

YinYang: “Positive/Negative”  

“complementary opposites”  

| +1 | +1 | +1 | +1 |   |
| +1 | +1 | -1 | -1 |   |
| +1 | +1 | +1 | -1 |   |
| +1 | -1 | +1 | +1 |   |
| +1 | -1 | -1 | -1 |   |

Discrete **NormalMatter** (NM) Lorentz Transform Type  

| NM-Minkowski-Identity : AM-Flip-xyz=AM-ComboPT  
| NM-Flip-z  
| NM-Flip-y  
| NM-Flip-t  
| NM-Flip-xz=NM-Rotate-xz(\pi)  
| NM-Flip-xy=NM-Rotate-xy(\pi)  
| NM-Flip-xyz=NM-ParityInverse:AM-Flip-t=AM-TimeReversal |

| -1 | +1 | +1 | +1 |   |
| -1 | +1 | +1 | -1 |   |
| -1 | +1 | -1 | +1 |   |
| -1 | -1 | +1 | +1 |   |
| -1 | -1 | -1 | -1 |   |

AM-Flip-xyz=AM-ParityInverse:NM-Flip-t=NM-TimeReversal  

| AM-Flip-xyz=AM-ParityInverse:NM-Flip-t=NM-TimeReversal  
| AM-Flip-xy=AM-Rotate-xy(\pi)  
| AM-Flip-xz=AM-Rotate-xz(\pi)  
| AM-Flip-x  
| AM-Flip-y  
| AM-Flip-z |

Trace : Determinant  

| Tr = +4 : Det = +1 Proper  
| Tr = +2 : Det = -1 Improper  
| Tr = +2 : Det = -1 Improper  
| Tr = 0 : Det = +1 Proper  
| Tr = -2 : Det = -1 Improper  |

Det[$\Lambda^\nu_\mu$]±1, $\Lambda^\mu_\nu \Lambda^\nu_\beta = \eta_{\beta\gamma}$  

Tr[$\Lambda^\nu_\mu$]={-∞, +∞} = Lorentz Transform Type  

NormalMatter  

AntiMatter

Combination of (P)arityInverse & (T)imeReversal  

take  

4-Vectors of Physical 4-Vectors
I ran across another variation of the YinYang symbol ☯, also known as the T'ai chi symbol, on the internet. I like the \(1+3=4\)-level symmetries. There are 8 total circles, with an overall even balance of \{white:black\}, \{+:-\}, so 4D in the two dual realms.

It also reminds me a bit of a Penrose Diagram, which is an extension of the Minkowski Diagram.
SRQM Lorentz Transforms $\Lambda^\mu_\nu = \partial_\nu [X^\mu]$

Lorentz Transform Connection Map – Trace Identification

CPT, Big-Bang, [Matter↔AntiMatter], Arrow(s)-of-Time

All Lorentz Transforms have Tensor Invariants: Determinant = ±1 and InnerProduct = 4. However, one can use the Tensor Invariant Trace to Identify CPT Symmetry & AntiMatter

Discrete NormalMatter (NM) Lorentz Transform Type
NM-Minkowski-Identity : AM-Flip-bxyz=AM-ComboPT=AM-NegateIdentity
NM-Flip-t=NM-TimeReversal, NM-Flip-x, NM-Flip-y, NM-Flip-z
AM-Flip-xyz=AM-ParityInverse
NM-Flip-xy=NM-Rotate-xy(π),NM-Flip-xz=NM-Rotate-xz(π),NM-Flip-yz=NM-Rotate-yz(π)
AM-Flip-xy=AM-Rotate-xy(π), AM-Flip-xz=AM-Rotate-xz(π), AM-Flip-yz=AM-Rotate-yz(π)
NM-Flip-xyz=NM-ParityInverse
AM-Flip-t=AM-TimeReversal, AM-Flip-x, AM-Flip-y, AM-Flip-z

AM-Minkowski-Identity : NM-Flip-bxyz=NM-ComboPT=NM-NegateIdentity
Discrete AntiMatter (AM) Lorentz Transform Type

Trace : Determinant
Tr = +4 : Det = +1 Proper
Tr = +2 : Det = -1 Improper
Tr = 0 : Det = +1 Proper
Tr = -2 : Det = -1 Improper
Tr = -4 : Det = +1 Proper

Two interesting properties of (1,1)-Tensors, of which the Lorentz Transform is an example:

1. Trace = Sum (Σ) of EigenValues : Determinant = Product (Π) of EigenValues
2. Create an Anti-Transform which has all EigenValue Tensor Invariants negated.

The Trace Invariant identifies a "Dual" Negative-Side for all Lorentz Transforms.
SRQM Lorentz Transforms \( \Lambda^\mu_\nu = \partial_\nu [X^\mu] \)

**Lorentz Transform Connection Map - Interpretations**

**CPT, Big-Bang, [Matter↔AntiMatter], Arrow(s)-of-Time**

Based on the Lorentz Transform properties of the last few pages, here is interesting observation about Lorentz Transforms:

They all have Determinant of \((±1)\), and Inner Product of \(+4 = 4D\), but the Trace varies depending on the particular Transform.

The Trace of the Identity is at \(+4\). Assume this applies to normal matter particles.

The Trace of normal-matter particle Rotations varies continuously from \(0..+4\).

The Trace of the normal-matter particle Boosts varies continuously from \(+4..+\infty\)

So, one can think of Trace \(= +4\) being the connection point between normal-matter Rotations and Boosts.

Now, various Flip Transforms (inc. the Time Reversal and Parity Transforms, and their combination as PT transform), take the Trace in discrete steps from \(−4,−2,0,+2,+4\). Applying a bit of symmetry:

The Trace of the Negative Identity is at \(−4\). Assume this applies to anti-matter particles.

The Trace of anti-matter particle Rotations varies continuously from \(0..−4\).

The Trace of the anti-matter particle Boosts varies continuously from \(−4..−\infty\)

So, one can think of Trace \(= −4\) being the connection point between anti-matter Rotations and Boosts.

This observation would be in agreement with the CPT Theorem (Feynman-Stueckelberg) idea that (normal/anti)-matter particles moving backward in SpaceTime are CPT symmetrically equivalent to (anti-normal)-matter particles moving forward in SpaceTime.

Now, scale this up to Universe size: The Baryon Asymmetry problem (aka. The Matter-AntiMatter Asymmetry Problem).

If the Universe was created as a huge chunk of energy, and matter-creating energy is always transformed into matter-antimatter mirrored pairs, then where is all the antimatter?

Turns out this is directly related to the Arrow-of-Time Problem as well.

Answer: It is temporally on the “Other/Dual-Side” of the Big-Bang! The antimatter created at the Big-Bang is travelling in the negative-time \((-t)\) direction from the Big-Bang creation point, and the normal matter is travelling in the positive-time direction \((+t)\).

**Universal CPT Symmetry**. So, what happened “before” the Big-Bang? It “is” the AntiMatter Dual to our normal matter universe!

Pair-production is creation of \(\Lambda\) mirror particles within SpaceTime. The Big-Bang is the creation of SpaceTime itself.

This also resolves the Arrow-of-Time Problem. If all known physical microscopic processes are time-symmetric, why is the flow of time experienced as uni-directional? (see Wikipedia “CPT Symmetry”, “CP Violation”, “Andrei Sakharov”)

Answer: Time flow on This-Side of the Universe is \((+t)\) direction, while time flow on the Dual-Side of the Universe is \((-t)\) direction.

The math all works out. Time flow is bi-directional, but on opposite sides of the BB/Origin-Singularity. **Universal CPT Symmetry**!

This gives total CPT Symmetry to all of the possible Lorentz Transforms \((AM=AntiMatter, NM=NormalMatter)\):

Trace Various \((AM \_Flips)\): Trace Various \((NM \_Flips)\):

\(-\infty...AM\_Boosts)... AM\_Identity=-4)... AM\_Rotations)...0)...NM\_Rotations)...(+4)...NM\_Identity)...NM\_Boosts)...+\infty

This solves the: Baryon (Matter-AntiMatter) Asymmetry Problem & Arrow(s)-of-Time Problem \((+t/−t)\)
This idea of Universal CPT Symmetry also gives a Universal Dimensional Symmetry as well.

Consider the well-known “balloon” analogy of the universe expansion. The “spatial” coordinates are on the surface of the balloon, and the expansion is in the (+t) direction. There is symmetry in the (+/-) directions of the spatial coordinates, but the time flow is always uni-directional, (+t), as the balloon gets bigger→inflates.

By allowing a “Dual-Side”, it provides a universal dimensional symmetry. One now has (+/-) symmetry for temporal (t) directions.

The “center” of the Universe is, literally, the Big Bang Singularity. It is the “center = zero = origin” point of both time and space directions. There are some people who prefer to say the BB is after inflation, but I am simply referring the “Origin:Singularity”.

The expansion gives time-flow always AWAY FROM the Big Bang singularity in both the Normal-Side (+t) and the Dual-Side (-t). All spatial coordinates expand in both the (+/-) directions on both temporal sides of the singularity.

Note that this gives an unusual interpretation of what came “before” the Big Bang.

The “past” on either side extends only to the BB singularity, not beyond. Time flow is always away from this creation singularity.

This is also in accord with known black hole physics, in that all matter entering a BH event horizon ends at the BH singularity. Time and space coordinates both come to a stop at either type of singularity, from the point of view of an observer that is in the spacetime but not at one of these singularities.

So, the Big Bang is a “starting” singularity, and black holes are “ending” singularities. This also provides for idea of “white holes” actually just being black holes on the Dual-Side. White hole = time-reversed black hole. Always confusion about stuff coming out. This way, the mass is still attractive. Time-flow is simply reversed on the alternate side so stuff still goes INTO the hole… which makes way more sense than stuff that can only come out of the “massive→attractive” white hole.

So, Universal CPT Symmetry = Universal Dimensional Symmetry.

And, going even further, I suspect this is the reason there is a duality in Metric conventions.

In other words, physicists have wondered why one can use Metric signature (+++,--) or (+---).

I submit that one of these metrics applies to the Normal Matter side, while the other complementarily applies to the Dual side. This would allow correct causality conditions to apply on either side.

Again, this is similar to the Dirac prediction of antimatter based on a duality of possible solutions.

This gives total CPT Symmetry to all of the possible Lorentz Transforms (AM=AntiMatter , NM=NormalMatter):

Trace Various (AM_Flips) : Trace Various (NM_Flips)

-Infinity...(AM_Boosts)...(AM_Identity=-4)...(AM_Rotations)...0...(NM_Rotations)...(+4)...(NM_Identity)...(NM_Boosts)...+Infinity

This solves the:
Baryon (Matter/AntiMatter) Asymmetry Problem & Arrow(s)-of-Time Problem (+t / -t )
## SRQM Study: Model SpaceTimes

A Klein geometry is a pair \((G,H)\) where \(G\) is a Lie group and \(H\) is a closed Lie subgroup of \(G\) such that the (left) coset space \(X=G/H\) is connected.

\(G\) acts transitively on the homogeneous space \(X\).

We may think of \(H\triangleleft G\) as the stabilizer subgroup of a point in \(X\).

\(d\) is the dimension of the SpaceTime, which is 4D for our universe.

### Klein Geometry G/H

<table>
<thead>
<tr>
<th>Lorentzian pseudo-Riemannian</th>
<th>Riemannian</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anti de Sitter (\text{SO}(3,2)/\text{SO}(3,1))</td>
<td>Hyperbolic (\text{SO}(4,1)/\text{SO}(4))</td>
</tr>
<tr>
<td>Minkowski (\text{ISO}(3,1)/\text{SO}(3,1)) (ds^2 = (cdt)^2 - dx^2)</td>
<td>Euclidean (\text{ISO}(4)/\text{SO}(4)) (ds^2 = (cdt)^2 + dx^2)</td>
</tr>
<tr>
<td>De Sitter (\text{SO}(4,1)/\text{SO}(3,1))</td>
<td>Spherical (\text{SO}(5)/\text{SO}(4))</td>
</tr>
</tbody>
</table>

### Geometric Context

<table>
<thead>
<tr>
<th>Geometric Context</th>
<th>Gauge Group</th>
<th>Stabilizer Subgroup</th>
<th>Local Model Space</th>
<th>Local Geometry</th>
<th>Global Geometry</th>
<th>Differential Cohomology</th>
<th>First Order Formulation of Gravity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Differential geometry</td>
<td>Lie group/algebraic group (G)</td>
<td>subgroup (monomorphism) (H\triangleleft G)</td>
<td>quotient (&quot;coset space&quot;) (G/H)</td>
<td>Klein geometry</td>
<td>Cartan geometry</td>
<td>Cartan connection</td>
<td></td>
</tr>
</tbody>
</table>

**Examples:**

- Euclidean group \(\text{Iso}(d)\)
- Lorentz group \(\text{O}(d-1,1)\)
- Minkowski spacetime \(\mathbb{R}^{d-1,1}\)
- Lorentzian geometry
- Pseudo-Riemannian geometry
- Spin connection
- Einstein gravity

**Fits known observational data**

- Anti de Sitter group \(\text{O}(d-1,2)\)
- Anti de Sitter spacetime \(\text{AdS}^d\)
- AdS gravity

- de Sitter group \(\text{O}(d,1)\)
- de Sitter spacetime \(\text{dS}^d\)
- de Sitter gravity

- Linear algebraic group
- Parabolic subgroup/flag variety
- Parabolic geometry

- Conformal group \(\text{O}(d,t+1)\)
- Conformal parabolic subgroup
- Möbius space \(\mathbb{S}^{d,1}\)
- Conformal geometry
- Conformal connection
- Conformal gravity
### Classical Transforms: Venn Diagram

**Full Galilean = Galilean + Translations**

- **(10)** Galilean Transformation Group
- **(6)** General Linear, Affine Transform
- **(4)** Translation Transform

#### Transformations

<table>
<thead>
<tr>
<th># of independent parameters</th>
<th># continuous symmetries</th>
<th># Lie Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3+3=6)</td>
<td>(1+3=4)</td>
<td>(0)</td>
</tr>
</tbody>
</table>

#### Galilean Transform

- 4-Tensor {mixed type-(1,1)}

<table>
<thead>
<tr>
<th>Discrete</th>
<th>Continuous</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time-reversal</strong> $G^{\mu\nu}_v \rightarrow T^{\mu\nu}_v$ (0)</td>
<td><strong>SpatialFlipCombos</strong> $G^{\mu\nu}_v \rightarrow F^{\mu\nu}_v$ (0)</td>
</tr>
<tr>
<td>$t \rightarrow -t^*$ time parity anti-unitary</td>
<td>${x</td>
</tr>
<tr>
<td><strong>Parity-Inversion</strong> $G^{\mu\nu}_v \rightarrow P^{\mu\nu}_v$ (0)</td>
<td><strong>Identity</strong> $I_{4(4)}$ $G^{\mu\nu}_v \rightarrow \eta^{\mu\nu}=\delta^{\mu\nu}$ (0)</td>
</tr>
<tr>
<td>$r \rightarrow -r$ space parity unitary</td>
<td><strong>no mixing unitary</strong></td>
</tr>
<tr>
<td><strong>Rotation</strong> $G^{\mu\nu}_v \rightarrow R^{\mu\nu}_v$ (3)</td>
<td><strong>4-Zero Motion</strong> $\Delta X^{\mu} \rightarrow (0,0)$ (0)</td>
</tr>
<tr>
<td>$x:y</td>
<td>x:z</td>
</tr>
<tr>
<td><strong>Motion-Shear</strong> $G^{\mu\nu}_v \rightarrow S^{\mu\nu}_v$ (3)</td>
<td><strong>Spatial</strong> $\Delta X^{\mu} \rightarrow (0,\Delta x)$ (3)</td>
</tr>
<tr>
<td>$t:x</td>
<td>t:y</td>
</tr>
<tr>
<td><strong>Isotropy</strong> {same all directions}</td>
<td><strong>Homogeneity</strong> {same all points}</td>
</tr>
</tbody>
</table>

#### Translation Transform

- 4-Vector $\Delta X^{\mu}$

<table>
<thead>
<tr>
<th>Discrete</th>
<th>Continuous</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Temporal</strong> $\Delta X^{\mu} \rightarrow (c \Delta t,0)$ (1)</td>
<td><strong>Spatial</strong> $\Delta X^{\mu} \rightarrow (0,\Delta x)$ (3)</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>$\Delta x</td>
</tr>
</tbody>
</table>

---

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

A Tensor Study of Physical 4-Vectors
**SRQM Transforms: Venn Diagram**

**Poincaré = Lorentz + Translations**

(10) (6) (4)

<table>
<thead>
<tr>
<th>Transformations</th>
<th>Lorentz Transform</th>
<th>Translation Transform</th>
<th>4-Tensor (mixed type-(1,1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discrete</td>
<td>Time-reversal</td>
<td>4-Zero</td>
<td>(0)</td>
</tr>
<tr>
<td></td>
<td>( \Lambda^\mu_\nu \rightarrow T^\mu_\nu )</td>
<td>( \Delta X^\mu \rightarrow (0,0) )</td>
<td>(0) no motion</td>
</tr>
<tr>
<td>Continuous</td>
<td>SpatialFlipCombos</td>
<td>Temporal</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \Lambda^\mu_\nu \rightarrow F^\mu_\nu )</td>
<td>( \Delta X^\mu \rightarrow (c \Delta t,0) )</td>
<td>(1) ( \Delta t )</td>
</tr>
<tr>
<td></td>
<td>{x</td>
<td>y</td>
<td>z} \rightarrow -{x</td>
</tr>
<tr>
<td></td>
<td>Identity ( I_{4(1)} )</td>
<td>( \Delta X^\mu \rightarrow (0,0) )</td>
<td>(0)</td>
</tr>
<tr>
<td></td>
<td>( \Lambda^\mu_\nu \rightarrow \eta^\mu_\nu = 0 )</td>
<td>Isotropy</td>
<td></td>
</tr>
<tr>
<td></td>
<td>no mixing</td>
<td>Isotropy</td>
<td></td>
</tr>
<tr>
<td></td>
<td>unitary</td>
<td>Homogeneity</td>
<td></td>
</tr>
<tr>
<td></td>
<td>{x</td>
<td>y</td>
<td>z} \rightarrow -{x</td>
</tr>
<tr>
<td></td>
<td>4-AngularMomentum</td>
<td>4-LinearMomentum</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( M^\mu_\nu = X^\mu \times P = X^\mu P^\nu - X^\nu P^\mu )</td>
<td>4-Vector</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parity-Inversion</td>
<td>( \Lambda^\mu_\nu \rightarrow P^\mu_\nu )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>r \rightarrow -r</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>space parity unitary</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \Lambda^\mu_\nu \rightarrow \eta^\mu_\nu = 0 )</td>
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</tr>
<tr>
<td></td>
<td>no mixing</td>
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<td></td>
<td>unitary</td>
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<td></td>
<td>( {x</td>
<td>y</td>
<td>z} \rightarrow -{x</td>
</tr>
<tr>
<td></td>
<td>CPT Symmetry</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Charge) R \rightarrow -R^*, q \rightarrow -q</td>
<td></td>
<td></td>
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<td>( \Lambda^\mu_\nu \rightarrow C^\mu_\nu )</td>
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<td>Charge-Conjugation</td>
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<td>( \eta^\mu_\nu = 0 )</td>
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**Stress-Energy(Density)-Tensor** \( T^\mu_\nu \) (symmetric)

- Covariant Decomposition
- Tensor Invariants: Symmetry, AntiSymmetry, Trace---Isotropy, Anisotropy
- \( T^\mu_\nu \) = flux of \( \delta^\mu_\nu \)-component of 4-Momentum along \( \delta^\mu \).
- (temporal:mixed:spatial) splitting
  - 1 Temporal:Temporal EnergyDensity \( (\rho) = V^\mu_\nu T^\mu_\nu \)
  - 3 Temporal:Spatial HeatEnergy Flux \( (Q^\mu) = c T^\nu \)
  - 1 Spatial:Spatial Isotropic Pressure \( (p_o) = (1/3) H^\mu_\nu T^\mu_\nu \)
  - 5 Spatial:Spatial Anisotropic Stress \( (\Pi^\mu_\nu) = (1/2) H^\mu_\nu T^\mu_\nu + (p_o) H^\mu_\nu \)
- 10 Total Independent components

**Poincaré Transformation Group = 10 Invariances**
The group of all isometries of SR:Minkowski Spacetime \( (6+4 = 10) \)

- conserve quadratic form

**4-Vector SRQM Interpretation of QM**

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http://scirealm.org/SRQM.pdf

Amusingly, Inhomogeneous Lorentz adds homogeneity.
SRQM Study:
Lie Groups and Generators

Homogeneous Lorentz Transformations (6)
\[ \Lambda^\nu{}_{\mu'} \rightarrow R^\nu{}_{\nu'} \]

Generators

\[ \Lambda^\nu{}_{\mu'} = e^{\Omega} = e^{(\omega \cdot \mathbf{J} + \zeta \cdot \mathbf{K})} \]

4-Vectors SRQM
\[ \Lambda^\nu{}_{\mu'} = (\Lambda^{-1})_{\mu'}{}^{\nu} : \Lambda^\nu{}_{\mu'} (\Lambda^{-1})_{\nu'}{}^{\mu} = \eta^\nu{}_{\mu'} = \delta^\nu{}_{\mu'} \]

\[ \eta_{\mu\nu} \Lambda^\mu{}_{\alpha} \Lambda^\nu{}_{\beta} = \eta_{\alpha\beta} \]

4-Vectors SRQM Interpretation of QM

\[ \text{Det}[\Lambda^\nu{}_{\mu'}] = +1 \text{ for Proper Lorentz Transforms} \]
\[ \text{Det}[\Lambda^\nu{}_{\mu'}] = -1 \text{ for Improper Lorentz Transforms} \]

Lorentz Matrices can be generated by a matrix \( M \) with \( \text{Tr}[M] = 0 \) which gives:
\[ \{ \Lambda = e^{\mathbf{M}} = e^{\mathbf{M}^T}, \mu \neq \nu \} \]
\[ \{ \Lambda^T = (e^{\mathbf{M}})^T = e^{\mathbf{M}^T} \} \]
\[ \{ \Lambda^{-1} = (e^{\mathbf{M}})^{-1} = e^{\mathbf{M}^T} \} \]

4-Angular Momentum
\[ P^\mu = X^\mu \ ^{\mathbf{P}} = X^\mu P^\nu - X^\nu P^\mu \]

Generators of Lorentz Transformations:
\[ \{ \Lambda^\nu{}_{\mu'}, \text{Rotations} (3) + \Lambda^\nu{}_{\mu'} \rightarrow B^\nu{}_{\nu'}, \text{Boosts} (3) \} \]

SR: Lorentz Transform
\[ \partial_i[R^\nu{}_{\nu'}] = \partial R^\nu{}_{\nu'} / \partial \nu' = \Lambda^\nu{}_{\mu'} \]
\[ \Lambda^\nu{}_{\mu'} = (\Lambda^{-1})_{\mu'}{}^{\nu} : \Lambda^\nu{}_{\mu'} (\Lambda^{-1})_{\nu'}{}^{\mu} = \eta^\nu{}_{\mu'} = \delta^\nu{}_{\mu'} \]

\[ \eta_{\mu\nu} \Lambda^\mu{}_{\alpha} \Lambda^\nu{}_{\beta} = \eta_{\alpha\beta} \]

\[ \text{Det}[\Lambda^\nu{}_{\mu'}] = \pm 1 \]
\[ \Lambda^\nu{}_{\mu'} = 4 \]

Rotations \( J_i = -\varepsilon_{imn} M_{mn} / 2 \), Boosts \( K_i = M_{0i} \)
Review of SR Transforms

10 Poincaré Symmetries, 10 Conservation Laws
10 Generators: Noether’s Theorem

Lagrange “Shift Operator” version of Taylor’s Theorem: \( e^{a\partial_x}f(x) = f(x+a) \)
Bloch Theorem: Translation Operator: \( e^{a^\mu}\psi(X) = \psi(X+R) \), with \( K \) as reciprocal lattice

Poincaré Transformation Group aka. Inhomogeneous Lorentz Transformation
The group of all isometries of SR:Minkowski Spacetime \((6+4=10)\)
(preserve quadratic form)

General Linear, Affine Transform \( X^\nu = \Lambda^\nu_\nu X^\nu + \Delta X^\nu \) with \( \text{Det}[\Lambda^\nu_\nu] = \pm 1 \)

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Review of SR Transforms

Poincaré Algebra & Generators

Casimir Invariants

The (10) one-parameter groups can be expressed directly as exponentials of the generators:

$$ U[1, (a^0)] = e^{i(a^0 \cdot H)} = e^{i(a^0 \cdot p^0)}; \quad (1) \text{ Hamiltonian (Energy) = Temporal Momentum } H $$

$$ U[1, (0, \lambda \hat{a})] = e^{i(-i(\lambda \hat{a} \cdot p))}; \quad (3) \text{ Angular Momentum } \lambda = \lambda_j J_j $$

$$ U[\Lambda(\lambda \hat{b} \hat{c}), 0] = e^{i(\lambda \hat{b} \hat{c} \cdot J)}; \quad (3) \text{ Angular Momentum } J = J^k K^k $$

$$ U[\Lambda(\lambda \hat{p} \hat{k}), 0] = e^{i(\lambda \hat{p} \hat{k} \cdot k J)}; \quad (3) \text{ Dynamic Mass Moment } k = n $$

The Poincaré Algebra is the Lie Algebra of the Poincaré Group:

Total of \((1+3+3+3 = (1+3)+(3+3) = 4+6 = 10)\) Invariances from Poincaré Symmetry

Covariant form:

These are the commutators of the the Poincaré Algebra:

$$ [X^\mu, X^\nu] = 0^\nu $$

$$ [P^\mu, P^\nu] = -ihq(F^\nu) \text{ if interacting with EM field; otherwise } = 0^\nu \text{ for free particles } $$

$$ M^{\mu\nu} = (X^\mu P^\nu - X^\nu P^\mu) = i\hbar (X^\mu \phi^\nu - X^\nu \phi^\mu) $$

$$ [M^\mu_\nu, M^\nu_\rho] = i\hbar (\eta^\rho M^\mu_\nu + \eta^\nu M^\rho_\nu + \eta^\mu M^\rho_\nu + \eta^\nu M^\mu_\rho) $$

Component form: Rotations \( J = -\epsilon_{\mu\nu\rho\sigma}M^{\mu\nu}/2 \), Boosts \( K_i = M_{i0} \)

$$ [J_{\mu}, J_{\nu}] = i\epsilon_{\mu\nu\rho\sigma}P^\rho $$

$$ [J_{\mu}, K_{\nu}] = 0 $$

$$ [K_{\mu}, P_{\nu}] = i\epsilon_{\mu\nu\rho\sigma} $$

$$ [K_{\mu}, K_{\nu}] = i\epsilon_{\mu\nu\rho\sigma}K^\rho $$

$$ [J_{\mu}, K_{\nu}] = i\epsilon_{\mu\nu\rho\sigma}J^\rho $$

$$ [K_{\mu}, K_{\nu}] = i\epsilon_{\mu\nu\rho\sigma}J^\rho $$

$$ [K_{\mu}, J_{\nu}] = i\epsilon_{\mu\nu\rho\sigma}J^\rho \text{ a Wigner Rotation resulting from consecutive boosts} $$

$$ [J_{\mu} + iK_{\mu}, J_{\nu} - iK_{\nu}] = 0 $$

Poincaré Algebra has 2 Casimir Invariants = Operators that commute with all the Poincaré Generators

These are \( [P^2 = p^\mu p_\mu = (m c)^2, W^2 = W_\mu W^\mu = -(m c)^2 (j + 1) ] \), with \( W^2 = (-1/2)\epsilon_{\mu\nu\rho\sigma}J^\mu P_\nu \) as the Pauli-Lubanski Pseudovector

$$ [P^2, P^\mu] = [P^\mu, P] = [P^2, J] = [P^2, K] = 0: \text{ Hence the 4-Momentum Magnitude squared commutes with all Poincaré Generators} $$

$$ [W^2, P^\mu] = [W^\mu, P] = [W^2, J] = [W^2, K] = 0: \text{ Hence the 4-SpinMomentum Magnitude squared commutes with all Poincaré Generators} $$

\( \text{Trace}[T^\tau] = \eta_{\mu\nu} T^{\tau\nu} = T^\tau = T \)

\( V \cdot V = V^\mu V^\nu = ((V^\mu)^2 - V^\mu \nu) = (V^\mu)^2 \)

\( \text{ Lorentz Scalar Invariant} \)
**SRQM Study:**

### 10 Poincaré Symmetry Invariances

Noether’s Theorem: 10 SR Conservation Laws

**d’Alembertian Invariant Wave Equation:** \( \partial^2 - (\partial/c)^2 - \nabla \cdot \nabla = (\partial/c)^2 \)

**Time Translation:**
Let \( X_t = (ct+c\Delta t, x) \), then \( \partial[X_t] = (\partial/c, -\nabla)(ct+c\Delta t, x) = \text{Diag}[1,-1] = \partial[X] = \eta^{\mu \nu} \)
so \( \partial[X] = \partial[X] \) and \( \partial[K] = [0] \)
\( (\partial-\partial)[K \cdot X] = (\partial-\partial)[K \cdot X] \)
\( \partial-\partial)[K \cdot X + K \cdot \partial[X] = 0] + K \cdot \partial[X] = \partial[K] \cdot X + K \cdot \partial[X] = \partial(\partial[K \cdot X]) = \partial(\partial)[K \cdot X] \):

**Space Translation:**
Let \( X_s = (ct+x, \Delta x) \), then \( \partial[X_s] = (\partial/c, \nabla)(ct+x, \Delta x) = \text{Diag}[1,-1] = \partial[X] = \eta^{\mu \nu} \)
so \( \partial[X] = \partial[X] \) and \( \partial[K] = [0] \)
\( (\partial-\partial)[K \cdot X_s] = (\partial-\partial)[K \cdot X_s] + K \cdot \partial[X_s] = 0 + K \cdot \partial[X] = \partial[K] \cdot X + K \cdot \partial[X] = \partial(\partial[K \cdot X]) = \partial(\partial)[K \cdot X] \):

**Lorentz Space-Space Rotation:**
Let \( X_R = (ct,R[x]) \), then \( \partial[X_R] = (\partial/c, -\nabla)(ct,R[x]) = \text{Diag}[1,-1] = \partial[X] = \eta^{\mu \nu} \)
so \( \partial[X] = \partial[X] \) and \( \partial[K] = [0] \)
\( (\partial-\partial)[K \cdot X_R] = (\partial-\partial)[K \cdot X_R] + K \cdot \partial[X_R] = 0 + K \cdot \partial[X] = \partial[K] \cdot X + K \cdot \partial[X] = \partial(\partial[K \cdot X]) = \partial(\partial)[K \cdot X] \):

**Lorentz Time-Space Boost:**
Let \( X_B = \gamma(ct-\beta \cdot x, -\beta t+x) \), then \( \partial[X_B] = (\partial/c, -\nabla)\gamma(ct-\beta \cdot x, -\beta t+x) = [\gamma, -\gamma \beta, -\gamma \beta, \gamma] = \Lambda^{\mu \nu} \)
\( \partial[K \cdot X_B] = \partial[K] \cdot X_B + K \cdot \partial[X_B] = \Lambda^{\mu \nu} K = K_B \) as a Lorentz Boosted \( K \), as expected
\( (\partial-\partial)[K \cdot X_B] = (\partial-\partial)[K \cdot X_B] + K \cdot \partial[X_B] = 0 + K \cdot \partial[X] = \partial[K] \cdot X + K \cdot \partial[X] = \partial(\partial[K \cdot X]) = \partial(\partial)[K \cdot X] \):

**SR Waves:**
Let \( \Psi = ae^{-i(K \cdot X)}, \Psi_T = ae^{-i(K \cdot X_T)}, \Psi_B = ae^{-i(K \cdot X_B)} \)
\( (\partial-\partial)[K \cdot X_T] = (\partial-\partial)[K \cdot X_S] = (\partial-\partial)[K \cdot X] = (\partial-\partial)[K \cdot X] = (\partial-\partial)[K \cdot X] \): Wave Equation Invariant under all Poincaré transforms
Total of \((1+3+3+3 = 10)\) Invariances from Poincaré Symmetry
SRQM Study:

10 Poincaré Symmetry Invariances

Noether's Theorem: 10 SR Conservation Laws

4-AngularMomentum $L^\mu = \frac{1}{c} \int_V \mathbf{r} \times \mathbf{g} \, dV$

4-LinearMomentum $P^\mu = \mathbf{p}^\mu$

4-Displacement $\Delta X = (\Delta \mathbf{x}, \Delta t)$

Interpretation

SciRealm.org

SR → QM

4-Vector SRQM Interpretation of QM

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SRQM Study: 4-Vector Operations

Lorentz Scalar Product $A \cdot B = A_\mu B^\mu$

Exterior Product $A \wedge B = A^\mu B^\nu - A^\nu B^\mu$

There are at least three 4-Vector relations which use the Exterior (Wedge=$\wedge$) Product.

$$\partial^\wedge A = \partial^\mu A^\nu - \partial^\nu A^\mu = F_{\mu\nu} : \text{the Faraday EM 4-Tensor}$$

$$R^P = R_\mu^\nu P^\nu = R_\nu^\mu P^\mu : \text{the 4-Angular-Momentum Tensor}$$

$$R^F = R_\mu^\nu F^\nu = R_\nu^\mu F^\mu : \text{the 4-(Angular-)Torque Tensor}$$

This gives the components of each remarkably similar properties.

Likewise, each of these has a physical (Dot=$\cdot$) Product relation as well.

$$\partial A = \partial^\mu A_\mu = 0 : \text{the Lorenz Gauge, a conservation of 4-EMVectorPotential}$$

$$R \cdot P = R_\mu P^\mu = -S_{\text{action,free}} : \text{the Action Scalar}$$

$$R \cdot F = R_\mu F^\mu = ??? : \text{probably something important}$$

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$$R \cdot F = R_\mu F^\mu = ??? : \text{probably something important}$$
SRQM Study:

4-Momentum → 4-Force

4-AngularMomentum → 4-Torque

Linear:
4-Force is the ProperTime Derivative of 4-Momentum.

Angular:
4-Torque is the ProperTime Derivative of 4-AngularMomentum.

\[
\frac{d}{d\tau}[M^{\mu\nu}] = \frac{d}{d\tau}[X^\mu P^\nu - X^\nu P^\mu] = [U^\mu P^\nu + U^\mu F^\nu - U^\nu P^\mu - X^\nu F^\mu] = [U^\mu m_0 U^\nu + U^\mu F^\nu - U^\mu m_0 U^\nu - X^\mu F^\nu] = [m_0 (U^\mu U^\nu - U^\mu U^\nu) + X^\mu F^\nu - X^\mu F^\nu] = [m_0 (0^{\mu\nu}) + X^\mu F^\nu - X^\mu F^\nu] = [X^\mu F^\nu - X^\mu F^\nu]
\]

\[
\frac{d}{d\tau}[P^\mu] = T^{\mu\nu} = [X^\mu F^\nu - X^\nu F^\mu] = X^\mu F
\]

4-Velocity
\[
U = \gamma(c, u) = dX/d\tau
\]

4-Momentum
\[
P^\mu = P = (mc, p) = (E/c, p)
\]

4-Force
\[
F = \gamma(E/c, f^p) = dP/d\tau
\]

4-Gradient
\[
\partial[\Lambda] = \partial(\partial/c, -V)
\]

4-Displacement
\[
\Delta X = (c\Delta t, \Delta x)
\]

4-Position
\[
X = (ct, x)
\]

SpaceTime
\[
\partial X = \partial_\mu X^\mu = 4 \text{ Dimension}
\]

Minkowski
\[
\partial[X] = \partial^\mu [X^\nu] = \eta^{\mu\nu}
\]

Lorentz
\[
\partial_\nu [X^\mu] = \Lambda^\nu_\nu
\]

4-Vector SRQM Interpretation of QM

Transform

\[
\text{4-Gradient}
\]

\[
\text{4-Force Tensor}
\]

\[
T^{\mu\nu} = X^\mu F^\nu - X^\nu F^\mu = X^\mu F
\]

\[
\text{4-Force}
\]

4-Displacement

\[
\Delta X = (\gamma(c, t), \Delta x)
\]

4-Position

\[
X = (ct, x)
\]

4-Force

\[
F = \gamma(E/c, f^p)
\]

4-Torque

\[
\frac{d}{d\tau}[m_0 E/c^2]
\]

4-Torque

\[
\Lambda = \gamma(E/c, f^p)
\]

4-Gradient

\[
\partial[\Lambda] = \partial(\partial/c, -V)
\]

4-Force

\[
F = \gamma(E/c, f^p)
\]

4-Torque

\[
\Lambda = \gamma(E/c, f^p)
\]

4-Gradient

\[
\partial[\Lambda] = \partial(\partial/c, -V)
\]
SR 4-Vectors & 4-Tensors

Lorentz Scalar Product & Tensor Trace

Invariants: Similarities

All {4-Vectors:4-Tensors} have an associated {Lorentz Scalar Product:Trace}

Each 4-Vector has a “magnitude” given by taking the Lorentz Scalar Product of itself.

\[ V \cdot V = V^\mu \eta_{\mu\nu} V^\nu = (v_0^2 + v_1^2 + v_2^2 + v_3^2) = (v_0^2 - v \cdot v) = (v_0^2) \]

The magnitude of \( V \) is \( V \cdot V \), which can be \((+/0/-)\)

Each 4-Tensor has a “magnitude” given by taking the Tensor Trace of itself.

\[ \text{Trace}[T_{\mu\nu}] = \text{Tr}[T_{\mu\nu}] = \eta_{\mu\nu} T^\mu_{\nu} = T^0_0 = T^{11} + T^{22} + T^{33} = (T^{00} - T^{11} - T^{22} - T^{33}) = T \]

Notice the similarities. In both cases there is a tensor contraction with the Minkowski Metric Tensor \( \eta_{\mu\nu} -> \text{Diag}[+1,-1,-1,-1] \) (Cartesian basis)

\[ V \cdot V = (v_0^2 - v \cdot v) = (v_0^2) \]

\[ 4 \text{-Vector} \]

\[ V = V^\mu = (v^0, v) \]

\[ \text{Trace Tensor Invariant} \]

\[ \text{4-Tensor} \]

\[ \eta_{\mu\nu} T^\mu_{\nu} = (T^{00} - T^{11} - T^{22} - T^{33}) = T \]

Notice the similarities. In both cases there is a tensor contraction with the Minkowski Metric Tensor \( \eta_{\mu\nu} \rightarrow \text{Diag}[+1,-1,-1,-1] \)

\[ \text{ex.} \quad P \cdot P = (E/c)^2 - p \cdot p = (E_0/c)^2 = (m_0 c)^2 \]

which says that the “magnitude” of the 4-Momentum is the RestEnergy/c = RestMass*c

\[ \text{ex. Trace}[\eta_{\mu\nu}] = (\eta^{00} - \eta^{11} - \eta^{22} - \eta^{33}) = 1 -(-1) -(-1) -(-1) = 1+1+1+1 = 4 \]

which says that the “magnitude” of the Minkowski Metric = SpaceTime Dimension = 4

\[ \text{Minkowski Metric} \]

\[ \partial[R] = \eta_{\mu\nu} \rightarrow \text{Diag}[+1,-1,-1,-1] \]

\[ \text{ex.} \quad \text{Trace}[T_{\mu\nu}] = \eta_{\mu\nu} T^\mu_{\nu} = T^0_0 = T \]

\[ V \cdot V = V^\mu \eta_{\mu\nu} V^\nu = [v_0^2 - v \cdot v] = (v_0^2) \]

\[ = \text{Lorentz Scalar Invariant} \]

\[ \text{SR 4-Vector SRQM Interpretation of QM} \]

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Some other SR Invariants include:

- **4-Vector**
  \[ V^\mu = \begin{pmatrix} v^0 \\ v \end{pmatrix} \]

- **4-Momentum**
  \[ P = (mc, p) = \begin{pmatrix} E/c \\ p \end{pmatrix} \]

- **Lorentz Scalar Invariant**
  \[ P \cdot P = (mc)^2 = (E/c)^2 \]

- **Phase Space Invariant**
  \[ \frac{d^3p}{E} \] if \( V \cdot V \) is (constant)

- **Particle #**
  \[ N = (\gamma V/c) \int dT \cdot N = \int n_0 d^3x \rightarrow n_0 V_0 \]

- **EM Charge**
  \[ Q = (\gamma V/c) \int dT \cdot J = \int \rho_0 d^3x \rightarrow \rho_0 V_0 \]

\[ d^4x = c dt \cdot dx \cdot dy \cdot dz = \gamma (dr) \cdot (dA) = \gamma d^3x \]

\[ d^4p = \int \frac{dE}{c} d\mathbf{p} = \int \frac{dE}{c} d^3\mathbf{p} \]

\[ d^4K = \int \frac{d\omega}{c} d\mathbf{k} = \int \frac{d\omega}{c} d^3\mathbf{k} \]

\[ \text{Rest Volume} \ V_0 = \int \gamma dV = \int \gamma d^3x = -cN/\int dT \cdot N = -cQ/\int dT \cdot J \]

\[ \text{Trace}[T^{\mu\nu}] = \eta^{\mu\nu} T^{\mu\nu} = T^{\mu\nu} = T \]

**SR 4-Vectors & 4-Tensors**

**More 4-Vector-based Invariants**
Some 4-Vectors have an alternate form of Tensor Invariant: \((\frac{dv'}{v'} = \frac{dv}{v})\) or \((\frac{d^3v'}{v'} = \frac{d^3v}{v})\) in addition to the standard Lorentz Invariant \(V \cdot V = V^\mu V_\mu = (v^0v^0 - v \cdot v) = (v_0^o)^2\)

If \(V \cdot V = \text{constant}\), with \(V = (v^0, v)\) then \(d(V \cdot V) = 2d(V \cdot dv) = d(\text{constant}) = 0\)

hence \((V \cdot dv) = 0 = v^0dv^0 - v \cdot dv\)

\(dv^0 = v \cdot dv/v^0\)

Generally: with \(\Lambda = \Lambda_{\nu}^\mu\) = Lorentz Boost Transform in the \(\beta\)-direction
\(V' = \Lambda V\) : from which the temporal component \(v_0' = (\gamma v_0 - \gamma \beta \cdot v)\)
\(dv' = \Lambda dv\) : from which the spatial component \(dv' = (\gamma dv - \gamma \beta dv^0)\)

Combining:
\(dv' = (\gamma dv - \gamma \beta (v \cdot dv/v^0))\)
\(dv' = (1/v^0)^{1/2}(v^0 dv - \gamma \beta (v \cdot dv))\)
\(dv' = (1/v^0)^{1/2}(v^0 dv - \gamma \beta \cdot v)dv\)
\(dv' = (v^0 - \gamma \beta \cdot v)\frac{1}{2}dv\)
\(dv' = (\gamma v^0/dv^0)dv\)
\(dv' = dv' and = Invariant of V = (v^0, v) for V \cdot V = \text{constant}\)

So, for example:
\(P \cdot P = (m_c)^2\) = (constant)

Thus, \(dp'(E/c) = dp/(E/c)\) = Invariant
Or, \(dp'/E' = dp/E \rightarrow d^3p/E = dp^*dp/dp^*E = \text{Invariant, usually seen as } \int F(\text{various invariants})*d^3p/E = \text{Invariant}\)
Invarient \( d^4X = -(V_0) dT \cdot dX = -(dV_0) T \cdot dX = cdt \, d^3x \)  

The 4D Position coords that are integrated to give a 4D volume: SI units \([m^4]\)

4-Differential \( dX = (cdt, dx) \)  
4-UnitTemporal \( T = \gamma(1, \beta) = (\gamma, \gamma \beta) \)  
4-UnitTemporalDifferential \(dT = d[\gamma, \gamma \beta] = (d\gamma, d\gamma \beta) \)

\[ V = \int dV = \int dx \cdot dy \cdot dz = \int \int \int dx \cdot dy \cdot dz = \int d^3x \]

\( V = V_0/\gamma \)

\( dV = d^3x \) 3D Spatial Volume Element  
\( \gamma \) is Lorentz Scalar

\[ d^4X = cdt \cdot dx \cdot dy \cdot dz = cdt \cdot d^3x \]

And, this makes sense.  
\( T \) is a temporal 4-Vector with fixed magnitude: \( T \cdot T = 1 \)  
\( d(T \cdot T) = d(1) = 0 = 2(dT \cdot T) \)

Since \((dT \cdot T) = 0\), \(dT\) must orthogonal to \(T\) and thus must be a spatial 4-Vector.  
If \(dX\) is also spatial, then the Lorentz scalar product \(\langle dT \cdot dX \rangle = -\text{magnitude}\) will be negative with this choice of Minkowski Metric.  
Thus, multiplying by \(-V_0\) gives a positive volume element \((cdt \cdot dx \cdot dy \cdot dz = d^4x\)

It is sort of quirky though, that the temporal \((cdt)\) comes from the \(dX\) part, and the spatial \((d^3x)\) comes from the \(dT\) part.
More 4-Vector-based Invariants

Phase Space Integration

\[ \rho \, d^4x = \rho' \, d^4x = (-V/c) dT \cdot J = \text{Lorentz Scalar Invariant} \]
\[ n \, d^4x = n' \, d^4x = (-V/c) dT \cdot N = \text{Lorentz Scalar Invariant} \]

4-CurrentDensity \( J = (pc, j) = \rho, U \)
4-NumberFlux \( N = (nc, n) = n, U \)
4-UnitTemporal \( T = \gamma(1, \beta) = (\gamma, \beta) \)
4-UnitTemporalDifferential \( dT = d[(\gamma, \beta)] = (d[\gamma], d[\beta]) \)

\[ V = V/c \]
\[ d\gamma = -(V/c)dV \]

\((-V/c)\, dT \cdot J = \text{Invariant, because (Rest Scalar } \times \text{ Lorentz Scalar Product) = Invariant} \]
\[ = (-V/c)(\gamma, \beta)(pc, j) \]
\[ = (-V/c)(d[\gamma]; pc - d[\beta]j) \]
\[ = (-V/c)(-\gamma/V^3)(dv/pc) - d[\beta]j \]
\[ = (-V/c)(-\gamma/V^3)(dv/pc) - d[\beta](1)j \]
\[ = (-V/c)(-\gamma/V^3)(dv/pc) \]
\[ = (dv/pc) \]
\[ = (pc)(dv/c) \]
\[ = (p)(dv) \]
\[ = p \, d^4x \]

Total Charge \( Q = [\gamma p, d^4x = [p, d^4x = \text{Lorentz Scalar Invariant} \]
Total Particle # \( N = [\gamma n, d^4x = [n, d^4x = \text{Lorentz Scalar Invariant} \]
Total RestVolume \( V_o = \gamma(1) d^4x = \text{Lorentz Scalar Invariant} \]

This also gives an alternate way to define the RestVolume Invariant \( V_o \).

\[ (-V/c) \, dT \cdot N = n^d x \]
\[ N = [\gamma n^2 = [(-V/c) \, dT \cdot N \]
\[ cN/V_o = -dT \cdot N \]
\[ V_o = cn/dT \cdot N \]

SR 4-Tensor

(2,0)-Tensor \( T^{\mu}_\nu \)
(1,1)-Tensor \( T^{\mu}_\nu \) or \( T^{\mu}_\nu \)
(0,2)-Tensor \( T_{\mu}^{\nu} \)

SR 4-Vector

\( V = V^\gamma = \gamma(c, u) \)

SR 4-CoVector: OneForm

\( (1,0)-\text{Tensor} \)

SR 4-Scalar

(0,0)-Tensor \( S \) or \( S_o \)

Lorentz Scalar

\[ \text{Trace}[T^{\mu}_\nu] = \eta^{\mu}_\nu \, T^{\nu}_\mu = T \]
\[ V \cdot V = V^\gamma \eta^\gamma_\nu V^\nu = (V^\gamma)^2 - V \cdot V = (V^\gamma)^2 \]
\[ = \text{Lorentz Scalar Invariant} \]
The 4D Momentum coords that are integrated to give a 4D Momentum Volume: SI Units 

\[ [(kg \cdot m/s)^4] \]

The 4D WaveVector coords that are integrated to give a 4D WaveVector Volume: SI Units 

\[ [(1/m)^4] \]

The 4D Differential Momentum 

\[ dP = (dE/c) \, dp \]

The 4D Differential WaveVector 

\[ dK = (d\omega/c) \, dk \]

4-Unit Temporal Differential 

\[ dT = (d[\gamma], d[\gamma\beta]) \]

4-Unit Temporal 

\[ T = \gamma(1, \beta) = (\gamma, \gamma\beta) \]

4-Momentum Differential 

\[ dP = dP^\mu = (dE/c, dp) \]

Phase Space Integration 

\[ \int \text{F[Various Invariants]} \, d^4P \]

\[ \int \text{F[Various Invariants]} \, d^4K \]

SR 4-Vectors & 4-Tensors

More 4-Vector-based Invariants

Phase Space Integration

SR 4-Vector

(2,0)-Tensor \( T^\mu_\nu \)

(1,1)-Tensor \( V^\mu = V = (v^0, v) \)

SR 4-CoVector: OneForm

(0,1)-Tensor \( V_\mu = (\omega_0, \omega) \)

SR 4-Scalar

(0,0)-Tensor \( S \) or \( S_0 \)

Lorentz Scalar
SR 4-Vectors & 4-Tensors

More 4-Vector-based Invariants

Phase Space Integration

\[ \text{4-Unit Temporal Differential} \quad dT = (d[\gamma], d[\gamma \beta]) \]

\[ \int \text{F[various Invariants]} \, d^3p \, d^3x \]

\[ \int \text{F[various Invariants]} \, d^3k \, d^3x \]

Likewise, \( d^3k \, d^3x = \text{Invariant} \)

**SR 4-Vector**
- (1,0)-Tensor \( \mathbf{V} = (v^0, \mathbf{v}) \)
- (1,1)-Tensor \( \mathbf{V} \circ \mathbf{V} = \mathbf{V} \times \mathbf{V} \)
- (0,2)-Tensor \( \mathbf{S} \) or \( \mathbf{S}_0 = \text{Lorentz Scalar} \)

**SR 4-Scalar**
- (0,0)-Tensor \( \mathbf{S}_0 = \text{Lorentz Scalar} \)

**SR 4-Tensor**
- (2,0)-Tensor \( \mathbf{T}_{iv} \)
- (2,1)-Tensor \( \mathbf{T}_{iv} \circ \mathbf{T}_{iv} \)
- (1,1)-Tensor \( \mathbf{T} \)
- (0,2)-Tensor \( \mathbf{T}_{iv} \circ \mathbf{T}_{iv} \)

---

\[ \text{Trace}[T_{iv}] = \eta_{iv} T_{iv} = T_i^i = T \]
\[ \text{V-V} = (v^0, \mathbf{v}) \times (v^0, \mathbf{v}) = (v^0)^2 - \mathbf{v} \cdot \mathbf{v} = (v^0)^2 \]

\[ = \text{Lorentz Scalar Invariant} \]
SRQM Study: SR 4-Tensor Properties

General → Symmetric & Anti-Symmetric

Any SR Tensor $T^{\mu\nu} = (S^{\mu\nu} + A^{\mu\nu})$ can be decomposed into parts:

- Symmetric: $S^{\mu\nu} = (T^{\mu\nu} + T^{\nu\mu})/2$ with $S^{\mu\nu} = +S^{\nu\mu}$
- Anti-Symmetric: $A^{\mu\nu} = (T^{\mu\nu} - T^{\nu\mu})/2$ with $A^{\mu\nu} = -A^{\nu\mu}$

$S^{\mu\nu} + A^{\mu\nu} = (T^{\mu\nu} + T^{\nu\mu})/2 + (T^{\mu\nu} - T^{\nu\mu})/2 = T^{\mu\nu}/2 + T^{\nu\mu}/2 - T^{\nu\mu}/2 - T^{\mu\nu}/2 = T^{\mu\nu} + 0 = T^{\mu\nu}$

Max 16 possible

Independent components: \{ $4^2 = 16 = 10 + 6$ \}

Max 10 possible

Symmetric 4-Tensor $S^{\mu\nu} = \begin{bmatrix} S^{00},& S^{01},& S^{02},& S^{03} \\ S^{10},& S^{11},& S^{12},& S^{13} \\ S^{20},& S^{21},& S^{22},& S^{23} \\ S^{30},& S^{31},& S^{32},& S^{33} \end{bmatrix}$

Tr$[S^{\mu\nu}] = S^{\mu\mu}$

Max 6 possible

Anti-Symmetric 4-Tensor $A^{\mu\nu} = \begin{bmatrix} A^{00},& A^{01},& A^{02},& A^{03} \\ A^{10},& A^{11},& A^{12},& A^{13} \\ A^{20},& A^{21},& A^{22},& A^{23} \\ A^{30},& A^{31},& A^{32},& A^{33} \end{bmatrix}$

Tr$[A^{\mu\nu}] = 0$

Importantly, the Contraction of any
Symmetric tensor
with any
Anti-Symmetric tensor
on the same pair of indices is always 0.

*Note* These don’t have to be composed from a single general tensor.

$S^{\mu\nu}A_{\mu\nu} = 0$

Proof:

$S^{\mu\nu}A_{\mu\nu} = S^{\mu\nu}A^{\nu\mu}$ because we can switch dummy indices
$= (+S^{\mu\nu})A^{\nu\mu}$ because of symmetry
$= S^{\mu\nu}(-A^{\nu\mu})$ because of anti-symmetry
$= -S^{\mu\nu}A^{\nu\mu}$
$= 0$: because the only solution of \{$c = -c$\} is 0.

Physically, the anti-symmetric part contains rotational information and the symmetric part contains information about isotropic scaling and anisotropic shear.

**Maximum Degrees of Freedom (DoF)**

- SR 4-Tensor $(2,0)$-Tensor $T^{\mu\nu}$
- (1,1)-Tensor $T^{\mu\nu}$, or $T_{\mu\nu}$
- (0,2)-Tensor $T_{\mu\nu}$

- SR 4-Vector $(1,0)$-Tensor $V = \begin{bmatrix} \eta_{00},& \eta_{01},& \eta_{02},& \eta_{03} \end{bmatrix}$
- $V_{\mu\nu}V^{\mu\nu} = (V_{\mu})^2$

- SR 4-Scalar $(0,0)$-Tensor $S$ or $S_{\mu\nu}$
  - Lorentz Scalar

- Trace of Tensor $Tr[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu\mu} = T$

- Lorentz Scalar Invariant

$V_{\mu\nu}V^{\mu\nu} = (V_{\mu})^2$ = Lorentz Scalar Invariant
Any Symmetric SR Tensor \( S^{\mu \nu} = (T^{\mu \nu}_{\text{iso}} + T^{\mu \nu}_{\text{aniso}}) \) can be decomposed into parts:

**Isotropic**  \( T^{\mu \nu}_{\text{iso}} = (1/4)\text{Trace}[S^{\mu \nu}] \)  \( \eta^{\mu \nu} = (T) \eta^{\mu \nu} \)

**Anisotropic**  \( T^{\mu \nu}_{\text{aniso}} = S^{\mu \nu} - T^{\mu \nu}_{\text{iso}} \)

The Anisotropic part is Traceless by construction, and the Isotropic part has the same Trace as the original Symmetric Tensor. The Minkowski Metric is a symmetric, isotropic 4-tensor with \( T=1 \).

### Independent components:

**Max 10 possible**

<table>
<thead>
<tr>
<th>Symmetric 4-Tensor</th>
<th>Independent components:</th>
<th>Max 1 possible</th>
<th>Max 9 possible</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S^{\mu \nu} = )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( [S^{00}, S^{01}, S^{02}, S^{03}] )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( +S^{01}, +S^{12}, +S^{23} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( [T, 0, 0, 0] )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 0, 0, -T, 0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 0, 0, 0, -T )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Symmetric Isotropic 4-Tensor**

\( T^{\mu \nu}_{\text{iso}} = (1/4)\text{Trace}[S^{\mu \nu}] \)

**Symmetric Anisotropic 4-Tensor**

\( T^{\mu \nu}_{\text{aniso}} = S^{\mu \nu} - T^{\mu \nu}_{\text{iso}} \)

\( \text{with } T = (1/4)\text{Trace}[S^{\mu \nu}] \)

**Maximum Degrees of Freedom (DoF)**

\( = \) # of possible independent components \( = (\text{Tensor dimension})^\text{rank(4-tensor)} \)

Importantly, the Contraction of any Symmetric tensor with any Anti-Symmetric tensor on the same index is always 0.

*Note* These don’t have to be composed from a single general tensor.

**Proof:**

\( S^{\mu \nu} A_{\mu \nu} = 0 \) because we can switch dummy indices

\( = (+S^{\mu \nu})A_{\nu \mu} \) because of symmetry

\( = S^{\mu \nu}(-A_{\mu \nu}) \) because of anti-symmetry

\( = -S^{\mu \nu}A_{\mu \nu} \) = 0; because the only solution of \( c = -c \) is 0

Physically, the isotropic part represents a direction independent transformation (e.g., a uniform scaling or uniform pressure); the deviatoric part represents the distortion

An Isotropic Tensor has the same components in all possible coordinate-frames.

**Rank 0:** All Scalars are isotropic

**Rank 1:** There are no non-zero isotropic vectors

**Rank 2:** Most general isotropic 2\(^{nd}\) rank tensor must equal to \( \lambda_{\mu \nu} = \lambda \eta_{\mu \nu} \) for some scalar \( \lambda \)

**Rank 3:** Most general isotropic 3\(^{rd}\) rank tensor must equal to \( \lambda_{\mu \nu} \delta_{\nu \mu} \) for some scalar \( \lambda \)

**Rank 4:** Most general isotropic 4\(^{th}\) rank tensor must equal to \( a_{\mu \nu} \delta_{\nu \mu} \) for some scalars \( a, b, c, d \)

Maximum Degrees of Freedom (DoF)

\( = \) # of possible independent components

\( = (\text{Tensor dimension})^\text{rank(4-tensor)} \)

\( \text{Lorentz Scalar Invariant} \)

\( \text{Trace}[T^{\mu \nu}] = \eta_{\mu \nu} T^{\mu \nu} = T_{\nu \nu} = T \)

\( V\cdot V = V^\mu \nu V^\nu = (V^\mu V^\nu) - (V^\mu) (V^\nu) = (V^\mu V^\nu) \)

\( = \text{Lorentz Scalar Invariant} \)
SRQM Study: SR 4-Tensors

4-Tensor Decomposition based on Tensor Invariants

General (rank=2) 4-Tensor $T^{\mu\nu}$

$= T_{\text{symm}}^{\mu\nu} + T_{\text{anti-symm}}^{\mu\nu}$

Symmetric 4-Tensor

$T_{\text{symm}}^{\mu\nu} = (T^{\mu\nu} + T^{\nu\mu})/2$

$= T_{\text{iso}}^{\mu\nu} + T_{\text{aniso}}^{\mu\nu}$

Uniform scaling info

max DoF = 1

Tr[$T_{\text{iso}}^{\mu\nu}$] = 0

Anisotropic Symm

4-Tensor

$T_{\text{aniso}}^{\mu\nu} = T_{\text{symm}}^{\mu\nu} - T_{\text{iso}}^{\mu\nu}$

Shearing: distortion info

max DoF = 9

Tr[$T_{\text{aniso}}^{\mu\nu}$] = 0

Anti-Symmetric 4-Tensor $T_{\text{anti-symm}}^{\mu\nu}$

$= (T^{\mu\nu} - T^{\nu\mu})/2$

Rotational info

max DoF = 6

Tr[$T_{\text{anti-symm}}^{\mu\nu}$] = 0

max DoF = (dim)$^2$(rank) = 4$^2$ = 4x4 = 16 = (10+6)

SR 4-Tensor

(2,0)-Tensor $T^{\mu\nu}$

(1,1)-Tensor $T^{\mu\nu}$, or $T^{\mu}_{\nu}$

(0,2)-Tensor $T^{\mu}_{\nu}$

SR 4-Vector

(1,0)-Tensor $V^{\mu} = V = (v^0,v)$

SR 4-CoVector: OneForm

(0,1)-Tensor $V_{\mu} = (v_0,-v)$

SR 4-Scalar

(0,0)-Tensor $S$ or $S_o$

Lorentz Scalar

Maximum Degrees of Freedom (DoF)

=# of possible independent components

= (Tensor dimension)$^2$(Tensor rank)

$\text{Trace}[T^{\mu\nu}] = \eta^{\mu\nu}T^{\mu\nu} = T_{\mu\nu} = T$

$V \cdot V = V^{\mu}\eta_{\mu\nu}V^{\nu} = [(v^0)^2 - v_0v] = (v^0)^2$

= Lorentz Scalar Invariant
SRQM Study: SR 4-Tensors

4-Tensor Temporal: Mixed: Spatial (1+3) Decomposition

General (rank=2) 4-Tensor $T_{\mu \nu} = T_{\text{sym}}^{\mu \nu} + T_{\text{anti-sym}}^{\mu \nu}$

$\text{max DoF} = (\text{dim})^{\text{rank}} = 4^2 = 4 \times 4 = 16 = (10 + 6)$

Symmetric 4-Tensor:

$T_{\text{sym}}^{\mu \nu} = \frac{(T^{\mu \nu} + T^{\nu \mu})}{2}$

$\text{max DoF} = 10$

$\text{Trace}[T_{\text{sym}}^{\mu \nu}] = \eta^{\mu \nu} T_{\text{sym}}^{\mu \nu} = \frac{(T^{\mu \nu} + T^{\nu \mu})}{2}$

Spatial Symmetric 4-Tensor:

$T_{\text{sym}}^{\mu \nu} = \eta^{\mu \nu} T_{\text{sym}}^{\mu \nu}$

$\text{max DoF} = 6$

$\text{Trace}[T_{\text{sym}}^{\mu \nu}] = \eta^{\mu \nu} T_{\text{sym}}^{\mu \nu} = \frac{(T^{\mu \nu} + T^{\nu \mu})}{2}$

Isotropic Spatial Symmetric 4-Tensor:

$T_{\text{iso}}^{\mu \nu} = \frac{(T_{\text{sym}}^{\mu \nu} + T_{\text{iso}}^{\mu \nu})}{2}$

$\text{max DoF} = 1$

$\text{Trace}[T_{\text{iso}}^{\mu \nu}] = \eta^{\mu \nu} T_{\text{iso}}^{\mu \nu}$

Anisotropic Spatial Symmetric 4-Tensor:

$T_{\text{aniso}}^{\mu \nu} = T_{\text{sym}}^{\mu \nu} - T_{\text{iso}}^{\mu \nu}$

$\text{max DoF} = 5$

$\text{Trace}[T_{\text{aniso}}^{\mu \nu}] = 0$

Effective 3-vector mixed time: space info

$Q_{\mu} = \rho_{0} c^{2} Q_{\mu}^{0} / c + \Pi_{\mu}$

Effective scalar

$\rho_{0} = \rho_{m} c^{2}$

Effective symm 3-tensor spatial info

$\text{max DoF} = 6$

Mixed Symmetric 4-Tensor:

$T_{\text{mixed}}^{\mu \nu} = V_{\mu}^{\alpha} H_{\nu}^{\beta} (T_{\text{sym}}^{\alpha \beta})$

$\text{max DoF} = 3$

Effective 3-tensor uniform scaling info ex. pressure ($p$)

$\text{max DoF} = 3$

Effective scalar temporal info ex. energy density ($\rho_{e}$)

$\text{max DoF} = 1$

Temporal Symmetric 4-Tensor:

$T_{\text{temp}}^{\mu \nu} = V_{\mu}^{\alpha} V_{\nu}^{\beta} (T_{\text{sym}}^{\alpha \beta})$

$\text{max DoF} = 1$

$\text{max DoF} = (\text{dim})^{\text{rank}} = 4^2 = 4 \times 4 = 16 = (10 + 6)$

$\text{SR 4-Tensor (2,0)-Tensor } T_{\mu}^{\nu}$

$\text{SR 4-Vector (1,0)-Tensor } V = (V^{\mu}, V_{\nu})$

$\text{SR 4-CoVector: OneForm (0,1)-Tensor } V_{\mu} = (V_{\mu}^{\nu})$

$\text{SR 4-Scalar (0,0)-Tensor or } S_{\mu}$

$L_{\text{O}}$ or Lorentz Scalar

4-Vector SRQM Interpretation of QM

$\text{SciRealm.org}$

John B. Wilson
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http://scirealm.org/SRQM.pdf
SRQM Study: SR 4-Tensors

SR Tensor Invariants

(0,0)-Tensor = Lorentz Scalar S: Has either (0) or (1) Tensor Invariant, depending on exact meaning (S) itself is Invariant

(1,0)-Tensor = 4-Vector V^\mu: Has (1) Tensor Invariant = The Lorentz Vector Product

V\cdot V = \eta_{\mu\nu}V^\mu V^\nu = \text{Tr}[V^\mu V^\nu] = V^\mu V^\nu = (V^0 v^0 + v^1 v^1 + v^2 v^2 + v^3 v^3) = (v^0)^2

V\cdot V = (v^0 v^0 - v\cdot v) = (v^0)^2

(2,0)-Tensor = 4-Tensor

(3,0)-Tensor = 4-Tensor

(4,0)-Tensor = 4-Tensor

(1,1)-Tensor T^\alpha_\beta: Has (4+) Tensor Invariants (though not all independent)

a): T^\mu_\nu T^\nu_\nu = \text{Trace}[T^\mu_\nu] = \eta_{\mu\nu}T^\mu_\nu = T^\mu_\nu = (T^0_0 + T^1_1 + T^2_2 + T^3_3) = (T^0_0 - T^1_1 - T^2_2 - T^3_3) = (T)

for anti-symmetric: = 0

b): InnerProduct T^\mu_\nu T^\nu_\nu = T^0_0 T^0_0 + T^1_0 T^1_0 + T^2_0 T^2_0 + T^3_0 T^3_0 = (T^0_0)^2 - \Sigma[T^0_0]^2 = (T^0_0)^2 - \Sigma[T^0_0]^2 + \Sigma[T^0_0]^2 + \Sigma[T^0_0]^2

for symmetric | anti-symmetric: = (T^0_0)^2 - 2\Sigma[T^0_0]^2 + \Sigma[T^0_0]^2 + \Sigma[T^0_0]^2 + 2\Sigma[T^0_0]^2

c): Antisymmetric Triple Product T^\alpha_\beta T^\beta_\gamma T^\gamma_\delta = (T^0_\gamma T^\gamma_\delta) - 3(T^0_\gamma T^\gamma_\delta)(T^0_\gamma T^\gamma_\delta) + T^\beta_\gamma T^\gamma_\delta + T^\gamma_\delta T^\beta_\gamma - T^\gamma_\delta T^\beta_\gamma

for anti-symmetric: = 0

d): Determinant Det[T^\mu_\nu] = (1/2)\epsilon_{\mu\nu\rho\sigma}T^{\rho\sigma}

(1,0)-Tensor V= V^\mu = (v^0, v^1, v^2, v^3)

\text{Trace}[T^\mu_\nu] = \eta_{\mu\nu}T^\mu_\nu = T^\mu_\nu = (T^0_0 + T^1_1 + T^2_2 + T^3_3) = (T)

V\cdot V = \eta_{\mu\nu}V^\mu V^\nu = (v^0)^2

V\cdot V = (v^0)^2

\text{Determinant Tensor Invariant}

4-Tensor

\text{Asymmetric Tri-Product Tensor Invariant}

\text{Inner Product Tensor Invariant}

\text{Trace Tensor Invariant}

4-Vector SRQM Interpretation of QM

\text{Phys}

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http://scirealm.org/SRQM.pdf

\text{SR 4-Vector Interpretation of Physical 4-Vectors}

Det[T^\mu_\nu] = \Pi_\alpha[\lambda_\alpha]; \text{ with } \{\lambda_\alpha\} = \text{Set of Eigenvalues}

\text{Characteristic Eqns: Det[T^\mu_\nu - \lambda I_{(4)}] = 0}
SRQM Study: SR 4-Tensors

SR Tensor Invariants

Tensor Gymnastics

Some Tensor Gymnastics:

Matrix $A = \text{Tensor } A^i_j$
with rows denoted by “r”, columns by “c”

Example with dim=4: $r,c = (0..3)$
Matrix $A = [A_{r,c} = \delta_{r,c}]$
\[
\sum_{r=0}^{3} \sum_{c=0}^{3} A_{r,c} = 4
\]

If we have sums over both indices:
$A_{a,b} B_{a,b} = M_{a,b} = \text{Trace}[M]$
The sum over “d” gives the matrix multiplication and then the sum over “d” gives the Trace of the resulting matrix $M$

$A_{a,b} A_{a,b} = (\text{Trace}[A])^2 - \text{Trace}[A]^2$

with brackets [ ] around the indices indicating anti-symmetric product

The Trace formula's are independent of tensor dimension.

$A_{r,c} = \text{Tr}[A]$

$A_{r,a} A_{a,b} = A_{r,b} - A_{r,a} = (\text{Tr}[A])^2 - \text{Tr}[A]^2$

$A_{r,a} A_{a,b} = A_{r,b} - A_{r,a} = (\text{Tr}[A])^2 - \text{Tr}[A]^2$

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$A_{r,a} A_{a,b} = A_{r,b} - A_{r,a} = (\text{Tr}[A])^2 - \text{Tr}[A]^2$
A Tensor Study
Physics

(1,1)-Tensor $T_{\mu \nu}$

$\dim = 4$

The following are the Principle Tensor Invariants for dimensions 1..4

**General Cayley-Hamilton Theorem**

$A^4+c_3A^3+c_2A^2+c_1A+c_0 = 0_{(d)}$, with $A$ = square matrix, $d$ = dimension, $A^0 = Identity(d) = I_{(d)}$

Characteristic Polynomial: $p(\lambda) = \text{Det}[A - \lambda I_{(d)}]$

**4D Invariants**

$I_4 = \Sigma[\text{Unique Eigenvalue Naughts}] = 1$  \hspace{1cm} (1)

$I_3 = \Sigma[\text{Unique Eigenvalue Singles}] = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$  \hspace{1cm} (4)

$I_2 = \Sigma[\text{Unique Eigenvalue Doubles}] = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4$  \hspace{1cm} (6)

$I_1 = \Sigma[\text{Unique Eigenvalue Triples}] = \lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4$  \hspace{1cm} (4)

$I_0 = \Sigma[\text{Unique Eigenvalue Quadruples}] = \lambda_1\lambda_2\lambda_3\lambda_4$  \hspace{1cm} (1)

Each dimension gives the number of elements from it's row in Pascal's Triangle :)

$\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are Eigenvalues

$\lambda_1 \lambda_2 \lambda_3 \lambda_4$ are Unique Eigenvalue Naughts

$\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are Unique Eigenvalue Singles

$\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4$ are Unique Eigenvalue Doubles

$\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4$ are Unique Eigenvalue Triples

$\lambda_1\lambda_2\lambda_3\lambda_4$ are Unique Eigenvalue Quadruples

$\Sigma[\text{Eigenvalues}] = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$

$\Sigma[\text{Unique Eigenvalue Naughts}] = 1$

$\Sigma[\text{Unique Eigenvalue Singles}] = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$

$\Sigma[\text{Unique Eigenvalue Doubles}] = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4$

$\Sigma[\text{Unique Eigenvalue Triples}] = \lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4$

$\Sigma[\text{Unique Eigenvalue Quadruples}] = \lambda_1\lambda_2\lambda_3\lambda_4$

Characteristics Eqns: $\text{Det}[T_{\mu \nu} - \lambda I_{(d)}] = 0$
### General Cayley-Hamilton Theorem

\[ A^d + c_{d-1}A^{d-1} + \ldots + c_0A = 0 \_\_\_\_0 \]  
with \( A = \text{square matrix} \),  
\( d = \text{dimension} \), \( A^0 = \text{Identity}(d) = I_d \).  
\( I_d A^4 - I_d A^3 + I_d A^2 - I_d A + I_d = 0 : \text{for 4D} \)

**Characteristic Polynomial:**  
\( p(\lambda) = \det[A - \lambda I_d] \)

### Tensor Invariants \( I_n \)

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Formula</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( I_0 = 1/0! = 1 )</td>
<td>( I_0 = \left( \begin{array}{cccc} a \ b \ c \end{array} \right) )</td>
</tr>
<tr>
<td>2</td>
<td>( I_1 = \text{tr}[A]/1! = A^a_b )</td>
<td>( I_1 = \left( \begin{array}{cccc} a \ b \ c \end{array} \right) )</td>
</tr>
<tr>
<td>2</td>
<td>( I_2 = (\text{tr}[A]^2 - \text{tr}[A^2])/2! = A^a_{\alpha \beta} A^\alpha_{\beta 1} / 2 )</td>
<td>( I_2 = \left( \begin{array}{cccc} a \ b \ c \end{array} \right) )</td>
</tr>
<tr>
<td>3</td>
<td>( I_3 = [\text{tr}(A^3) - 3 \text{tr}(A^2)(\text{tr} A) + 2 \text{tr}(A^3)]/3! = A^a_{\alpha \beta \gamma} A^\alpha_{\beta \gamma 1} / 6 )</td>
<td>( I_3 = \left( \begin{array}{cccc} a \ b \ c \end{array} \right) )</td>
</tr>
<tr>
<td>4</td>
<td>( I_4 = [(\text{tr}(A)^4 - 6 \text{tr}(A^2)(\text{tr} A)^2 + 3(\text{tr}(A^2))^2 + \text{tr}(A^4)]/4! )</td>
<td>( I_4 = \left( \begin{array}{cccc} a \ b \ c \end{array} \right) )</td>
</tr>
</tbody>
</table>
SRQM Study: SR 4-Tensors

SR Tensor Invariants for Faraday EM Tensor

The Faraday EM Tensor $F^\mu_\nu = \partial^\mu A^\nu - \partial^\nu A^\mu = \partial ^\wedge A$ is an anti-symmetric tensor that contains the Electric and Magnetic Fields, defined by the Exterior “Wedge” Product (^). The 3-electric components ( $e = e_i = eF_i^3$ ) are in the temporal-spatial sections. The 3-magnetic components ( $b = b^i = -(1/2)e_{ij}F_i^j$ ) are in the only-spatial section.

(2,0)-Tensor = 4-Tensor $T^\mu_\nu$ Has (4+) Tensor Invariants (though not all independent)

- $T^\mu_\nu = $ Trace = Sum of EigenValues for (1,1)-Tensors (mixed)
- $T^\mu_\nu T^\nu_\mu = $ Asym Tri-Product $\rightarrow$ Inner Product
- $T^\mu_\nu T^\nu_\gamma = $ Asym Tri-Product $\rightarrow$ ?Name?
- $T^\mu_\nu T^\nu_\gamma T^\gamma_\alpha = $ Asym Quad-Product $\rightarrow$ 4D Determinant $\rightarrow$ Product of EigenValues for (1,1)-Tensors

Fundamental EM Invariants:
- $P = (1/2)F^\mu_\nu F^\nu_\mu = (1/2)F_i^\mu F_i^\nu = 0$ (Asym, Inner Product Tensor Invariant)
- $Q = (1/4)F^\mu_\nu F^\nu_\mu = (1/8)e_{ijk}F_i^\mu F_j^\nu F_k^\rho = 0$ (Inner Product Tensor Invariant)

The 4-Gradient $\partial = \partial^\mu = (\partial/c, \nabla)$

$\text{Det}[F_{\mu\nu}]= (e-b)(e\cdot e/c^2)$

Asymmetric Tri-Product Tensor Invariant

$\text{AsymmTri}[F_{\mu\nu}]= 0$

4-Gradient Tensor Invariant

$\text{Tr}[F_{\mu\nu}]= F_{\nu\nu} = 0$

$\text{Trace Tensor Invariant}$

Importantly, the Faraday EM Tensor has only (2) linearly-independent Lorentz invariants:
- $2((b\cdot b)\cdot (e\cdot e/c^2))$
- $2((e\cdot e/c)^2)$

The 4-Gradient and 4-EMVectorPotential have (4) independent components each, for total of (8).

Subtract (2) due to the invariants which provide constraints to get a total of (6) independent components $= (6)$ independent components of a 4x4 anti-symmetric tensor = (3) 3-electric $e$ and (3) 3-magnetic $b$ = (6) independent EM field components

Note: It is possible to have non-zero $e$ and $b$, yet still have zeroes in the Tensor Invariants. If $e$ is orthogonal to $b$, then $\text{Det}[F_{\mu\nu}]= (b\cdot e/c^2) = 0$.

If $\text{InnerProd}[F_{\mu\nu}] = 2((b\cdot b)\cdot (e\cdot e/c^2)) = 0$.

These conditions lead to the properties of EM waves = photons = null 4-vectors, which have fields $|b| = |e|/c$ and $b$ orthogonal to $e$, travelling at velocity $c$.

4-Vector: SRQM Interpretation of QM
SRQM Study: SR 4-Tensors

SR Tensor Invariants for 4-AngularMomentum Tensor

The 4-AngularMomentum Tensor \( M^a_{\beta} = X^a P^\beta - X^\beta P^a \) is an anti-symmetric tensor. The 3-mass-moment components \( (n = n_i = M_{i}/c) \) are in the temporal-spatial sections. The 3-angular-momentum components \( (I = I^i = +\frac{1}{2}c_i^i M_{i}/c) \) are in the only spatial sections.

\[
2\text{-Tensor} = 4\text{-Tensor } T^{ij} \text{ has (4+) Tensor Invariants (though not all independent)}
\]

a) \( T^{ij} = \text{Trace } = \text{Sum of EigenValues for (1,1)-Tensors (mixed)} \)
b) \( T^{ij} = \text{Asyrm Bi-Product } \rightarrow \text{Inner Product} \)
c) \( T^{ij}T^{ij} = \text{Asyrm Tri-Product } \rightarrow ? \text{Name?} \)
d) \( T^{ij}T^{ji} = \text{Asyrm Quad-Product } \rightarrow 4\text{D Determinant } = \text{Product of EigenValues for (1,1)-Tensors} \)

\[
a) 4\text{-AngMom } \text{Trace} [M^{\alpha\beta}] = M_{\alpha}^{\alpha} = (M^{00}+M^{11}-M^{22}-M^{33}) = 0
b) 4\text{-AngMom Inner Product } M_{\alpha\beta} M^{\gamma\delta} = \Sigma [M_{\alpha\beta}^{\gamma\delta}] = 2 \Sigma [M_{\alpha\beta}^{\gamma\delta}] + 2 \Sigma [M_{\alpha\beta}^{\gamma\delta}] = 0
\]

\[
c) 4\text{-AngMom AsymTri}[M^{\alpha\beta}] = T^{\alpha\beta} - 3(T^{\alpha\beta})(M_{\alpha}^{\gamma\delta} + M_{\beta}^{\alpha\gamma} + M_{\gamma}^{\alpha\beta} + M_{\delta}^{\alpha\gamma}) = 0
\]

\[
d) 4\text{-AngMom Det[anti-symmetric } M^{\alpha\beta}] = Pfaffian[M^{\alpha\beta}] = [(c(\n!)^2)]
\]

Importantly, the 4-AngularMomentum Tensor has only (2) linearly-independent Lorentz Invariants:

b) \( 2((t)(c^2 n^n)) \), see Wikipedia Laplace-Runge-Lenz_vector, sec. Casimir Invariants
a) \( \{c(n)\} \), and do not provide additional constraints

The 4-Position and 4-Momentum have (4) independent components each, for total of (8).

Subtract (2) due to the invariants which provide constraints to get a total of (6) independent components:

\( = (6) \) independent components of a 4x4 anti-symmetric tensor

\( = (3) \) 3-mass-moment \( n = \) and (3) 3-angular-momentum \( I = (6) \) independent 4-AngularMomentum components

3-massmoment \( n = x m - t p = m(x - t u) = m (r - t (\omega x r)) \) : Tangential velocity \( u_{T} = (\omega x r) \)

\( (-k/r)n = -m(k^f - t (\omega x f)) = m k^f - t^* d/dt(p) x L = m k^f \)

\( n \) is related to the LRL = Laplace-Runge-Lenz 3-vector: \( A = p x L - \omega \times R \)

which is another classical conserved vector. The invariance is shown here to be relativistic in origin. Wikipedia article: Laplace-Runge-Lenz vector shows these as Casimir Invariants.

See Also: Relativistic Angular Momentum.

\[
4\text{-Position } X = X^\mu = (c, t) \in \text{Event}
\]

\[
4\text{-AngularMomentum Tensor } M^a_{\beta} = X^a P^\beta - X^\beta P^a
\]

\[
\text{Trace } [T^{ij}] = \eta_{ij} T^{ij} = T_{ij}^i = T
\]

\[
\text{V-V} = V^\gamma \eta_{ij} V^\beta = (V^\gamma)^2 = (V^\gamma)^2
\]

\[
\text{Lorentz Scalar Invariant, } \text{SR 4-Scalar}\]

SR 4-Vector
\[
(2,0)\text{-Tensor } T^{ij}, (1,1)\text{-Tensor } T^{ij} \text{ or } T_{ij}, (0,2)\text{-Tensor } T^{ij}
\]

SR 4-Vector
\[
(1,0)\text{-Tensor } V^i = V = (V^i, V)
\]

SR 4-Scalar
\[
(0,0)\text{-Tensor } S or S_{ij} \text{ Lorentz Scalar}
\]
The Minkowski Metric Tensor $\eta^{\mu\nu}$ is the tensor all SR 4-Vectors are measured by.

(2,0)-Tensor = 4-Tensor $T^{\mu\nu}$: Has (4+) Tensor Invariants (though not all independent)

a) $T^\alpha_\alpha = \text{Trace} = \text{Sum of EigenValues for (1,1)-Tensors (mixed)}$

b) $T^\alpha_\beta T^\beta_\gamma = \text{Asymm Bi-Product} \rightarrow \text{Inner Product}$

c) $T^\alpha_\beta T^\beta_\gamma T^\gamma_\delta = \text{Asymm Tri-Product} \rightarrow \text{？Name?}$

d) $T^\alpha_\beta T^\beta_\gamma T^\gamma_\delta = \text{Asymm Quad-Product} \rightarrow 4D \text{ Determinant} = \text{Product of EigenValues for (1,1)-Tensors}$

Det $\det(\text{Exp}[A]) = \text{Exp}(\text{Trace}[A])$

$\Delta = \text{Asymm Tri-Product}$

$\Lambda = \text{Asymm Bi-Product}$

Eigenvalues for the Lorentz Transforms since they are type (1,1)-Tensors, mixed indices

Eigenvalues not defined for the standard Minkowski Metric Tensor since it is a type (2,0)-Tensor, all upper indices. However, they are defined for the mixed form (1,1)-Tensor $[\eta^{\mu\nu}] = 1/|\eta^{\mu\nu}| : \eta^{\mu\nu} = \delta^{\mu\nu}$

SR: Minkowski Metric

"Particle Physics" Convention

Determinant Tensor Invariant

Trace Tensor Invariant

$\det[A] = \prod_{i=1}^4 \lambda_i$; with $\{\lambda_i\} = \text{EigenValues}$

Characteristic Eqns: $\det[T^{\mu\nu} - \lambda_k I_{4\times4}] = 0$

Trace $[T^{\mu\nu}] = \lambda_{\mu} T^{\mu\nu} = T^{\mu}_{\mu} = T$

$V \cdot V = V^\dagger \eta^{\mu\nu} V = \left( (v^\mu)^2 \right) - (v^0)^2 = (v^\mu)^2$

= Lorentz Scalar Invariant

$\eta^{\mu\nu} T^{\mu\nu} = \eta^{00} T^{00}$

SR 4-Vector

$V = (V^0, V^1, V^2, V^3)$

SR 4-CoVector

$V^\dagger = (V_0, V_1, V_2, V_3)$

SR 4-Scalar

$S = S_0$ Lorentz Scalar

SR 4-Tensor

$T^{\mu\nu}$, or $T_{\mu\nu}$

SR 4-Scalar

$\Pi_4[\lambda_4]$: with $\{\lambda_4\} = \text{EigenValues}$
SRQM Study: SR 4-Tensors

SR Tensor Invariants

for Perfect Fluid Stress-Energy Tensor

The Perfect Fluid Stress-Energy Tensor $T^{\mu\nu}$ is the tensor of a non-torsional relativistic fluid.

(2,0)-Tensor = 4-Tensor $T^{\mu\nu}$: Has (4+) Tensor Invariants (though not all independent)

a) Perfect Fluid Trace $\text{Tr}[T^{\mu\nu}] = \rho_0 - 3p_o$

b) Perfect Fluid Inner Product $T_{\mu\nu}T^{\mu\nu} = (\rho_0)^2 + 3(p_o)^2$

c) Perfect Fluid Asymmetry $T_{\mu\nu} - T_{\nu\mu}$

d) Perfect Fluid Det $\text{Det}[T^{\mu\nu}] = \rho_0 - 3p_o$

SR Conservation of Stress-Energy $T^{\mu\nu}$

In Cartesian form

$\text{Tr}[T^{\mu\nu}] = (\rho_0) - 3(p_o) - (p_0)$

$\text{Det}[T^{\mu\nu}] = T^{\mu\nu} = \rho_0 - 3p_o$

$\text{Trace Tensor Invariant}$

$T_{\mu\nu}T^{\mu\nu} = (\rho_0)^2 + 3(p_o)^2$

$\text{Diag}[\rho_0, p_o, p_o, p_o]$ =

$[\rho_0, 0, 0, 0]$

$[0, p_o, 0, 0]$

$[0, 0, p_o, 0]$

$[0, 0, 0, p_o]$

$\text{Equation of State}$

$\text{EoS}[T^{\mu\nu}] = w = \rho_0 / \rho_0$

$\text{Asymmetry}$

$\text{Asymmetry Tensor Invariant}$

$\text{Determinant Tensor Invariant}$

$\text{Eigenvalues}$

$\text{Trace}$

$\text{Symmetric}$

$\text{Units}$

$\text{Characteristic Eqns}$: $\text{Det}[T^{\mu\nu} - \lambda_k I_{4\times4}] = 0$

EigenValues not defined for the standard Perfect Fluid Tensor since it is a type (2,0)-Tensor, all upper indices. However, they are defined for the mixed form (1,1)-Tensor, mixed indices.
SRQM Study: SR 4-Tensors

SR Tensor Invariants for Maxwell 4D EM Stress-Energy Tensor

SR Perfect Fluid 4-Tensor
\[ T_{\text{perfect fluid}}^{\mu\nu} = (\rho_{eo}) V^{\nu} + (-p_{o}) g^{\nu\mu} \]

SR Conservation of Stress-Energy \( T^{\mu\nu} \)
if \( F_{\text{density}}^{\mu\nu} = 0 \)

4-Force Density \( F_{\text{density}} = \partial T^{\mu\nu} \)
Note: this is positive-definite

Trace Tensor Invariant
\[ \text{Tr}[T^{\mu\nu}] = 0 \]
\[ = T^{00} - T^{11} - T^{22} - T^{33} \]
\[ = T^{00} - 3T^{00}(1 \text{ from } \delta^{ii}) + (2T^{00} \text{ from } T^{xx} + T^{yy} + T^{zz}) \]
\[ = T^{00} - 3T^{00} + 2T^{00} \]
\[ = 0 \]
= Sum of EigenValues

SR 4-Tensor
\[ (2,0)-\text{Tensor } T^{\mu\nu}, \text{ or } T^{\nu}_{\mu} \]
SR 4-Vector
\[ (1,0)-\text{Vector } V = (V^{\nu},V) \]
SR 4-Vector
\[ (0,1)-\text{Vector } V_{\mu} = (V^{\nu})_{\mu} \]
SR 4-Scalar
\[ (0,0)-\text{Scalar } S \text{ or } S_{0} \]

Det[T_{\mu\nu}] = \Pi_{i}\lambda_{i} \; \text{with } \{ \lambda_{i} \} = \text{EigenValues} \]

Characteristic Eqns: \( \text{Det}[T_{\mu\nu} - \lambda_{i} I_{4\times4}] = 0 \)

\[ T_{\text{MaxwellEM}}^{\mu\nu} \]
\[ \rightarrow \text{3D Maxwell Stress Tensor} \]
\[ T^{0j} = s = (e_{b} b_{i}/\mu_{0}) \]

\[ \text{Poynting Vector} \]
\[ \text{Trace}[T^{\mu\nu}] = \eta_{\mu\nu} T^{\mu\nu} = T^{\nu}_{\nu} = T \]
\[ V \cdot V = [V^{\nu} V^{\nu}] = (V^{\nu})^{2} = \text{Lorentz Scalar Invariant} \]
**SRQM Study: SR 4-Tensors**

**SR Tensor Invariants for Maxwell 4D EM Stress-Energy Tensor**

**Maxwell 4D EM Stress-Energy Tensor**

\[ T_{\mu\nu} = \frac{1}{2}(\epsilon_{\mu\nu}e^2 + b^2/\mu_0) \text{ s}^0/c \]

\[ \text{Note this is positive-definite} \]

**Trace Tensor Invariant**

\[ \text{Tr}[T_{\mu\nu}] = 0 = T_{00} - T_{11} - T_{22} - T_{33} \]

\[ = T_{00} - 3T_{00}(1) + (2T_{00} \text{ from } T^{xx} + T^{yy} + T^{zz}) \]

\[ = T_{00} - 3T_{00} + 2*T_{00} = 0 \]

**Sum of EigenValues**

**Faraday EM Tensor**

\[ F_{\mu\nu} = \frac{1}{2}(b_{\mu}b_{\nu} - b_{\nu}b_{\mu}) \]

\[ = (c\epsilon_{\mu\nu}e^2 + b^2/\mu_0) \text{ s}^0/c \]

\[ = (1/c)\frac{\partial F_{\mu\nu}}{\partial x^\alpha} \]

\[ \text{Det}[F_{\mu\nu}] = \epsilon_{\mu\nu}e^2 + b^2/\mu_0 \]

\[ \text{Trace: } \text{Tr}[F_{\mu\nu}] = 0 \]

**Symmetric 4-Tensor**

\[ \frac{1}{2}(\epsilon_{\mu\nu}e^2 + b^2/\mu_0) \text{ s}^0/c \]

\[ = (c^2/v)\epsilon_{\mu\nu}e^2 + b^2/\mu_0 \]

**Poynting Vector**

\[ P_{\mu\nu} = \epsilon_{\mu\nu}e^2 + b^2/\mu_0 \]

**Interpretation of QM**

\[ \text{SR 4-Tensor: } (2,0) \text{-Tensor } T^{4\nu} \]

\[ \text{SR 4-Vector: } V = (v^0, v) \]

\[ \text{SR 4-CoVector: OneForm } (0,1) \text{-Tensor } V_0 = (v_0, v) \]

\[ \text{SR 4-Scalar: } (0,0) \text{-Tensor } V_0 \text{ or } \mu_0 \]

\[ \text{Lorentz Scalar: } \text{EoS}[T_{\mu\nu}] = \Pi_\mu(\lambda) \text{ with } \lambda_{\mu} = \text{EigenValues} \]

\[ \text{Characteristic Eqns: } \text{Det}[T_\alpha - \lambda_{\alpha}(I_{4\mu})] = 0 \]

\[ \text{Trace } [T^{\mu\nu}] = \frac{1}{16\pi}T^{\mu\nu} = T_{00} = T_{00} = T_{00} = T_{00} = 0 \]

\[ \text{w/ 3D Poynting Vector} \]

**Traces**

\[ \frac{1}{2}(\epsilon_{\mu\nu}e^2 + b^2/\mu_0) \text{ s}^0/c \]

\[ = (c^2/v)\epsilon_{\mu\nu}e^2 + b^2/\mu_0 \]

\[ = (c^2/v)\epsilon_{\mu\nu}e^2 + b^2/\mu_0 \]

\[ \text{Poynting Vector} \]

\[ \text{SR 4-Tensor: } (2,0) \text{-Tensor } T^{4\nu} \]

\[ \text{SR 4-Vector: } V = (v^0, v) \]

\[ \text{SR 4-CoVector: OneForm } (0,1) \text{-Tensor } V_0 = (v_0, v) \]

\[ \text{SR 4-Scalar: } (0,0) \text{-Tensor } V_0 \text{ or } \mu_0 \]

\[ \text{Lorentz Scalar: } \text{EoS}[T_{\mu\nu}] = \Pi_\mu(\lambda) \text{ with } \lambda_{\mu} = \text{EigenValues} \]

\[ \text{Characteristic Eqns: } \text{Det}[T_\alpha - \lambda_{\alpha}(I_{4\mu})] = 0 \]

\[ \text{Trace } [T^{\mu\nu}] = \frac{1}{16\pi}T^{\mu\nu} = T_{00} = T_{00} = T_{00} = T_{00} = 0 \]

\[ \text{w/ 3D Poynting Vector} \]

**Traces**

\[ \frac{1}{2}(\epsilon_{\mu\nu}e^2 + b^2/\mu_0) s^0/c \]

\[ = (c^2/v)\epsilon_{\mu\nu}e^2 + b^2/\mu_0 \]

\[ = (c^2/v)\epsilon_{\mu\nu}e^2 + b^2/\mu_0 \]

\[ \text{Poynting Vector} \]

\[ \text{SR 4-Tensor: } (2,0) \text{-Tensor } T^{4\nu} \]

\[ \text{SR 4-Vector: } V = (v^0, v) \]

\[ \text{SR 4-CoVector: OneForm } (0,1) \text{-Tensor } V_0 = (v_0, v) \]

\[ \text{SR 4-Scalar: } (0,0) \text{-Tensor } V_0 \text{ or } \mu_0 \]

\[ \text{Lorentz Scalar: } \text{EoS}[T_{\mu\nu}] = \Pi_\mu(\lambda) \text{ with } \lambda_{\mu} = \text{EigenValues} \]

\[ \text{Characteristic Eqns: } \text{Det}[T_\alpha - \lambda_{\alpha}(I_{4\mu})] = 0 \]

\[ \text{Trace } [T^{\mu\nu}] = \frac{1}{16\pi}T^{\mu\nu} = T_{00} = T_{00} = T_{00} = T_{00} = 0 \]

\[ \text{w/ 3D Poynting Vector} \]
SRQM Study: SR 4-Tensors
Properties of Lorentz Transform Tensors
Relation to 4D Kronecker Delta

The Lorentz Transform Tensor \( \Lambda_{\mu}^{\nu} = \delta x^{\mu}/\delta x^{\nu} = \delta_{\mu}^{\nu} \) is the tensor all SR 4-Vectors must transform by.

For Rotations:
- 0\(^\text{th}\) with 0\(^\text{th}\) : \((1)(1)+(0)(0)+(0)(0)+(0)(0) = 1\)
- 0\(^\text{th}\) with 1\(^\text{st}\) : \((1)(0)+(0)(\cos[\theta])+(0)(-\sin[\theta])+(1)(0) = 0\)
- 0\(^\text{th}\) with 2\(^\text{nd}\) : \((1)(0)+(0)(\cos[\theta])+(0)(\sin[\theta])+(1)(0) = 0\)
- 0\(^\text{th}\) with 3\(^\text{rd}\) : \((1)(0)+(0)(0)+(0)(0)+(0)(1) = 0\)
  ...
- 1\(^\text{st}\) with 1\(^\text{st}\) : \((0)(0)+(\cos[\theta])(\cos[\theta])+(\sin[\theta])(-\sin[\theta])+(0)(0) = \cos^2+\sin^2 = 1\)
- 1\(^\text{st}\) with 2\(^\text{nd}\) : \((0)(0)+(\cos[\theta])(\sin[\theta])+(\sin[\theta])(\cos[\theta])+(0)(0) = \cos^2\sin^2\sin = 0\)
  etc.

For Boosts:
- 0\(^\text{th}\) with 0\(^\text{th}\) : \((\gamma)(\gamma)+(-\beta)(\gamma)(\gamma)+(0)(0)+(0)(0) = \gamma^2-\gamma^2 = 1\)
- 0\(^\text{th}\) with 1\(^\text{st}\) : \((\gamma)(\beta)+(-\beta)(\gamma)(\gamma)+(0)(0)+(0)(0) = \gamma^2\beta-\beta\gamma = 0\)
- 0\(^\text{th}\) with 2\(^\text{nd}\) : \((\gamma)(0)+(-\beta)(0)(0)+(0)(0)+(0)(1) = 0\)
  ...
- 2\(^\text{nd}\) with 3\(^\text{rd}\) : \((0)(0)+(0)(0)+(1)(0)+(0)(1) = 0\)
  etc.

The fact that each row gives a single (1) leads to the overall inner product \( \Lambda_{\mu}^{\nu}\Lambda_{\nu}^{\mu} = 4 \equiv \delta_{\mu}^{\nu} \) for 4D Lorentz Transforms \((1+1+1+1 = 4)\).

SR 4-Tensor
\[
(2,0)-\text{Tensor} \ T^{iv} = \Lambda_{\mu}^{\nu} V^{\nu} = V^{\nu} v_{\nu} = (v^{\nu} v_{\nu})^2 \]

SR 4-Vector
\[
(1,0)-\text{Tensor} \ V^{\nu} = V = (\nu, \nu) \]

SR 4-Scalar
\[
(0,0)-\text{Tensor} \ \delta_{\nu}^{\nu} = \delta_{\nu}^{\nu} = 4 \equiv \Lambda_{\mu}^{\nu} \Lambda_{\nu}^{\mu} \]

SR 4-Vector
\[
(1,0)-\text{Tensor} \ \Lambda_{\mu}^{\nu} \rightarrow R^{\nu}_{\nu} \]

SR 4-Scalar
\[
\Lambda_{\mu}^{\nu} \Lambda_{\nu}^{\mu} = 4 \equiv \delta_{\mu}^{\nu} \Lambda_{\nu}^{\mu} \]

Properties of Lorentz Transform Tensors

- Lorentz SR Tensor \( \Lambda_{\mu}^{\nu} \rightarrow R^{\nu}_{\nu} \)
- Lorentz SR Identity Tensor \( \eta_{\mu}^{\nu} \rightarrow \delta_{\mu}^{\nu} \)
- Lorentz SR Boost Tensor \( \Lambda_{\mu}^{\nu} \rightarrow B^{\nu}_{\nu} \)

The properties of the Lorentz Transforms give interesting relations to the 4D Kronecker Delta.

The Lorentz Transform \( \eta_{\mu}^{\nu} \) is the tensor all SR 4-Vectors must transform by.

Inner Product Tensor Invariant

\[ \Lambda_{\mu}^{\nu} \Lambda_{\nu}^{\mu} = 4 \equiv \Lambda_{\mu}^{\nu} \Lambda_{\nu}^{\mu} \]

4D Lorentz Tensor \( \Lambda_{\mu}^{\nu} \) is the tensor all SR 4-Vectors must transform by.

For Boosts:
- \( \gamma - \beta \gamma 0 0 \)
- \( \beta \gamma \gamma 0 0 \)
- \( 0 0 1 0 \)
- \( 0 0 0 1 \)

For Rotations:
- \( 1 0 0 0 \)
- \( 0 \cos[\theta] - \sin[\theta] 0 \)
- \( 0 \sin[\theta] \cos[\theta] 0 \)
- \( 0 0 0 1 \)

Double Minkowski-Metric Indexed-Altered Tensors (1 raised, 1 lowered)

Lorentz SR Boost Tensor \( \Lambda_{\mu}^{\nu} \rightarrow B^{\nu}_{\nu} \)

\[ B^{\nu}_{\nu} \]

Lorentz SR Boost Tensor \( \Lambda_{\mu}^{\nu} \rightarrow B^{\nu}_{\nu} \)

\[ B^{\nu}_{\nu} \]

Lorentz SR Identity Tensor \( \eta_{\mu}^{\nu} \rightarrow \delta_{\mu}^{\nu} \)

\[ \delta_{\mu}^{\nu} \]

Lorentz SR Rotation Tensor \( \Lambda_{\mu}^{\nu} \rightarrow R^{\nu}_{\nu} \)

\[ R^{\nu}_{\nu} \]

SRQM Study: SR 4-Tensors

Properties of Lorentz Transform Tensors

Relation to 4D Kronecker Delta

The fact that each row gives a single (1) leads to the overall inner product \( \Lambda_{\mu}^{\nu}\Lambda_{\nu}^{\mu} = 4 \equiv \delta_{\mu}^{\nu} \) for 4D Lorentz Transforms \((1+1+1+1 = 4)\).
SR Rotation EigenValues = \{1, 1, \, e^{i\theta}, \, e^{-i\theta}\}
SRQM Study: SR 4-Tensors

SR Tensor Invariants for Continuous Lorentz Transform Tensors

The Lorentz Transform Tensor \( \{ \Lambda^{\mu}_{\nu} = \partial x^{\mu}/\partial x^{\nu} = \delta^{\mu}_{\nu} \} \) is the tensor all SR 4-Vectors must transform by.

(2,0)-Tensor = 4-Tensor \( T^{\nu}_{\mu} \); Has (4+) Tensor Invariants (though not all independent)

a) Lorentz Trace[\( \Lambda^{\mu}_{\nu} \) = \{0..4..Infinity\}] Lorentz Boost meets Rotation at Identity of 4
b) Lorentz Inner Product \( \Lambda^{\mu}_{\nu} \Lambda^{\nu}_{\mu} = 4 \) from \( \{ \eta_{\mu\nu}, \Lambda^{\mu}_{\nu}, \Lambda^{\nu}_{\mu} \} \) and \( \{ \eta_{\mu\nu} = 4 \} \)
c) Lorentz Asymmetry of \( \Lambda^{\mu}_{\nu} \)
d) Lorentz Det[\( \Lambda^{\mu}_{\nu} \) = +1 for Proper Transforms, Continuous Transforms Proper

An even more general version would be with \( a \) & \( b \) as arbitrary complex values:

- 2 boosts, 2 rotations, or a boost+rotation combo

\( \text{Trace}[\Lambda^{\mu}_{\nu}] = \{ -\infty, +\infty \} \)

- Lorentz Transform Type

\( \text{Product of} \) \( \Lambda^{\mu}_{\nu} \) \( \Lambda^{\nu}_{\mu} = 4 \)

- Lorentz Transform Scalar

\( \text{Product of} \) \( \Lambda^{\mu}_{\nu} \) \( \Lambda^{\nu}_{\mu} = \Lambda^{\mu}_{\nu} \Lambda^{\nu}_{\mu} = 4 \)

- Lorentz Transform Scalar

\( \text{Det}[\Lambda^{\mu}_{\nu}] = \pm 1 \) Proper Transform always +1

\( \text{EigenValues}[\Lambda^{\mu}_{\nu}] = \{ e^{\theta a}, e^{\theta b}, e^{\theta a}, e^{\theta b} \} \)

- Lorentz Transform Type

\( \text{Trace Tensor Invariant} \)

\( \text{Tr}[\Lambda^{\mu}_{\nu}] = \{ 0..4..\infty \} \)

- Lorentz Transform Type

\( \text{Sum of} \) \( \text{EigenValues}[\Lambda^{\mu}_{\nu}] = \{ 1, 1, 1, 1 \} \)

- Lorentz Transform Type

\( \text{Determinant Tensor Invariant} \)

\( \text{Det}[\Lambda^{\mu}_{\nu}] = \pm 1 \) Proper Transform always +1

\( \text{Rotation Tensor} \)

\( \Lambda^{\mu}_{\nu} \rightarrow \Lambda^{\mu}_{\nu} = \Lambda^{\mu}_{\nu} \)

- Lorentz Transform Type

\( \text{Identity Tensor} \)

\( \Lambda^{\mu}_{\nu} \rightarrow \Lambda^{\mu}_{\nu} = \Lambda^{\mu}_{\nu} \)

- Lorentz Transform Type

\( \text{EigenValues}[\Lambda^{\mu}_{\nu}] = \{ e^{\theta a}, e^{\theta b}, e^{\theta a}, e^{\theta b} \} \)

- Lorentz Transform Type

\( \text{EigenValues}[\Lambda^{\mu}_{\nu}] = \{ e^{\theta a}, e^{\theta b}, e^{\theta a}, e^{\theta b} \} \)

- Lorentz Transform Type
SRQM Diagram:
The Basis of Classical SR Physics
The Lorentz-Transform $\partial_v [R^\mu_v] = \partial R^\mu_v / \partial R^v = \Lambda^\mu_v$

SR: Lorentz Transform
$\partial[R^\mu] = \partial R^\mu / \partial R^v = \Lambda^\mu_v$
$\Lambda^\mu_v = (\Lambda^\mu_v)^\nu$
$\eta_{\mu\nu} \Lambda^\mu_v \Lambda^\nu_p = \eta_{\nu p}$
$\Lambda^\mu_v (\Lambda^\mu)^v = \Lambda^{\mu(\Lambda)}_{\nu v} = \eta_{\nu v} = \delta^\mu_v$

for $\nu = 0, v = 1$; similar result for $v = 2, v = 3$

$\Lambda^\mu_v = (\Lambda^\mu_v)^\nu$
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The Trace of various discrete Lorentz transforms varies in steps from \{-4, -2, 0, 2, 4\}. This includes Mirror Flips, Time Reversal, and Parity Inverse – essentially taking all combinations of ±1 on the diagonal of the transform.

<table>
<thead>
<tr>
<th>Trace Tensor Invariant</th>
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</thead>
<tbody>
<tr>
<td>Tr[Discrete (\Lambda^\nu)] = {-4, -2, 0, 2, 4} Depends on transform</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Determinant Tensor Invariant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Det[(\Lambda^\nu)] = ±1 Proper Transform = +1 Improper Transform = -1</td>
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<thead>
<tr>
<th>Inner Product Tensor Invariant</th>
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<tbody>
<tr>
<td>(\delta[\Lambda^\nu] = \partial R^\nu / \partial R^\nu = \Lambda^\nu_\nu)</td>
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<table>
<thead>
<tr>
<th>Asymm Tri-Product Tensor Invariant</th>
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<tbody>
<tr>
<td>(\Lambda^\nu \Lambda^\mu \Lambda^\alpha = 4)</td>
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<thead>
<tr>
<th>Lorentz SR TP Combo Tensor (\Lambda^\nu \rightarrow TP^\nu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Lambda^\nu = \eta^\nu_\nu = -\delta^\nu_\nu)</td>
</tr>
<tr>
<td>(\Lambda^\nu = \left[\begin{array}{ccc} -1 &amp; 0 &amp; 0 \ 0 &amp; -1 &amp; 0 \ 0 &amp; 0 &amp; -1 \end{array}\right]) = Flip-xyz</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Lorentz SR Parity-Inversion Tensor (\Lambda^\nu \rightarrow \Pi^\nu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Lambda^\nu = \eta^\nu_\nu = -\delta^\nu_\nu)</td>
</tr>
<tr>
<td>(\Lambda^\nu = \left[\begin{array}{ccc} 1 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 1 \end{array}\right]) = Rotation-(z(\pi))</td>
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</table>

<table>
<thead>
<tr>
<th>Lorentz SR Flip-x-y-Combo Tensor (\Lambda^\nu \rightarrow Fxy^\nu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Lambda^\nu = \eta^\nu_\nu = -\delta^\nu_\nu)</td>
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<tr>
<td>(\Lambda^\nu = \left[\begin{array}{ccc} 1 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 1 \ 0 &amp; 1 &amp; 0 \end{array}\right]) = Flip-t</td>
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</tbody>
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<table>
<thead>
<tr>
<th>Lorentz SR Time-Reversal Tensor (\Lambda^\nu \rightarrow \overline{T}^\nu)</th>
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<tbody>
<tr>
<td>(\Lambda^\nu = \eta^\nu_\nu = -\delta^\nu_\nu)</td>
</tr>
<tr>
<td>(\Lambda^\nu = \left[\begin{array}{ccc} 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 \end{array}\right]) = Minkowski Delta</td>
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<tr>
<th>Lorentz SR Identity Tensor (\Lambda^\nu \rightarrow \eta^\nu_\nu)</th>
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<tbody>
<tr>
<td>(\Lambda^\nu = \eta^\nu_\nu = -\delta^\nu_\nu)</td>
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<tr>
<td>(\Lambda^\nu = \left[\begin{array}{ccc} 1 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 1 \end{array}\right])</td>
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<tr>
<th>Lorentz SR SRQM Interpretation of SRQM</th>
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<tr>
<td>(\det[\Lambda^\nu] = \pm 1) Proper Transform = +1 Improper Transform = -1</td>
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<tr>
<th>Lorentz SR SRQM Invariants for Discrete Lorentz Transform Tensors</th>
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<tbody>
<tr>
<td>(\text{Trace}[\Lambda^\nu] = \eta^\nu_\nu)</td>
</tr>
<tr>
<td>(\det[\Lambda^\nu] = \left[\begin{array}{ccc} 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 \end{array}\right]) = Lorentz Scalar Invariant</td>
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<tr>
<th>SR 4-Vector</th>
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<tbody>
<tr>
<td>((2,0)-\text{Tensor } T^{\mu\nu})</td>
</tr>
<tr>
<td>((1,0)-\text{Tensor } V^\nu = (\overline{v}^\nu, v^\nu))</td>
</tr>
<tr>
<td>((0,1)-\text{Tensor } V_\nu = (v_\nu, \overline{v}_\nu))</td>
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<tr>
<th>SR 4-Scalar</th>
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<tr>
<td>((0,0)-\text{Tensor } S \text{ or } S_\nu) Lorentz Scalar</td>
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<td>(T^{\nu\nu} = \left[\begin{array}{ccc} 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 \end{array}\right])</td>
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<td>(V^\nu = (v^\nu, \overline{v}^\nu))</td>
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The Trace of various discrete Lorentz transforms varies in steps from \{-4, -2, 0, 2, 4\}. This includes Mirror Flips, Time Reversal, and Parity Inverse – essentially taking all combinations of ±1 on the diagonal of the transform.

Inner Product Tensor Invariant:
\(\delta[\Lambda^\nu] = \partial R^\nu / \partial R^\nu = \Lambda^\nu\_\nu\)

Asymm Tri-Product Tensor Invariant:
\(\Lambda^\nu \Lambda^\mu \Lambda^\alpha = 4\)

Determinant Tensor Invariant:
\(\det[\Lambda^\nu] = \pm 1\) Proper Transform = +1 Improper Transform = -1

Trace Tensor Invariant:
\(\text{Tr}[	ext{Discrete } \Lambda^\nu] = \{-4, -2, 0, 2, 4\}\)

Product of Eigenvalues:
\(\text{Product of Eigenvalues}[\Lambda^\nu] = \text{Det}[\Lambda^\nu] = -1 \cdot -1 \cdot -1 = 1\)

Eigenvalues:
\(\text{Eigenvalues}[\Lambda^\nu] = \{-1, -1, -1\}\)

Sum of Eigenvalues:
\(\text{Sum of Eigenvalues}[\Lambda^\nu] = \text{Tr}[\Lambda^\nu] = \eta^\nu\_\nu = -\delta^\nu\_\nu\)

Proper: \(\text{Det}[\Lambda^\nu] = 1\)

Improper: \(\text{Det}[\Lambda^\nu] = -1\)

Proper: \(\text{Trace}[\Lambda^\nu] = \eta^\nu\_\nu = -\delta^\nu\_\nu\)

Improper: \(\text{Trace}[\Lambda^\nu] = 0\)

Proper: \(\text{Trace}[\Lambda^\nu] = 1\)

Improper: \(\text{Trace}[\Lambda^\nu] = 0\)

Proper: \(\text{Trace}[\Lambda^\nu] = 1\)

Improper: \(\text{Trace}[\Lambda^\nu] = 0\)
SRQM Study: SR 4-Tensors

More SR Tensor Invariants for Discrete Lorentz Transform Tensors

The Flip-xy-Combo is the equivalent of a \(\pi\)-Rotation-z.

I suspect that this may be related to exchange symmetry and the Spin-Statistics idea that a particle-exchange is the equivalent of a spin-rotation.

A single Flip would not be an exchange because it leaves a mirror-inversion of <right-[left>.

But the extra Flip along an orthogonal axis corrects the mirror-inversion, and would be an overall exchange because the particle is in a different location.

SR: Lorentz Transform
\[ \frac{\partial[R^\mu]}{\partial r^\nu} = \Lambda^\mu_\nu \]
\[ \Lambda^\nu_\nu = (\Lambda^{-1})^\nu_\mu \Rightarrow \Lambda^\nu_\mu \Lambda^\mu_\nu = \eta^\nu_\nu \Rightarrow \delta^\nu_\nu \]
\[ \det[\Lambda^\mu_\nu] = \pm 1 \quad \Lambda^\mu_\nu \Lambda^\nu_\mu = 4 \]
\[ \text{Tr}[\Lambda^\mu_\nu] = \left\{ \begin{array}{ll} \infty & \text{Lorentz Transform Type} \\
-\infty & \text{non-Lorentz Transform Type} \end{array} \right. \]

Lorentz SR 0-Rotation-z Tensor
\[ \Lambda^\nu_\nu \rightarrow R^\nu_\nu = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \text{Identity} \]

Lorentz SR Flip-x Tensor
\[ \Lambda^\nu_\nu \rightarrow Fx^\nu_\nu = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \text{Flip-x} \]

More SR Tensor Invariants for Discrete Lorentz Transform Tensors

\[ \text{Trace}[T^\mu_\nu] = \eta^\nu_\nu T^\mu_\mu = T^\nu_\nu = T \]
\[ V \cdot V = V^\nu \eta^\nu_\nu V^\mu = (V^\nu)^2 \quad \text{Lorentz Scalar Invariant} \]

\[ V \cdot V = (\nu^\nu) V^\nu = (V^\nu)^2 \]

The four SR Tensors: Lorentz SR 4-Tensor

SR 4-Vector
\[ (2.0)-\text{Tensor} T^\mu_\nu \]
\[ (1.0)-\text{Tensor} V^\nu = V = (v^\nu) \nu \]
\[ (0.1)-\text{Tensor} V_\nu = (v_\nu) \]

SR 4-Scalar
\[ (0.0)-\text{Tensor} S \text{ or } S_\nu \text{ or Lorentz Scalar} \]

SR 4-CoVector: OneForm
\[ (0,1)-\text{Tensor} V_\nu = (v_\nu) \]

Characteristic Eqns: \[ \det[T^\mu_\nu - \lambda^\mu_\nu I_{(4)}] = 0 \]
SRQM Study: More Tensor Invariants

Riemann Curvature Tensor = Riem
\[ R_{\rho\sigma\mu\nu} = \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda} \Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda} \Gamma^\lambda_{\mu\sigma} \]
\[ \to 0_{\rho\sigma\mu\nu} \text{ for SR "Flat" Minkowski SpaceTime} \]

Contraction 1 pair of indices

Ricci Tensor = Ric
\[ R_{\mu\nu} = R^{\rho\mu\rho\nu} \]

Contraction 2 pair of indices

Kretschmann scalar
\[ K = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \]
\[ \to 48G^2M^2(\text{c}^4t^4) \text{ for Schwarzschild BH} \]

Carminati–McLenaghan invariants or CM scalars
Below are just the 5 real ones, there are some complex ones too

\[ R = g^{\nu\rho} R_{\mu\nu} = R_{\nu\mu} \]
\[ R_1 = (1/4) S^a_b S^b_a \]
\[ R_2 = (-1/8) S^a_b S^b_c S^c_a \]
\[ R_3 = (1/16) S^{ab} S_{ac} S^c_b S^b_d \]
\[ M_3 = (1/16) S^{ac} S_{bd} (C_{abcd} C_{eefg} + *C_{abcd} *C_{eefg}) \]
\[ M_4 = (-1/32) S^{ac} S^c_e S^d_f (C_{ac}^{db} C_{eefg} + *C_{ac}^{db} *C_{eefg}) \]

see also: Zakhary–McIntosh curvature invariants

\[ \text{Trace}[T^\rho_{\mu}] = \eta_{\rho\nu} T^{\nu}_{\mu} = T^\rho_{\mu} = T \]
\[ V \cdot V = V^\rho \eta_{\rho\nu} V^\nu = \left[(v^\rho)^2 - v \cdot v\right] = (v^\rho)^2 \]
\[ = \text{Lorentz Scalar Invariant} \]
SR 4-Scalars, 4-Vectors, 4-Tensors beautifully and elegantly display the relations between lots of different physical properties and relations. Their notation makes navigation through the physics very simple.

They also devolve very nicely into the limiting/approximate Newtonian cases of \( |v| \ll c \) by letting \( \gamma \rightarrow 1 \) and \( \gamma' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \).

SR tells us that several different physical properties are actually dual aspects of the same thing, with the only real difference being one's point of view, or reference frame.

Examples of 4-Vectors = (1,0)-Tensors include:

One can also examine 4-Tensors, which are type (2,0)-Tensors. The Faraday EM Tensor similarly combines EM fields:
\[
F^{\alpha\beta} = \begin{bmatrix}
0 & -e^0/c \\
+e^0/c & -(\epsilon^\beta_k b^k)
\end{bmatrix}
\]

Also, things are even more related than that. The 4-Momentum is just a constant times 4-Velocity. The 4-WaveVector is just a constant times 4-Velocity.

In addition, the very important conservation/continuity equations seem to just fall out of the notation. The universe apparently has some simple laws which can be easy to write down by using a little math and a super notation.
SRQM Study:
SR Gradient 4-Vectors = 4D (1,0)-Tensors
SR Gradient One-Forms = 4D (0,1)-Tensors

4-Vector = Type 4D (1,0)-Tensor

<table>
<thead>
<tr>
<th>Temporal</th>
<th>Spatial</th>
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<tbody>
<tr>
<td>Components</td>
<td></td>
</tr>
<tr>
<td>[Time (t) : Space (r)]</td>
<td></td>
</tr>
<tr>
<td>[Time Differential ((\partial_t)) : Spatial Gradient((\nabla))]</td>
<td></td>
</tr>
</tbody>
</table>

Standard 4-Vector

| 4-Position \(R = R^\mu = (ct, r)\) |
| 4-Velocity \(U = U^\mu = \gamma(c, u)\) |
| 4-Momentum \(P = P^\mu = (E/c, p)\) |
| 4-WaveVector \(K = K^\mu = (\omega/c, k)\) |

Related Gradient 4-Vector (from index-raised Gradient One-Form)

| 4-PositionGradient \(\partial_R \equiv \partial/\partial R = (\partial/\partial R^t/c, \nabla_R)\) |
| 4-VelocityGradient \(\partial_U \equiv \partial/\partial U = (\partial/\partial U^t/c, \nabla_U)\) |
| 4-MomentumGradient \(\partial_P \equiv \partial/\partial P = (\partial/\partial P^t/c, \nabla_P)\) |
| 4-WaveGradient \(\partial_K \equiv \partial/\partial K = (\partial/\partial K^t/c, \nabla_K)\) |

In each case, the (Whichever)Gradient 4-Vector is derived from a 4D SR One-Form or 4-CoVector, which is a type (0,1)-Tensor

ex. One-Form PositionGradient \(\partial_{R^\nu} \equiv \partial/\partial R^\nu = (\partial/\partial R^t/c, \nabla_R^\nu)\)

The (Whichever)Gradient 4-Vector is the index-raised version of the SR One-Form (Whichever)Gradient

ex. 4-PositionGradient \(\partial_{R^\nu} = \partial/\partial R^\nu = (\partial/\partial R^t/c, \nabla_R)\) = \(\eta^{t\nu}\partial_{R^\nu} = \eta^{t\nu}\partial/\partial R^\nu = \eta^{t\nu}(\partial_{R^\nu}/c, \nabla_R)\) = \(\eta^{t\nu}(One-Form\ PositionGradient)\).

This is why the 4-Gradient is commonly seen with a minus sign in the spatial component, unlike the other regular 4-Vectors, which have all positive components.

4-Tensors can be constructed from the Tensor Outer Product of 4-Vectors
Some Basic 4-Vectors

Minkowski SpaceTime Diagram

Events & Dimensions

- past
- future
- elsewhere

$c$, $-c$

Special Relativity

Classical Mechanics

$\Delta t$, time-like interval

$\Delta r$, space-like interval

Note the separate dimensional units: (time + 3D space)

\(\Delta t\) is [time, SI→s], \(|\Delta r|\) is [length, SI→m]

4-Displacement

\[\Delta R_{\text{CM}} = (c\Delta t, \Delta r)\]

Note the matching dimensional units: (4D Time·Space)

\(c\Delta t\) is [length/time]·[time] = [length], \(|\Delta r|\) is [length], \(|\Delta R|\) is [length]

\(\tau\) is the Proper Time = “rest-time”, time as measured by something not moving spatially

The Minkowski Diagram provides a great visual representation of SpaceTime
Some Basic 4-Vectors

Minkowski SpaceTime Diagram, WorldLines, LightSpeed to the Future!

\[ \Delta t \quad \text{time-like interval (+)} \]
\[ \Delta R = (c \Delta t, \Delta r) \]

\[ \text{at-rest WorldLine (} u = 0 \text{)} \]
\[ \text{inertial motion WorldLine (} 0 < u < c \text{)} \]

An Event (*) is a point in SpaceTime
The 4-Position points to an Event.

A WorldLine is a series of connected Events which trace out a path in SpaceTime, such as the track of a moving particle.

\[ \Delta r \quad \text{space-like interval (−)} \]

\[ \Delta R \cdot \Delta R = [(c \Delta t)^2 - \Delta r \cdot \Delta r] = 0 \]
for light-like (0)
for time-like (+)
for space-like (−)

The 4-Position is a particular type of 4-Displacement, for which the vector base is at the <Origin> = (0, 0, 0, 0) = 4-Zero.

4-Position is Lorentz Invariant, but not Poincaré Invariant.
A standard 4-Displacement is both.

\[ U = \gamma (c, u) = dR/d\tau \]
\[ U \cdot U = c^2 \]
\[ U_o = (c, 0) \]
\[ U_c = \gamma c (c, c\hat{n}) \]

Massive particles move temporally into the future at the speed-of-light (c) in their own rest-frame.

Massless particles (photonic) move nullly into the future at the speed-of-light (c), and have no rest-frame.

\[ V \cdot V = \eta_{\mu \nu} V^\mu V^\nu = (c^2) \quad \text{Lorentz Scalar Invariant} \]
SR Invariant Intervals

Minkowski Diagram: Lorentz Transform

Since the SpaceTime magnitude of \( U \) is a constant \( (c) \), changes in the components of \( U \) are like rotating the 4-Vector without changing its length. It keeps the same magnitude. Rotations, purely spatial changes, (eg. along x,y) result in circular displacements. Boosts, or temporal-spatial changes, (eg. along x,t) result in hyperbolic displacements.

The interval between the origin and a given topograph-line is a Lorentz Invariant Constant.

\[
U \cdot U = \gamma(c,u) \cdot \gamma(c,u) = \gamma^2(c^2 - u \cdot u) = (c^2)
\]

Rotation (x,y): Purely Spatial

Boost (x,t): Spatial-Temporal

\[
\partial_v[R^\mu] = \partial R^\nu / \partial R^\nu = \Lambda^\mu_\nu
\]

\[
\Lambda^\mu_\nu = (\Lambda^{-1})^\mu_\nu : \Lambda^\mu_\alpha (\Lambda^{-1})^\alpha_\nu
\]

\[
\eta_{\mu\nu} \Lambda^\mu_\alpha \Lambda^\alpha_\beta = \eta_{\delta\epsilon}
\]

\[
\text{Det}[\Lambda^\mu_\nu] = \pm 1, \quad \Lambda^\mu_\nu \Lambda^\nu_\mu = 4
\]

\[
\eta_{\mu\nu} = \frac{1}{\eta_{\mu\nu}} \Rightarrow \eta^{\mu\nu} = \delta^{\mu\nu}
\]

\[
\text{Tr}[\eta^{\mu\nu}] = 4
\]

The Light Cone / Minkowski Diagram provides a great visual representation of SpaceTime
Since the SpaceTime magnitude of $\mathbf{U}$ is a constant ($c$), changes in the components of $\mathbf{U}$ are like rotating the 4-Vector without changing its length. It keeps the same magnitude ($c$). Rotations, purely spatial changes, {eg. along $x,y$} result in circular displacements. Boosts, or temporal-spatial changes, {eg. along $x,t$} result in hyperbolic displacements. The interval between the origin and a given topograph-line is a Lorentz Invariant Constant.

The Minkowski Diagram provides a great visual representation of SpaceTime.
SRQM: Some Basic 4-Vectors

4-Position, 4-Velocity, 4-Acceleration
SpaceTime Kinematics

ProperTime
\[ R \cdot U / U \cdot U = (ct, r) \cdot \gamma(c, u) / c^2 = \gamma(c^2 t - r \cdot u) / c^2 = (c^2 t_0) / c^2 \]

\[ t_0 = \tau \]

4-Position
\[ R = (ct, r) \]

4-Velocity
\[ U = \gamma(c, u) \]

4-Acceleration
\[ A = \gamma(c, u) (\partial / \partial t - \gamma u) \]

ProperTime Derivative
\[ U \cdot \partial = \gamma(c, u) (\partial / \partial \tau) = \gamma d/\gamma dt \]

4-Vectors:
\[ R \in < \text{Event} > \]
\[ U = dR / d\tau \]
\[ A = dU / d\tau \]

Classic Mechanics
\[ |v| = |u| << c \]
\[ \gamma = 1 / \sqrt{1 - (v/c)^2} \]
\[ \gamma \rightarrow 0 \]

The relativistic Gamma factor \( \gamma = 1 / \sqrt{1 - (v/c)^2} \)


4-Tensor
\[ S \]

3-Tensor
\[ T \]

SR 4-Tensor
\[ (2,0)-Tens \]
\[ (1,1)-Tens \]
\[ (0,2)-Tens \]
\[ \text{Tensor } V \]

SR 4-CoVector: OneForm
\[ (1,0)-Tens \]

Lorentz Scalar
\[ S \]

SR 4-Scalar
\[ (0,0)-Tens \]

Galilean Invariant
\[ \gamma' = d \gamma / d t = \gamma^2 (u \cdot a) / c^2 \]

Not Lorentz Invariant

Trace[T^\nu] = \eta_\nu^\mu T^\mu = T^\nu_{\nu} = T

V \cdot V = V^\nu \eta_\nu^\nu = (v^\nu) \cdot (v^\nu) = (v^2)^2

= Lorentz Scalar Invariant

For historical reasons, velocity can be represented by either (v) or (u)
SRQM: Some Basic 4-Vectors

4-Position, 4-Velocity, 4-Acceleration, (RestMass), 4-Momentum, 4-Force

SpaceTime Dynamics

4-Vectors:
\[ R \in \langle \text{Event} \rangle \]
\[ U = \frac{dR}{d\tau} \]
\[ A = \frac{dU}{d\tau} \]
\[ P = m_0 U \]
\[ F = \frac{dP}{d\tau} \]

4-Gradient
\[ \partial = (\partial_t, -\nabla) = \frac{d}{d\tau} \]

This group of 4-Vectors are the main ones that are connected by the ProperTime Derivative.
\[ U \cdot \partial = \frac{d}{d\tau} \]
\[ = \gamma (c, u) \cdot (\gamma c, -\nabla) \]
\[ = \gamma (\partial_t + u \cdot \nabla) = \gamma \frac{d}{d\tau} \]

The classical part of it, the convective derivative, \((\partial_t + u \cdot \nabla)\), is known by many different names:
The convective derivative is a derivative taken with respect to a moving coordinate system. It is also called the advective derivative, derivative following the motion, hydrodynamic derivative, Lagrangian derivative, material derivative, particle derivative, substantial derivative, substantive derivative, Stokes derivative, or total derivative.

Special Relativity
\[ |v| = |u| = \{0 \rightarrow c\} \]
\[ \gamma = \frac{1}{\sqrt{1 - (v/c)^2}} \]

Trace\[ T_{\mu\nu} = \eta_{\mu\nu} T^\mu_\nu = T_{\mu\nu} = T \]
\[ V \cdot V = V^\mu V_\mu = (v^\mu)^2 - v \cdot v = (v^\mu) \]

Lorentz Scalar Invariant

SR 4-Tensor
\[(2,0)-Tensor T^{\mu\nu}_{\mu\nu} \]
\[(1,1)-Tensor V^{\mu}_{\mu} = (\gamma v^\mu) \]
SR 4-Vector
\[(1,0)-Tensor V^{\nu} \]
\[(0,2)-Tensor T_{\mu\nu}^{\mu\nu} \]
SR 4-Scalar
\[(0,0)-Tensor S \]

SR 4-Vector: OneForm
\[ T_{\mu\nu}^{\mu\nu} \]
\[ V_{\mu} = (v_{\mu}) \]

Lorentz Scalar
SRQM: Some Basic 4-Vectors

**4-Velocity, 4-Momentum, \( \mathbf{E} = mc^2 \)**

### 4-Velocity

\[
\mathbf{U} = \gamma(\mathbf{c}, \mathbf{u})
\]

- Temporal part: \( \mathbf{U} \cdot \mathbf{U} = (c)^2 \)
- Spatial part: \( \mathbf{U} \cdot \mathbf{U} = \gamma(\mathbf{c}, \mathbf{u}) \cdot (\mathbf{c}, \mathbf{u}) = (c)^2 \)

### 4-Momentum

\[
\mathbf{P} = \left( \frac{\mathbf{E}}{c}, \mathbf{p} \right) = (mc, \mathbf{p})
\]

- Temporal part: \( \mathbf{P} \cdot \mathbf{P} = (\mathbf{E}/c)^2 = (mc)^2 \)
- Spatial part: \( \mathbf{P} \cdot \mathbf{U} = \gamma(\mathbf{E} - \mathbf{p} \cdot \mathbf{u}) = E_0 = mc^2 \)

### Special Relativity

\[
\gamma = \frac{1}{\sqrt{1 - (\mathbf{v}/c)^2}}
\]

- For historical reasons, velocity can be represented by either \( \mathbf{v} \) or \( \mathbf{u} \)

### Classical Mechanics

\[
\mathbf{U}_{\text{CM}} = (1 + (\mathbf{v}/c)^2)(\mathbf{c}, \mathbf{u})
\]

- Temporal part: \( \mathbf{U}_{\text{CM}} \cdot \mathbf{U}_{\text{CM}} = (c)^2 \)
- Spatial part: \( \mathbf{U}_{\text{CM}} \cdot \mathbf{U}_{\text{CM}} = (\mathbf{c}, \mathbf{u}) \cdot (\mathbf{c}, \mathbf{u}) = (c)^2 \)

### Newtonian/Classical Limit

\[
\mathbf{U}_{\text{CM}} \rightarrow (\mathbf{v}_0, \mathbf{u}_0, \mathbf{v}_0^2)
\]

- Temporal part: \( \mathbf{U}_{\text{CM}} \cdot \mathbf{U}_{\text{CM}} \rightarrow (1 + (\mathbf{v}/c)^2/2)m_0(c, u) \)
- Spatial part: \( \mathbf{p} \rightarrow m_0 \mathbf{u} \)

\[
\mathbf{P}_{\text{CM}} = \left( \frac{\mathbf{E}}{c}, \mathbf{p} \right) = (mc, \mathbf{p})
\]

- Temporal part: \( \mathbf{P}_{\text{CM}} \cdot \mathbf{P}_{\text{CM}} \rightarrow (1 + (\mathbf{v}/c)^2/2)m_0(c, u) \)
- Spatial part: \( \mathbf{p} \rightarrow (\mathbf{p}_0, \mathbf{p}_0, \mathbf{p}_0^2) \)

For historical reasons, velocity can be represented by either \( \mathbf{v} \) or \( \mathbf{u} \).
SRQM: Some Basic 4-Vectors

4-Velocity, 4-Acceleration, SpaceTime Orthogonality

4-Velocity
\[ U = \gamma(c, u) \]

4-Acceleration
\[ A = \gamma(c' \gamma, \gamma')u + \gamma a \]

4-Gradient
\[ d/d\tau \]

4-Vectors
\[ R \in \langle \text{Event} \rangle \]
\[ U = dR/d\tau = R' \]
\[ A = dU/d\tau = U' \]

The Lorentz Scalar Product can be used to show SpaceTime orthogonality when the result is zero.

\[ U \cdot U = c^2 \]
\[ d/d\tau[U \cdot U] = d/d\tau[c^2] = 0 \]
\[ d/d\tau[U \cdot U] = d/d\tau[U] \cdot U + U \cdot d/d\tau[U] = A \cdot U + U \cdot A = 2(U \cdot A) = 0 \]
\[ U \cdot A = U \cdot U' = 0 \]

4-Velocity is the direction along a WorldLine.
4-Acceleration is the thing which causes a WorldLine to bend/curve.

SpaceTime Orthogonality
\[ U \perp A \]

4-Vector SRQM Interpretation of QM
SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf
SRQM: Some Basic 4-Vectors

4-Displacement, 4-Velocity, Relativity of Simultaneity

If Lorentz Scalar \((\mathbf{U} \cdot \Delta \mathbf{X} = 0 = c^2 \Delta \tau)\), then the ProperTime displacement \((\Delta \tau)\) is zero, and the event separation \((\Delta \mathbf{X} = \mathbf{X}_2 - \mathbf{X}_1)\) is orthogonal to the worldline \(\mathbf{U}\).

\(\mathbf{X}_1\) and \(\mathbf{X}_2\) are therefore simultaneous for the observer on this worldline \(\mathbf{U}\).

Examining the equation we get \(\gamma(c^2 \Delta t - \mathbf{u} \cdot \Delta \mathbf{x}) = 0\). The coordinate time difference between the events is \((\Delta t = \mathbf{u} \cdot \Delta \mathbf{x}/c^2)\)

The condition for simultaneity in an alternate frame (moving at 3-velocity \(\mathbf{u}\) wrt. the worldline \(\mathbf{U}\)) is \(\Delta t = 0\), which implies \((\mathbf{u} \cdot \Delta \mathbf{x}) = 0\).

This can be met by:

1. \(|\mathbf{u}| = 0\), the alternate observer is not moving wrt. the events, i.e. is on worldline \(\mathbf{U}\) or on a worldline parallel to \(\mathbf{U}\).
2. \(|\Delta \mathbf{x}| = 0\), the events are at the same spatial location (co-local).
3. \((\mathbf{u} \cdot \Delta \mathbf{x} = 0)\), the alternate observer's motion is perpendicular (orthogonal) to the spatial separation \(\Delta \mathbf{x}\) of the events in that frame.

If none of these conditions is met, then the events will not be simultaneous in the alternate reference frame.

This is the mathematics behind the concept of Relativity of Simultaneity.
SR Diagram:

SR Motion * Lorentz Scalar = Interesting Physical 4-Vector

4-Displacement
\[ \Delta \mathbf{R} = (c \Delta t, \Delta r) \]
\[
\text{4-Position} \quad \mathbf{R} = (ct, r)
\]

4-Velocity
\[ \mathbf{U} = \gamma (c, \mathbf{u}) \]
\[
\gamma \frac{d}{dt} [\ldots] = \mathbf{U} \cdot \partial [\ldots]
\]

4-Acceleration
\[ \mathbf{A} = \gamma (c \mathbf{\gamma}', \gamma \mathbf{u} + \gamma \mathbf{a}) \]
\[
\gamma \frac{d}{d\tau} [\ldots] = \mathbf{A} \cdot \partial [\ldots]
\]

Interesting note:
Most 4-Vectors have 4 independent components. (1 temporal, 3 spatial)

The 4-Velocity has only the 3 spatial however, due to its invariant magnitude
\[ \mathbf{U} \cdot \mathbf{U} = c^2 \].

This fact allows one to multiply it by a Lorentz Scalar Invariant to make a new 4-Vector with 4 independent components, as shown in the diagram.

Proof of non-varying (c)
\[ \{ \mathbf{m}_0 = 0 \} \leftrightarrow \{ \mathbf{P} \cdot \mathbf{U} = 0 \} \leftrightarrow \{ \mathbf{P} \text{ is null} \} \]
\[ \{ \omega_0 = 0 \} \leftrightarrow \{ \mathbf{K} \cdot \mathbf{U} = 0 \} \leftrightarrow \{ \mathbf{K} \text{ is null} \} \]

4-Gradient
\[ \partial = (\partial/c, -\nabla) \]

4-ChargeFlux
4-CurrentDensity
\[ \mathbf{J} = (\rho c, \mathbf{j}) = \rho (c, \mathbf{u}) \]
\[
\mathbf{E} = \frac{\mathbf{c}}{\varepsilon_0} \mathbf{c}^2
\]

4-EMVectorPotential
4-Momentum
\[ \mathbf{A} = \gamma (c \mathbf{\gamma}', \gamma \mathbf{u} + \gamma \mathbf{a}) \]
\[ \mathbf{P} = m (c, \mathbf{u}) = (E/c, \mathbf{p}) \]
\[ \{ \mathbf{m}_0 = 0 \} \leftrightarrow \{ \mathbf{P} \cdot \mathbf{U} = 0 \} \leftrightarrow \{ \mathbf{P} \text{ is null} \} \]

Electric: Magnetic
\[ \varepsilon_0 c^2 / \mu_0 = c^2 \]

\[ \mathbf{E} = \frac{1}{\varepsilon_0} \mathbf{c} \]

Maxwell EM Wave Eqn
\[ \mathbf{E} \cdot \partial = \mathbf{0} \]

4-NumberFlux
\[ \mathbf{N} = (n, c) = n (c, \mathbf{u}) \]

4-Position
\[ \mathbf{R} = (ct, r) \]

SR 4-Tensor
\[ (2, 0) \text{-Tensor } \mathbf{T}^{uv} \]
\[ (1, 1) \text{-Tensor } \mathbf{T}^{\mu \nu} \text{ or } \mathbf{T}_{\mu \nu} \]
\[ (0, 2) \text{-Tensor } \mathbf{T}_{\mu \nu} \]

SR 4-Vector
\[ (2, 0) \text{-Vector } \mathbf{V} = \mathbf{v} = (v^0, \mathbf{v}) \]
\[ (1, 1) \text{-Vector } \mathbf{V}^{\mu} = \mathbf{v}^\mu = (v^0, \mathbf{v}) \]
\[ (0, 2) \text{-Vector } \mathbf{V} = (v_\mu, v^\mu) \]

SR 4-Scalar
\[ (0, 0) \text{-Scalar } \mathbf{S} \text{ or } \mathbf{S}_0 \]

Rest Number Density
\[ n_0 \]

Rest Charge Density
\[ \rho_0 \]

Rest Scalar Potential
\[ \phi_0 / c^2 \]

Rest Mass:Energy
\[ m_0 E_0 / c^2 \]

EM Wave Velocity
\[ v_{\text{group}} = v_{\text{phase}} = c^2 \]

Rest Angular Frequency
\[ \omega_0 / c^2 \]

Lorentz Scalar
\[ \omega_{\text{Lorentz}} = \omega_{\text{Lorentz}} / \sqrt{c^2} \]

Trace[\mathbf{T}^{\mu \nu}] = \eta_{\mu \nu} T^{\mu \nu} = T^{\mu}_{\mu} = T \]
\[ \mathbf{V} \cdot \mathbf{V} = \mathbf{v} \cdot \mathbf{v} = (v^0)^2 - \mathbf{v} \cdot \mathbf{v} = (v^0)^2 \]

Lorentz Scalar Invariant

SciRealm.org
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http://scirealm.org/SRQM.pdf
4-Velocity is the direction of an Event along a WorldLine.

4-Acceleration of an Event is the thing which causes a WorldLine to bend.

**4-Vector SRQM Interpretation of QM**

**SRQM Diagram:**

ProperTime Derivative

Very Fundamental Results

SR 4-Tensor

(2,0)-Tensor $T^\mu_\nu$

(1,1)-Tensor $T^\nu_\mu$, or $T_{\mu\nu}$

(0,2)-Tensor $T_{\nu\mu}$

SR 4-Vector

(1,0)-Tensor $V^\nu = (\mathbf{v}^\nu, \mathbf{v})$

SR 4-CoVector: OneForm

(0,1)-Tensor $V_\nu = (\mathbf{v}_\nu, \mathbf{v})$

SR 4-Scalar

(0,0)-Tensor $S$ or $S_\mu$

Lorentz Scalar

Trace[$T^\nu_\mu$] = $\eta^\nu_\mu T^\nu_\mu = T$

$V\cdot V = \mathbf{v}^\nu \eta^\nu_\nu T^\nu_\nu = (\mathbf{v}^\nu)^2 - \mathbf{v}^2 = (\mathbf{v})^2$

Lorentz Scalar Invariant
Local Continuity of 4-Velocity leads to all of the Conservation Laws

Conservation Laws:

All of the Physical Conservation Laws are in the form of a 4-Divergence, which is a Lorentz Invariant Scalar equation.

Conservation of Charge:
\[ \gamma \frac{d}{dt} [\rho] = 0 \]

Example:
\[ \delta (\rho_o) U = 0 \]
\[ \delta J = 0 \]
\[ \delta (\rho c \gamma + \nabla j) = 0 \]
\[ \delta \rho + \nabla j = 0 \]
Conservation of Charge = A Continuity Equation

Conservation of the 4-Velocity Flow:
\[ \partial \cdot U = 0 \]

Example:
\[ \delta (\rho_o) U = 0 \]
\[ \delta J = 0 \]
\[ \delta (\rho c \gamma + \nabla j) = 0 \]
\[ \delta \rho + \nabla j = 0 \]
Conservation of Charge = A Continuity Equation
4-Vector SRQM Interpretation of QM

SRQM Diagram:
SRQM Motion * Lorentz Scalar Conservation Laws, Continuity Eqns

Conservation Laws:
All of the Physical Conservation Laws are in the form of a 4-Divergence, which is a Lorentz Invariant Scalar equation.

These are local continuity equations which basically say that the temporal change in a quantity is balanced by the flow of that quantity into or out of a local spatial region.

Conservation of Charge:
\[ \partial \cdot J = (\partial / c, \vec{V}) = 0 \]

These are Fluid or Density -type Conservation/Continuity Laws

These are Individual Particle/Wave/Delta-function Conservation/Continuity Laws

Quantum Principles

Existing SR Rules

SR 4-Tensor
(2,0)-Tensor $T^{\mu
\nu}$
(1,1)-Tensor $T^{\mu
\nu}$, or $T^{\nu
\mu}$
(0,2)-Tensor $T^{\mu
\nu}$

SR 4-Vector
(1,0)-Tensor $V^\mu = \chi / \phi$
(0,1)-Tensor $V_{\nu} = c \chi$
SRQM: Some Basic 4-Vectors

4-Velocity, 4-Gradient, Time Dilation

The Minkowski Diagram provides a great visual representation of SpaceTime

at-rest worldline \( \mathbf{U}_0 \)
(at \( u=0 \))
fully temporal

const inertial motion worldline \( \mathbf{U} \)
(0<\( u < c \))
trades some time for space

\[ \mathbf{U} = \gamma (\mathbf{c}, u) \]
\[ \mathbf{U} \cdot \mathbf{U} = \gamma (\mathbf{c}, u) \cdot \gamma (\mathbf{c}, u) = \gamma^2 (c^2 - u \cdot u) = (c^2) \]
\[ \gamma = 1/\sqrt{1-(u/c)^2} = 1/\sqrt{1-\beta^2} \]

Everything moves into future (+t)

at the speed-of-light (c)
in its own spatial rest-frame

Since the SpaceTime magnitude of \( \mathbf{U} \) is a constant, changes in the components of \( \mathbf{U} \) are like “rotating” the 4-Vector without changing its length. However, as \( \mathbf{U} \) gains some spatial velocity, it loses some “relative” temporal velocity. Objects that move in some reference frame “age” more slowly relative to those at rest in the same reference frame.

Time Dilation!
\[ \Delta t = \gamma \Delta \tau = \gamma \Delta t_o \]
\[ dt = \gamma d\tau \]
\[ d/d\tau = \gamma d/dt \]

Each observer will see the other as aging more slowly; similarly to two people moving oppositely along a train track, seeing the other as appearing smaller in the distance.
SRQM: Some Basic 4-Vectors

Lorentz Invariant d’Alembertian ($\partial \cdot \partial$)

The d’Alembertian \( \{ \partial \cdot \partial = (\partial^t/c)^2 - \nabla \cdot \nabla \} \) is a 4D Lorentz Scalar Invariant.

It is used as the basis of many “wave-type” equations.

\[
(\partial \cdot \partial)\phi[\mathbf{X}] = \{ \partial \cdot \partial \phi[(t,\mathbf{x})] \} = 0 \text{ is the standard relativistic wave equation}
\]

\[
(\partial \cdot \partial)\mathbf{A}[\mathbf{X}] = \{ \partial \cdot \partial \mathbf{A}[(t,\mathbf{x})] \} = 0 \text{ is the Maxwell EM Wave equation in Lorenz Gauge} \ (\partial \cdot \mathbf{A})=0
\]

\[
(\partial \cdot \partial)\mathbf{A}[\mathbf{X}] = \{ \partial \cdot \partial \mathbf{A}[(t,\mathbf{x})] \} = \mu_0 J \text{ is the Maxwell EM Wave equation in Lorenz Gauge} \ (\partial \cdot \mathbf{A})=0 \text{ with a Source term } J
\]

\[
(\partial \cdot \partial)\phi[\mathbf{X}] = -(m_0 c/\hbar)^2 \phi[\mathbf{X}] \text{ is the standard relativistic quantum Klein-Gordon equation}
\]

\[
(\partial \cdot \partial)\mathbf{G}[\mathbf{X}-\mathbf{X}'] = \{ \partial \cdot \partial \mathbf{G}[(t,\mathbf{x})] \} \text{ is a 4D Green's Function, } \delta^{(4)}[\mathbf{X}-\mathbf{X}'] \text{ is a 4D Dirac Delta function}
\]

\[
\delta^{(4)}[\mathbf{X}-\mathbf{X}'] = \frac{1}{(2\pi)^4} \int d^4K \ e^{iK \cdot (\mathbf{X} - \mathbf{X}')} = \delta[ct - ct'] \delta^{(3)}[\mathbf{x} - \mathbf{x}'] \delta[y - y'] \delta[z - z']
\]

\[
\text{[Some 4D Volume] } \delta^{(4)}[\mathbf{X} - \mathbf{X}'] \ d^4\mathbf{X} = \{1 \text{ if } \mathbf{X}' \text{ in the 4D Volume, 0 otherwise} \}
\]

The Covariant 4D versions of the Green’s Function and the Dirac Delta Function. Given a linear ordinary differential equation (ODE), \( L(\text{solution}) = \text{source} \), one can first solve \( L(\text{green}) = \delta[s] \), for each \( s \), and realizing that, since the source is a sum of delta functions, the solution is a sum of Green’s functions as well, by linearity of \( L \).

\[
\text{Trace}[T^{\mu \nu}] = \eta_{\mu \nu} T^{\mu \nu} = T^\nu_\nu = T
\]

\[
\mathbf{V} \cdot \mathbf{V} = V^\nu V_\nu = (V^t)^2 - \mathbf{v} \cdot \mathbf{v} = (V^0)^2 = (\partial^t/c)^2
\]

= Lorentz Scalar Invariant
There are multiple ways of writing out the components of the 4-WaveVector, with each one giving an interesting take on what the 4-WaveVector means.

An SR wave \( \Psi \) is actually composed of two tensors:

1. 4-Vector propagation part = \( K^\mu \) (the engine), in \( e^\omega (\text{ik}^\nu x^\nu) \)
2. Variable amplitude part = \( A \) (the load), depends on what is waving...

There are multiple ways of writing out the components of the 4-WaveVector, each one giving an interesting take on what the 4-WaveVector means.

I believe the last one is correct: \((\partial /\partial \tau) [R] = 0\) = \((\partial /\partial c \partial \tau) [R] = A_c /c^2 = 0\). The 4-Acceleration seen in the ProperTime Frame = RestFrame = 0 Normally \((\partial /\partial \tau) [R] = A\), which could be non-zero. But that is for the total derivative, not the partial derivative.
SRQM: Some Basic 4-Vectors

4-Velocity, 4-WaveVector

Wave Properties, Relativistic Doppler Effect

Relativistic SR Doppler Effect

(\hat{n}) here is the unit-directional 3-vector of the photon

Choose an observer frame for which:

\[ K = (\omega/c, \hat{k}) \]

with \( k, \hat{n} \) pointing toward observer

\[ U_{\text{obs}} = (c, 0) \quad K \cdot U_{\text{obs}} = (\omega/c, \hat{n}) \cdot (c, 0) = \omega = \omega_{\text{obs}^p} \]

\[ U_{\text{emit}} = (\mu, \omega) \quad K \cdot U_{\text{emit}} = (\omega/c, \hat{k}) \gamma(c, \mu) = \gamma(\omega - k \cdot u) = \omega_{\text{emit}^p} \]

\[ K \cdot U_{\text{obs}} / K \cdot U_{\text{emit}} = \omega_{\text{obs}^p} / \omega_{\text{emit}^p} = \omega / [\gamma(\omega - k \cdot u)] \]

For photons, \( K \) is null → \( K \cdot K = 0 \rightarrow k = (\omega/c)\hat{n} \)

\[ \omega_{\text{obs}^p} / \omega_{\text{emit}^p} = \omega / [\gamma(\omega - (\omega/c)\hat{n} \cdot u)] = 1/[\gamma(1 - \hat{n} \cdot \beta)] = 1/[\gamma(1 - |\beta| \cos[\theta_{\text{obs}^p}])] \]

\[ \omega_{\text{obs}^p} / \omega_{\text{emit}^p} = \gamma \omega_{\text{obs}^p} / \omega_{\text{emit}^p} = \omega_{\text{obs}^p} / \omega_{\text{emit}^p} \]

\[ \omega_{\text{obs}} = \omega_{\text{emit}} / [\gamma(1 - \hat{n} \cdot \beta)] = \omega_{\text{emit}} / \sqrt{[1 + |\beta|] \cdot \sqrt{[1 - |\beta|]}} / (1 - \hat{n} \cdot \beta) \]

with \( \gamma = 1 / \sqrt{[1 - \beta^2]} = 1 / (\sqrt{[1 + |\beta|] \cdot \sqrt{[1 - |\beta|]}) \)

For motion of emitter \( \beta \): (in observer frame of reference)

Away from obs, \( \hat{n} \cdot \beta = -\beta \), \( \omega_{\text{obs}} = \omega_{\text{emit}} / \sqrt{[1 - |\beta|]} = \sqrt{[1 + |\beta|] / (1 - \hat{n} \cdot \beta)} \)

Toward obs, \( \hat{n} \cdot \beta = +\beta \), \( \omega_{\text{obs}} = \omega_{\text{emit}} / \sqrt{[1 + |\beta|]} = \sqrt{[1 - |\beta|] / (1 - \hat{n} \cdot \beta)} \)

Transverse, \( \hat{n} \cdot \beta = 0 \), \( \omega_{\text{obs}} = \omega_{\text{emit}} / \gamma = \text{Transverse Doppler Shift} \)

The Phase Velocity of a Photon \( \{v_{\text{phase}} = c\} \) equals the Particle Velocity of a Photon \( \{u = c\} \)

The Phase Velocity of a Massive Particle \( \{v_{\text{phase}} > c\} \) is greater than the Velocity of a Massive Particle \( \{u < c\} \)

\[ \text{SR 4-Tensor} \]
\[ (2,0)\text{-Tensor } T_{\mu}^\nu \quad (1,1)\text{-Tensor } V_{\mu} = (v_{\mu}, v) \quad (0,2)\text{-Tensor } T_{\mu \nu} \]

\[ \text{SR 4-Vector} \]
\[ (1,0)\text{-Tensor } V_{\mu} = (v_{\mu}, v) \quad (0,1)\text{-Tensor } V_{\mu} = (v_{\mu}, v) \quad (0,1)\text{-Tensor } S_0 \]

\[ \text{SR 4-Scalar} \]
\[ (0,0)\text{-Tensor } S_0 \quad \text{Lorentz Scalar} \]
SRQM: Some Basic 4-Vectors

4-Velocity, 4-WaveVector

Wave Properties, Relativistic Aberration

Relativistic SR Aberration Effect

\[ \omega_{\text{obs}} = \omega_{\text{emit}} \left/ [\gamma(1 - \hat{n} \cdot \beta)] \right. = \omega_{\text{emit}} \left/ [\gamma(1 - |\beta| \cos[\theta_{\text{obs}}]) \right. \]

Change reference frames with \{obs→emit\} & \{ \beta \rightarrow -\beta \}

\[ \omega_{\text{emit}} = \omega_{\text{obs}} \left/ [\gamma(1 + \hat{n} \cdot \beta)] \right. = \omega_{\text{obs}} \left/ [\gamma(1 + |\beta| \cos[\theta_{\text{emit}}]) \right. \]

\[ (\omega_{\text{obs}})^* (\omega_{\text{emit}}) = (\omega_{\text{emit}})^* (\gamma(1 - |\beta| \cos[\theta_{\text{obs}}]))^* (\omega_{\text{obs}})^* (\gamma(1 + |\beta| \cos[\theta_{\text{emit}}])) \]

\[ 1 = (1/[\gamma(1 - |\beta| \cos[\theta_{\text{obs}}]))^* (1/[\gamma(1 + |\beta| \cos[\theta_{\text{emit}}])) \]

\[ 1 = (\gamma(1 - |\beta| \cos[\theta_{\text{obs}}]))^* (\gamma(1 + |\beta| \cos[\theta_{\text{emit}}])) \]

\[ 1 = \gamma^2(1 - |\beta| \cos[\theta_{\text{obs}}])^* (1 + |\beta| \cos[\theta_{\text{emit}}]) \]

Solve for \(|\beta| \cos[\theta_{\text{obs}}]\) and use \((\gamma^2-1) = \beta^2/2\)

\[ \cos[\theta_{\text{obs}}] = (\cos[\theta_{\text{emit}}] + |\beta|) / (1 + |\beta| \cos[\theta_{\text{emit}}]) \]

The Phase Velocity of a Photon \(\{v_{\text{phase}} = c\}\) equals the Particle Velocity of a Photon \(\{u = c\}\)

The Phase Velocity of a Massive Particle \(\{v_{\text{phase}} > c\}\) is greater than the Velocity of a Massive Particle \(\{u < c\}\)

SR 4-Vector

\( (1,0)\)-Tensor \( V = V = (v^0, v) \)

SR 4-Vector

\( (0,1)\)-Tensor \( V = (v^0, v) \)

SR 4-Scalar

\( (0,0)\)-Tensor \( S \) or \( S^\nu_o \)

4-Vector SRQM Interpretation of QM

http://scirealm.org/SRQM.pdf
SRQM: Some Basic 4-Vectors

**4-Momentum, 4-WaveVector, 4-Position, 4-Velocity, 4-Gradient, Wave-Particle**

- **4-Momentum** $P = (mc, p) = (E/c, p)$
  - $P \cdot P = (mc)^2 = (E/c)^2$

- **4-Velocity** $U = \gamma(c, u)$
  - $U \cdot U = (c)^2$

- **4-WaveVector** $K = (\omega/c, k) = (\omega/c, \omega \eta \nu \phi_{phase})$
  - $K \cdot K = (\omega/c)^2$

- **4-Gradient** $\phi = (\partial/c, -\nabla) \rightarrow (\partial/c, -\partial_x, -\partial_y, -\partial_z)$

- **Treating motion like a particle**
  - Moving particles have a 4-Velocity
  - 4-Momentum is the negative 4-Gradient of the SR Action ($S$)

- **Treating motion like a wave**
  - Moving waves have a 4-Velocity
  - 4-WaveVector is the negative 4-Gradient of the SR Phase ($\phi$)

See Hamilton-Jacobi Formulation of Mechanics for info on the Lorentz Scalar Invariant SR Action. $\{P = (E/c, p) = -\partial[S] = (-\partial/c\partial[t](E, \nabla[S]))\}$

- **Spatial component** $p = \nabla[S]$
  - **Temporal component** $E = -\partial/c\partial[t][S] = -\partial[S]$

- **Note** This is the Action ($S_{action}$) for a free particle. Generally Action is for the 4-TotalMomentum $P_t$ of a system.

See SR Wave Definition for info on the Lorentz Scalar Invariant SR WavePhase. $\{K = (\omega/c, k) = -\partial[\phi] = (-\partial/c\partial[t]\phi, \nabla[\phi])\}$

- **Spatial component** $k = \nabla[\phi]$
  - **Temporal component** $\omega = -\partial/c\partial[t][\phi] = -\partial[\phi]$

- **Note** This is the Phase ($\phi$) for a single free plane-wave. Generally WavePhase is for the 4-TotalWaveVector $K_t$ of a system.

SR 4-Tensor $(2,0)$-Tensor $T^{ix}$ $(1,1)$-Tensor $T^{iv}$, or $T_{iv}$ $(0,2)$-Tensor $T_{ix}$

SR 4-Vector $(1,0)$-Tensor $V^i = V = (v^i, v)$

SR 4-CoVector:OneForm $(0,1)$-Tensor $V_i = (v_i, v)$

SR 4-Scalar $(0,0)$-Tensor $S$ or $S_0$

Lorentz Scalar

**Quantum Principles**

**See SRQM.org**

John B. Wilson
SciRealm.org
http://scirealm.org/SRQM.pdf
Some Cool Minkowski Metric Tensor Tricks

4-Gradient, 4-Position, 4-Velocity

SpaceTime is 4D

\[ \nabla \cdot (\mathbf{R}) = \eta^{\mu \nu} \rightarrow \text{Diag}[1,-1,-1,-1] \]

\[ \nabla \times (\mathbf{R}) = \text{Index-Lowered Minkowski Metric} \]

\[ \text{Tr}[\eta^{\mu \nu}] = \eta^{\mu \nu} = 4 \]

\[ \{\eta^{\mu \nu}\} = 1/(\eta_{\mu \nu}) \]

Lorentz Scalar Product \((\mathbf{U} \cdot \partial)\) = Derivative wrt. ProperTime \((d/d\tau)\) = Relativistic Factor * Derivative wrt. CoordinateTime \(\gamma(d/dt)\):

\[ \text{Trace}[T^\mu] = \eta_{\mu \nu} T^{\nu \nu} = T^\nu = T \]

\[ \mathbf{V} \cdot \mathbf{V} = V^\mu V^\nu \left(\eta^{\mu \nu}\right) = (V^\nu V^\nu)^{1/2} \]

\[ = \text{Lorentz Scalar Invariant} \]

A Tensor Study

of Physical 4-Vectors

4-Vector SRQM Interpretation

of QM

SciRealm.org

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http://scirealm.org/SRQM.pdf
**SRQM+EM Diagram:**

4-Vectors

4-Displacement
\[\Delta R = (c\Delta t, \Delta r)\]
4-Position
\[R = (ct, r)\]

4-Unit Temporal
\[T = \gamma(c, \beta)\]

4-Number Flux
\[N = (n, c) = n(c, u)\]

4-Probability Flux
\[J = (p, c) = p(c, u)\]

4-ProbCurr Density
\[\rho = \gamma(\rho, c)\]

4-Momentum
\[P = (mc, p) = (E/c, p)\]

4-Force
\[F = \gamma(E/c, f = p)\]

4-Charge Flux
\[J = (\rho c, q) = (\rho c, q)\]

4-EM Vector Potential
\[A = (\phi/c, a)\]

4-EM Potential Flux
\[Q = (U/c, q)\]

4-Momentum Field
\[P_f = (E/c, p)\]

4-Gradient
\[\partial = (\partial/c, \nabla)\]

4-Acceleration
\[A_f = \gamma(c, \gamma')\]

4-Gradient
\[\vec{\nabla} = (\partial t/c, \vec{\nabla})\]

4-Total Momentum
\[P_T = (E_T/c, p_T) = (H/c, p_T)\]

4-Position
\[R = (ct, r)\]

4-Mass Flux
\[G = (\rho m c, g) = (\rho e/c, g)\]

4-Gradient
\[\partial = (\partial/c, \nabla)\]

4-Unit Spatial
\[S = \gamma(n\hat{\beta}, n)\]

4-Displacement
\[\Delta R = (c\Delta t, \Delta r)\]

4-Force Density
\[F_{den} = \gamma(E_{den}/c, f_{den})\]

Trace of Tensor
\[\text{Trace}[T_{\mu
\nu}] = \eta_{\mu
\nu}T_{\mu
\nu} = T_{\mu
\mu} = T\]

\[V \cdot V = V \cdot V = (v \cdot v)^2 = (v^2)^2 = \text{Lorentz Scalar Invariant}\]
SRQM+EM Diagram:

4-Vectors, 4-Tensors

Lorentz Scalars / Physical Constants

SRQM and EM Interpretation

4-Vector SRQM Interpretation

SRQM Diagram

4-Vectors, 4-Tensors

Lorentz Scalars / Physical Constants
SRQM Diagram: Physical Constants Emphasized

- $K\cdot R = -\Phi_{phase}(i)$
- $P_T \cdot R = -S_{action}$
- $\partial \cdot A = 0$
- $\partial \cdot J = 0$
- $\partial \cdot N = 0$
- $\partial \cdot R = 4$

SpaceTime Dimension

Minkowski Metric

8$\pi$G/c$^4$

ProperTime

Derivative

Conservation of EM Field

Lorentz Gauge

Notice that all the main “Universal” or “Fundamental” Physical Constants are here: $G, c, \hbar, \varepsilon_0, \mu_0$. Some depend on the actual particle type: $q, m_0, \omega_o$. Some depend on regional conditions: $\tau, \rho_0, \phi_0, \psi^* \psi$. Some depend on interaction: $\Phi_{phase}, S_{action}$. Some are mathematical: $0, 4, \pi, [Diag[1, -1, -1, -1]], d/d\tau$. Conservation Laws are also a type of “zero” constant in this regard.

The majority of the constants are Lorentz Scalars, but some are 4-Vector or 4-Tensor, and all are valid for all inertial observers.

Fundamental Physical Constants are SR Lorentz Scalars

The fact that these “tie together” a network of 4-Vectors is a good argument for why their values are constant. Changing even one would change the relationship properties among all of the 4-Vectors.
SRQM Diagram: Projection Tensors
Temporal, Spatial, Null, SpaceTime

Projection Tensors act as follows:
Generic 4-Vector:
\( A^\mu = (a^0, a^1, a^2, a^3) \)

Temporal Projection:
\( V^\mu_\nu = \eta^\mu_\nu V^\nu_\mu \rightarrow \text{Diag}[1,0,0,0] \)
\( V^\mu_\nu A^\nu = (a^0,0,0,0) = (a^0,0) \)

Spatial Projection:
\( H^\mu_\nu = \eta^\mu_\nu H^\nu_\mu \rightarrow \text{Diag}[0,1,1,1] \)
\( H^\mu_\nu A^\nu = (0,a^1,a^2,a^3) = (0,a) \)

Space-Time Projection:
\( V^\mu_\nu + H^\mu_\nu A^\nu = \eta^\mu_\nu A^\nu = \delta^\mu_\nu A^\nu = (a^0,a) \)

The Minkowski Metric Tensor is the Sum of Temporal & Spatial Projection Tensors, all of which are dimensionless.

Trace\( [T^\mu_\nu] = \eta^\mu_\nu T^\nu_\mu = T^\mu_\mu = T \)
\( V\cdot V = V^\mu_\nu V^\nu_\mu = (\nu^\nu)^2 - \nu^\nu \nu^\nu = (\nu^\nu)^2 \)
Lorentz Scalar Invariant

SR 4-Tensor
(2,0)-Tensor \( T^{\mu\nu} \)
(1,1)-Tensor \( T^\nu_\nu \), or \( T^\nu_\nu \)
(0,2)-Tensor \( T^{\mu_\nu} \)

SR 4-Vector
(1,0)-Tensor \( V^\mu = (v^0,v) \)
SR 4-CoVector:OneForm
(0,1)-Tensor \( V_\mu = (v_0,-v) \)

SR 4-Scalar
(0,0)-Tensor \( S \) or \( S \)
Lorentz Scalar
SRQM Diagram: Projection Tensors & Perfect-Fluid Stress-Energy Tensor

Projection Tensors act as follows:

- **Spatial Projection**
  \[ T^\mu_\nu = (\rho_0, a^\mu, a^\nu) \]
  The rest-energy-density \( \rho_0 \) is the Temporal Projection.

- **Temporal Projection**
  \[ V^\mu_\nu = (\eta_\mu^\nu, a^\mu, a^\nu) \]
  The neg rest-pressure \( -p_0 \) is the Spatial Projection.

The Minkowski Metric Tensor is the Sum of Temporal & Spatial Projection Tensors, all of which are dimensionless.

\[ \eta_{\mu\nu} \rightarrow \text{Diag}[1,0,0,0] \text{, Temporal “Vertical” Projection Tensor} \]
\[ \eta_{\mu\nu} \rightarrow \text{Diag}[0,1,1,1] \text{, Spatial “Horizontal” Projection Tensor} \]

The rest-energy-density \( \rho_0 \) can be written in much simpler form using Projection Tensors:

\[ T^\mu_\nu = (\rho_0 + p_0)U^\mu U^\nu - (p_0)\eta^\mu_\nu \]
SRQM+EM Diagram: Projection Tensors & Stress-Energy Tensors: Special Cases

A Tensor Study of Physical 4-Vectors

SR 4-Tensor

Tr\[T]=\text{Trace Function} = \eta_{\mu\nu} \text{Trace } T_{\nu\mu} = \text{Null Projection Tensor} = N^{\nu} \rightarrow \text{Diag}[1,1/3,1/3,1/3] \text{ with } Tr[N^{\nu}] = 0

Equation of State

Trace[Tr\[T]] = \eta_{\mu\nu}T^{\mu\nu} = T^{\nu\nu} = T

4-Vector SRQM Interpretation of QM

SR 4-Scalar

Tr [ ] = Trace Function = \eta_{\mu\nu} N^{\nu} \rightarrow \text{Diag}[1,1/3,1/3,1/3] \text{ with } Tr[N^{\nu}] = 0

\eta\text{Lorentz Scalar}
**SRQM Study: Physical 4-Tensors**

**Matter-Dust vs. Null-Dust**

**Matter-Dust** is a special case of a perfect fluid with time-like 4-Velocity $U$, energy density $\rho$, and zero pressure $p$, described by the energy-momentum tensor $T_{\mu\nu} = (p+\rho)U^{\mu}U^{\nu}$. Because there is no pressure gradient, the fluid elements of the dust follow time-like geodesics.

**Null-Dust** corresponds to the limit in which the 4-Velocity $U$ becomes null, and is described by $T_{\mu\nu} = p_{\mu}K^{\mu}K^{\nu}$, $K\cdot K = 0$, with $p > 0$ and trace[$T$] = 0. The elements of dust follow null geodesics.

Null-Dust is interpreted as a coherent zero-rest-mass field propagating at the speed of light ($c$) in the null direction $K$. A null-dust can describe propagating electromagnetic (EM) or gravitational waves.

Note: There is an unfortunate slight notational clash between:

- 4-(Dust)NumberFlux $N = (nc,nu) = n_o U$
- 4-"Unit"Null $N = (1,\pm \eta)$

Use colored overbars on the "Unit" 4-Vectors $T = \text{Temporal} : N = \text{Null} : S = \text{Spatial}$

**4-Scalar**

- SR 4-Scalar
  - (0,1)-Tensor $V^\nu = (0,1)$ Lorentz Scalar

**4-Vector**

- SR 4-Vector
  - (1,0)-Tensor $V^\nu = V = (v^0,v)$
  - (0,2)-Tensor $T^\nu_{\nu} = (0,2)$ Lorentz Scalar

**4-Momentum**

$P = p^\mu = (E/c = mc, p = nu) = m_o U$

**4-UnitTemporal**

$T_{\nu} = T_{\nu} = \gamma(1,\beta) = cT$

**4-UnitNull**

$N = N^{\nu} = (1,\pm \eta)$

**4-UnitNull**

$N = N^{\nu} = (1,\pm \eta)$

- $K_{\text{photonic}} = (|\vec{k}|,k)$

**4-Momentum**

$P = P^\nu = (E/c = mc, p = nu) = m_o U$

**4-Velocity**

$U = U^\nu = cT$

- $V^\nu = T^\nu_{\nu} = (0,0,0)$
- $T^\nu_{\nu} = (0,0,0)$ Symmetric, Null

**Stress-Energy 4-Tensor**

Symmetric, Spatial Isotropic, Pressureless

- $(\text{Cold})$ Matter-Dust
  - $T^\nu_{\nu} = (\rho_o U^\nu U^\nu) = m_o U$

**Equation of State**

$EoS[T^\nu_{\nu}] = \frac{w}{p_o}$

**Null-Dust**: "Gravitational Wave"

Incoherent EM Mixture

$T^\nu_{\nu} \rightarrow \Phi_0 N^\nu N^\nu = \Phi_0 N^\nu N^\nu \rightarrow (\text{MCRF}_{\text{z-dir no rest frame}})$

**Tr[T^\nu_{\nu}] = 0**

Equation of State

$EoS[T^\nu_{\nu}] = \frac{w}{p_o}$

**4-Vector SRQM Interpretation of QM**

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http://scirealm.org/SRQM.pdf
**SRQM Study: Physical 4-Tensors**

**Vacuums vs. Fluids vs. Dusts**

**Lambda Vacuum**
- $T^{μν} \rightarrow (\rho_{vac})V^{μν} + (-p_{vac})H_{μν} \rightarrow$ (MCRF)
- Dark Energy?
- $EoS[T^{μν}] = w = -1$
- $Tr[T^{μν}] = 4ρ_{vac}$

**Zero: Nothing Vacuum**
- $T^{μν} \rightarrow 0^{μν} \rightarrow$ (MCRF)
- Symmetric, Spatial Isotropic

**Stress-Energy 4-Tensor**
- $\rho_{vac} = ρ_{vac}c^2$
- $p_{vac} = -(ρ_{vac}/3)$

**Perfect Fluid Stress-Energy**
- Symmetric, Spatial Isotropic
- $EoS[T^{μν}] = w = p_{vac}/ρ_{vac}$
- $Tr[T^{μν}] = ρ_{vac} - 3p_{vac}$

**PhotonGas = RadiationFluid**
- $T^{μν} \rightarrow (ρ_{rad})V^{μν} + (-p_{rad}/3)H_{μν} \rightarrow$ (MCRF)
- $EoS[T^{μν}] = w = p_{rad}/ρ_{rad}$
- $Tr[T^{μν}] = 0$

**Null-Dust: “Gravitational Wave”**
- $T^{μν} \rightarrow (0)\mathbb{N}^{μν} = (0)\mathbb{N}^{μν} \rightarrow$ (MCRF, z-dir rest frame)

**Isotropy Group**
- SO(1,3) = Full Lorentz Group
- SO(3) = Ordinary Rotation Group
- Segre Type (1,1,11)
- E(2) = Euclidean Plane

**Isotropy Group (2,0)**
- 4-Vector SRQM Interpretation of QM
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- John B. Wilson
- SciRealm.org
- SciRealm.org/SRQM.pdf
SRQM Diagram: 4-Tensors and 4-Scalars generated from 4-Vectors

All SR 4-Tensors can be generated from SR 4-Vectors:

\[ F^{\mu\nu} = \partial^\Lambda A^\mu - \partial^\nu A^\Lambda \] : Faraday EM 4-Tensor (from the 4-Gradient & 4-EMVectorPotential)

\[ M^{\mu\nu} = X^P = X^\mu P^\nu - X^\nu P^\mu \] : 4-AngularMomentum 4-Tensor (from the 4-Position & 4-Momentum)

\[ \eta^{\mu\nu} = \partial[R] = \partial^\nu[R^\Lambda] \] : SR:Minkowski Metric 4-Tensor (from the 4-Gradient & 4-Position)

\[ V^{\mu\nu} = T \otimes T = T^\mu T^\nu \] : (V)ertical:Temporal Projection 4-Tensor (from the 4-UnitTemporal)

\[ H^{\mu\nu} = \eta^{\mu\nu} - V^{\mu\nu} = \eta^{\mu\nu} - T^\mu T^\nu \] : (H)orizontal:Spatial Projection 4-Tensor (from previously made 4-Tensors above)

\[ T_{\text{cold\_dust}}^{\mu\nu} = P \otimes N = P^\mu N^\nu \] : (Cold)Dust Stress-Energy 4-Tensor (from the 4-Momentum & 4-DustNumberFlux)

\[ (\rho_{eo}) = T_{\text{Cold\_dust}}^{\mu\nu} V_{\mu\nu} \] : MCRF EnergyDensity 4-Scalar (from previously made 4-Tensors above)

\[ T_{\text{Lambda\_Vacuum}}^{\mu\nu} = (\rho_{eo})\eta^{\mu\nu} \] : LambdaVacuum (Dark Energy) Stress-Energy 4-Tensor (from previously made 4-Tensors above)

\[ (\rho) = (k)(1/3)T_{\text{Lambda\_Vacuum}}^{\mu\nu} H_{\mu\nu} \] : MCRF Pressure 4-Scalar (from previously made 4-Tensors above)

with the pressure initially set to the EnergyDensity and (k) an arbitrary constant which sets pressure level

\[ T_{\text{Perfect\_Fluid}}^{\mu\nu} = (\rho_{eo})V^{\mu\nu} + (-\rho)H^{\mu\nu} \] : PerfectFluid Stress-Energy 4-Tensor (from previously made 4-Tensors above)

\[ \text{Equation of State} \ EoS[T^{\mu\nu}] = w = \rho/p_{eo} \]

\[ \text{Trace}[T^{\mu\nu}] = \eta^{\mu\nu}T^{\nu\nu} = T^\mu T^\nu = \partial [\eta] \]

\[ V \cdot V = V^\mu \eta^{\mu\nu} V^\nu = \left((v^0)^2 - v \cdot v\right) = (v^0)^2 \]

\[ = \text{Lorentz Scalar Invariant} \]
A Tensor Study of Physical 4-Vectors

SRQM Study:
4D Stokes':Gauss' Theorem

4D Generalized Stokes':Gauss' Theorem in Special Relativity
\[ \int_{\Omega} d^4x \left( \partial_{\mu} V^\mu \right) = \oint_{\partial \Omega} (V^\mu N_\mu) \]
\[ \int_{\Omega} d^4x \left( \partial \cdot V \right) = \oint_{\partial \Omega} (V \cdot N) \]

with:
- \( V = V^\mu \) is a 4-Vector field defined in 4D Minkowski Region \( \Omega \)
- \( (\partial \cdot V) = (\partial_{\mu} V^\mu) \) is the 4-Divergence of \( V \)
- \( (V \cdot N) = (V^\mu N_\mu) \) is the component of \( V \) along the boundary normal \( N \)-direction
- \( \Omega \) is a 4D simply-connected region of Minkowski SpaceTime
- \( \partial \Omega = \partial \Omega = S \) is its 3D boundary with its own 3D Volume element \( dS \) and outward pointing normal \( N \).
- \( N = N^\mu \) is the outward-pointing normal of the boundary
- \( d^4x = c dt (dx dy dz) \) is the 4D differential volume element.

In vector calculus, the divergence theorem, also known as Gauss's theorem or Ostrogradsky's theorem, is a result that relates the flow (that is, flux) of a vector field through a surface to the behavior of the vector field inside the surface. More precisely, the divergence theorem states that the outward flux of a vector field through a closed surface is equal to the volume integral of the divergence over the region inside the surface.

Intuitively, it states that the sum of all sources minus the sum of all sinks gives the net flow out of a region.

In vector calculus, and more generally in differential geometry, the generalized Stokes' theorem is a statement about the integration of differential forms on manifolds, which both simplifies and generalizes several theorems from vector calculus.

In Special Relativity, we have:
- 0D Vector \( S = 0 \)-Vector Field
- 1D Vector \( V = V^\mu \) is a 1-Vector Field
- 2D Vector \( T^{\mu\nu} \) or \( T_{\mu\nu} \)
- 3D Vector \( V = (V^0, V) \)
- 4D Vector \( V = (V^0, V^1, V^2, V^3) \)
- 4D Vector \( V = (V^0, V^1, V^2, V^3) \)

4-Vector SRQM: A treatise of SR → QM by John B. Wilson (SciRealm@aol.com)
SRQM Diagram:
Minimal Coupling = (EM)Potential Interaction
Conservation of 4-TotalMomentum

4-Momentum
\[ \dot{\mathbf{r}} = \mathbf{J} \]

4-Velocity
\[ \mathbf{u} = \gamma (\mathbf{c}, \mathbf{u}) \]

Rest Mass:Energy
\[ E = mc^2 \]

4-Vector SRQM Interpretation of QM

http://scirealm.org/SRQM.pdf

SciRealm.org
John B. Wilson
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A Tensor Study of Physical 4-Vectors

4-Vector SRQM

\( T^u \) \( \rightarrow \) QM

Hamilton-Jacobi
\[ H = -\partial [S_{\text{action}}], \mathbf{V}[S_{\text{action}}] \]

Conservation of 4-TotalMomentum
\[ \mathbf{P}_t = \mathbf{P} + \mathbf{q} \]

Minimal Coupling Relation
\[ \mathbf{P}_t = \mathbf{P} + \mathbf{q} \]

SR 4-Scalar
(0,2)-Tensor S or \( S_\mathbf{\gamma} \)
(0,1)-Tensor \( V = (\mathbf{v}, \mathbf{v}) \)
(1,0)-Tensor \( V = (\mathbf{v}, \mathbf{v}) \)

SR 4-Vector

4-Mass:4-Energy
\[ (m, E) \]

SR 4-Tensor
\( (2,0)-\mathbf{T}^u \)
\( (1,1)-\mathbf{T}^u \)
\( (0,2)-\mathbf{T}^u \)

SR 4-EMVector:OneForm

\[ \mathbf{A} = (q/c, \mathbf{a}) \]

Conservation of 4-TotalMomentum
\[ \mathbf{P}_t = \mathbf{P} + \mathbf{q} \]

Minimal Coupling Relation
\[ \mathbf{P}_t = \mathbf{P} + \mathbf{q} \]

SR 4-Tensor

(2,0)-Tensor \( T \)
(1,1)-Tensor \( T^u \)
(0,2)-Tensor \( T^u \)

SR 4-TotalMomentum is the Sum over all 4-Momenta

\[ \mathbf{P}_t = \mathbf{P} + \mathbf{q} \]

SR 4-Scalar

\[ \varphi/c^2 \]

Conservation of 4-TotalMomentum
\[ \mathbf{P}_t = \mathbf{P} + \mathbf{q} \]

Minimal Coupling Relation
\[ \mathbf{P}_t = \mathbf{P} + \mathbf{q} \]

SR 4-Vector

\[ \mathbf{A} = (q/c, \mathbf{a}) \]

Conservation of 4-TotalMomentum
\[ \mathbf{P}_t = \mathbf{P} + \mathbf{q} \]

Minimal Coupling Relation
\[ \mathbf{P}_t = \mathbf{P} + \mathbf{q} \]
SRQM Study:
SRQM Hamiltonian:Lagrangian Connection

\[ H + L = (p_T \cdot u) = \gamma (P_T \cdot U) + \frac{-(P_T \cdot U)}{\gamma} \]

4-Momentum \( P = m_o U = (E_o/c^2) U \)

4-Vector Potential \( A = (\varphi_o/c^2) U \)

4-Total Momentum \( P_t = (P + qA) = (E/c = (E + q\varphi)/c, p_t = p + qa) \)

4-Total Momentum \( P_T = (E_i/c = H/c, p_T) = \sum_n [P_n] \)

\[ P \cdot U = \gamma(E - p \cdot u) = E_o = m_o c^2 \quad A \cdot U = \gamma(\varphi - a \cdot u) = \varphi_o \]

\( P_T \cdot U = (P \cdot U + qA \cdot U) = E_o + q\varphi_o = m_o c^2 + q\varphi_o \)

\( \gamma = 1/\sqrt{1 - \beta^2} \): Relativistic Gamma Identity

(\( \gamma - 1/\gamma \)) = (\( \gamma\beta \cdot \beta \)): Manipulate into this form... still an identity

(\( \gamma - 1/\gamma \))(P_T \cdot U) = (\( \gamma\beta \cdot \beta \))(P_T \cdot U): Still covariant with Lorentz Scalar

\[ \gamma(P_T \cdot U) + -(P_T \cdot U)/\gamma = (\gamma \beta \cdot \beta)(P_T \cdot U) \]

\[ \gamma(P_T \cdot U) + -(P_T \cdot U)/\gamma = (\gamma \beta \cdot \beta)(E_o + q\varphi_o) \]

\[ \gamma(P_T \cdot U) + -(P_T \cdot U)/\gamma = (\gamma \beta \cdot \beta)(E_o + q\varphi_o)/c^2 \]

\[ \gamma(P_T \cdot U) + -(P_T \cdot U)/\gamma = [\gamma(E_o/c^2 + q\varphi_o/c^2) \cdot u \cdot u] \]

\[ \gamma(P_T \cdot U) + -(P_T \cdot U)/\gamma = [(E_o/c^2 + q\varphi_o/c^2) \cdot u \cdot u] \]

\[ \gamma(P_T \cdot U) + -(P_T \cdot U)/\gamma = [(p + qa) \cdot u \cdot u] \]

\[ \gamma(P_T \cdot U) + -(P_T \cdot U)/\gamma = (p_T \cdot U) \]

\( \{ H \} + \{ L \} = (p_T \cdot u) \): The \{Hamiltonian:Lagrangian\} Connection

H = \( \gamma(P_T \cdot U) = \gamma((P+qA) \cdot U) \): The Hamiltonian with minimal coupling

L = \( -(P_T \cdot U)/\gamma = -(P+qA) \cdot U)/\gamma \): The Lagrangian with minimal coupling

H:L Connection in Density Format

H + L = (p_T \cdot u)

nH + nL = n(p_T \cdot u), with number density \( n = \gamma n_o \)

\( \mathcal{H} + \mathcal{L} = (g_T \cdot u) \), with

momentum density \{g_T = n p_T\}

Hamiltonian Density \( \mathcal{H} = nH \)

Lagrangian Density \( \mathcal{L} = nL = (\gamma n_o)(L_\gamma/c) = n_o L_\gamma \)

Lagrangian Density is Lorentz Scalar

for an EM field (photonic):

\( \mathcal{H} = (1/2)[\epsilon_o e \cdot e + b \cdot b/\mu_o] \)

\( \mathcal{L} = (1/2)[\epsilon_o e \cdot e - b \cdot b/\mu_o] = (-1/4\mu_o) F_{\mu\nu} F^{\mu\nu} \)

\( \mathcal{H} + \mathcal{L} = \epsilon_o e \cdot e = (g_T \cdot u) \)

\( |u| = c \)

\( |g_T| = \epsilon_o e \cdot e/c \)

Poynting Vector \( |s| = |g| c^2 \rightarrow c\epsilon_o e \cdot e \)

H_o + L_o = 0 Calculating the Rest Values

H_o = (P_T \cdot U) \quad H = \gamma H_o \)

L_o = -(P_T \cdot U) \quad L = L_\gamma/o \)

4-Vector notation gives a very nice way to find the Hamiltonian/Lagrangian connection:

(\( H \) ) + (\( L \) ) = (p_T \cdot u), where \( H = \gamma(P_T \cdot U) \) & \( L = -(P_T \cdot U)/\gamma \)
Thus, \((H) + (L) = (p_T \cdot u) = \gamma(P_T \cdot U) + -(P_T \cdot U)/\gamma\)

**SRQM Study:**

**SRQM Hamiltonian: Lagrangian Connection**

\[ H + L = (p_T \cdot u) = \gamma(P_T \cdot U) + -(P_T \cdot U)/\gamma \]

<table>
<thead>
<tr>
<th>Relativistic Hamiltonian</th>
<th>Relativistic Lagrangian</th>
<th>(p_T \cdot u = (\gamma^2) (P_T \cdot U))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(H = \gamma(P_T \cdot U))</td>
<td>(L = -(P_T \cdot U)/\gamma)</td>
<td>(H + L = p_T \cdot u = \gamma(P_T \cdot U) - (P_T \cdot U)/\gamma)</td>
</tr>
</tbody>
</table>

**Rules to manipulate the equations**

\[ P_T = P + Q \]
\[ P = m_U = (E/c) \Rightarrow Q = qA = q(\varphi/c^2)U \]
\[ p_T \cdot U = H_c, \quad U = c^2 \]
\[ P \cdot U = m_U^2 = E_0 \Rightarrow A \cdot U = \varphi, \quad m = \gamma m_0 = E_0 \Rightarrow \varphi = \gamma \varphi_0 \]

**4-Vector notations gives a very nice way to find the Hamiltonian/Lagrangian connection:**

\((H) + (L) = (p_T \cdot u), \text{ where } H = \gamma(P_T \cdot U) \& L = -(P_T \cdot U)/\gamma\)
SRQM Study:

SR Lagrangian, Lagrangian Density, and Relativistic Action (S)

Relativistic Action (S) is Lorentz Scalar Invariant

\[ S = [Ldt = [(L/\gamma)(dt) = [(L_0/\gamma)(\gamma dt) = [(L_0)(dt)] \]

Explicitly-Covariant Relativistic Action (S)

Particle Form

Density Form (\( n^c \text{Particle} \))

H-L Connection in Density Format for Photonic System (no rest-frame)

For an EM field (photonic):

\[ H = (1/2)[(\varepsilon_e e \cdot e + b \cdot b/\mu_0)] = \varepsilon_e E_0 = \rho_{\text{EM}} = \text{EM Field Energy Density} \]

\[ L = (1/2)[(\varepsilon_e e \cdot e + b \cdot b/\mu_0)] = (-1/4 \mu_0)F_{\mu\nu}F^{\mu\nu} = (-1/4 \mu_0)^*\text{Faraday EM Tensor Inner Product} \]

The Relativistic Action Equation is seen in many different formats
SR Hamilton-Jacobi Equation and Relativistic Action (S)

Lagrangian \( \{ L = (p_T \cdot u) - H = -(P_T \cdot U)/\gamma \} \) is *not* a Lorentz Scalar
Rest Lagrangian \( \{ L_0 = \gamma L = -(P_T \cdot U) \} \) is a Lorentz Scalar

Relativistic Action (S) is Lorentz Scalar
\[ S = \int L dt \]
\[ S = \int (L_0/\gamma)(\gamma d\tau) \]
\[ S = \int (L_0)(d\tau) \]

Explicitly Covariant
Relativistic Action (S)
\[ S = \int L_0 d\tau = -\int H_\gamma d\tau \]
\[ S = -\int (P_T \cdot U)d\tau \]
\[ S = \int (P_T \cdot d\mathbf{R}/d\tau)d\tau \]
\[ S = \int (P_T \cdot d\mathbf{R}) \]
\[ S = \int (P_T \cdot d\mathbf{R}) \]
\[ S = \int ((P + qA) \cdot U)d\tau \]
\[ S = \int (P \cdot U + qA \cdot U)d\tau \]
\[ S = \int (E_0 + q\varphi_0)d\tau \]
\[ S = \int (E_0 + V_\gamma)d\tau \text{ with } V_\gamma = q\varphi_0 \]
\[ S = \int (m_0 c^2 + V_\gamma)d\tau \]
\[ S = \int (H_\gamma)d\tau \]

The Hamilton-Jacobi Equation is incredibly simple in 4-Vector form
\[ -\partial[S_{\text{action}}] = P_T : \text{ gives temporal } (-\partial[S_{\text{action}}] = H = E_T) \text{ & spatial } (V[S_{\text{action}}] = p_T) \]
SRQM Study:
Relativistic Action ($S$), Rest Lagrangian ($L_0$)
Path to 4-TotalMomentum

4-Velocity
$$\mathbf{U} = \gamma(c, \mathbf{u}) = (\mathbf{U} \cdot \partial_R) \mathbf{R}$$
$$\mathbf{U}^0 = (d/dt) \mathbf{R}$$

4-PositionGradient: 4-Gradient
$$\partial_R \beta = \partial_R \mathbf{R} = \partial / \partial t - \partial / \partial x, -\partial / \partial y, -\partial / \partial z$$

4-TotalMomentum
$$\mathbf{P}_T = -\partial_S[\mathbf{L}_0] = -\partial_\mathbf{u}[\gamma] = -\partial_\mathbf{u}[(\mathbf{P}_T \cdot \mathbf{U})] = \mathbf{P}_T$$

4-VelocityGradient
$$\partial_\mathbf{u} \beta = \partial_\mathbf{u} \partial_\mathbf{u} \mathbf{U} = (\partial_\mathbf{u} / c, -\nabla)$$

Virtual Path to 4-TotalMomentum
$$\mathbf{R} = (ct, \mathbf{r}) = (\mathbf{R} \cdot \partial_\mathbf{u}) \mathbf{U}$$
$$\mathbf{U}^0 = (d/dt) \mathbf{R} / (d/dt) \mathbf{R} = (\mathbf{R} \cdot \partial_\mathbf{u}) \mathbf{U}$$

SRQM: A treatise of SR→QM by John B. Wilson (SciRealm@aol.com)
SRQM Diagram: Relativistic Action Equation

\[ (S = -\int (P_T \cdot dR)) \] Integral Format: 4-Scalars

4-Displacement \( \Delta R = (c\Delta t, \Delta r) \)
4-Position \( R = (ct, r) \)

\( \delta[R] = \eta^{\mu \nu} \rightarrow \text{Diag}[1, -1, -1, -1] \)
Minkowski Metric

Lagrangian Density
\[ L = nL = (\gamma n_o)(L_o/\gamma) = n_o L_o \]

Proper Time Derivative
\[ \mathbf{U} \cdot \partial \Gamma \]

Relativistic Number density
\[ n = \gamma n_o \]

\( \gamma = 1/\sqrt{1 - \beta \cdot \beta} \): Relativistic Identity
\( (\gamma - 1/\gamma) = (\gamma - \beta \cdot \beta) \): Alternate Form
\( (\gamma - 1/\gamma)(P_T - U) = (\gamma - \beta \cdot \beta)(P_T - U) \)
\( (\gamma + 1/\gamma)(P_T - U) = (\gamma + \beta \cdot \beta)(P_T - U) \)
\( H = (P_T - U)/\gamma = L \gamma \)

Legendre Factor
\[ (P_T - U) \]

Hamiltonian
\[ H = \gamma (P_T - U) = H_o \]

Proper Time Derivative
\[ \mathbf{U} \cdot \partial \Gamma \]

\( \mathbf{U} \cdot \partial \Gamma \)

Relativistic Coordinate Time
\[ t = \gamma t_o \]

\( \gamma = \gamma t_o \)

4-Force
\[ F = \gamma (E/c, f = \mathbf{p}) \]

4-Momentum
\[ P = (mc, p) = (E/c, p) \]

4-TotalMomentum
\[ \mathbf{P} = (E/c, \mathbf{p}) = (H/c, \mathbf{p}) \]


\[ \mathbf{T} \]

Minimal Coupling

SR 4-Vector
(2,0)-Tensor \( T^{\mu \nu} \)
(1,1)-Tensor \( T^\mu \), or \( T_{\mu} \)
(0,2)-Tensor \( T_{\mu \nu} \)

SR 4-Vector
\( 4 \)-CoVector: OneForm
(0,1)-Tensor \( V_\mu = (v_\mu, -) \)

SR 4-Scalar
(0,0)-Tensor \( \rho_o \) or \( \rho_o \)
Lorentz Scalar

SR Relativistic Scalar
(not Lorentz Invariant)

Lorentz Scalar Invariant

\[ \mathbf{V} \cdot \mathbf{V} = V^\mu V_\mu = \left( (\mathbf{v})^2 \right) - V^\mu - (\mathbf{v})^2 = \left( (\mathbf{v})^2 \right) \]

Trace\( [T^\mu] = \eta_{\mu \nu} T^{\mu \nu} = T^{\mu \nu} = T \)

\[ V \cdot V = V^\mu V_\mu = (v^\mu)^2 - v^\mu - (v^\mu)^2 = (v^\mu)^2 \]
The differentia\[\partial\]Relativistic Euler-Lagrange Equation

Note Similarity:
4-Velocity is Proper Time
Derivative of 4-Position
U = (d/dt)R [m/s] = [1/s][m]

SR Relativistic
γ = dt/d\tau = 1/\sqrt{1-β²}

Proper Time Derivative
U = (d/d\tau)R

4-Position: Proper Time
R = (ct, r)

Note Similarity:
4-Velocity is Proper Time
Derivative of 4-Position
U = (d/dt)R [m/s] = [1/s][m]

SR Relativistic
γ = dt/d\tau = 1/\sqrt{1-β²}

Proper Time Derivative
U = (d/d\tau)R

4-Position: Proper Time
R = (ct, r)

The differential form just inverts the dimensional units
SRQM Diagram: Relativistic Euler-Lagrange Equation

The Easy Derivation \((U = (d/dt)R) \rightarrow (\partial_R = (d/d\tau)\partial_U)\)

Note Similarity:
- 4-Velocity is ProperTime Derivative of 4-Position
- \(U = (d/d\tau)R\) \([m/s] = [1/s][m]\)

Relativistic Euler-Lagrange Eqn
\(\partial_{\alpha} = (d/d\tau)\partial_{\beta}\) \([m] = [1/s][s/m]\)

The differential form just inverses the dimensional units, so the placement of the \(R\) and \(U\) switch.

That is it: so simple! Much, much easier than how I was taught in grad school.

To complete the process and create the Equations of Motion, one just applies the base form to a Lagrangian.

This can be:
- a classical Lagrangian
- a relativistic Lagrangian
- a Lorentz scalar Lagrangian
- a quantum Lagrangian

SR 4-Vector
- \((1,0)-\)Tensor \(T_{\alpha\beta}\)
- \((0,1)-\)Tensor \(V_{\alpha} = (v_{\alpha}, v)\)

4-Position
\(R((ct, r)) = (R\cdot \partial_{ct})U = R^0((\dot{d}t)U)\)

\(\partial_{R} = 4\) SpaceTime Dimension

\(\partial_{R}[\ldots] \gamma d/d\tau [\ldots] d/d\tau [\ldots] \ldots\)

\(\partial_{U} = 4\) SpaceTime Dimension

\(U\cdot \partial_{R}[\ldots] \gamma d/d\tau [\ldots] d/d\tau [\ldots] \ldots\)

VelocityGradient One-Form
\(\partial_{\alpha} = \partial U = (\partial U/c, \nabla U)\)

\(\rightarrow (\partial U/c, \nabla U, \partial U/c, \nabla U)\)

Relativistic 4D Euler-Lagrange Eqn
\(\partial_{R} = (d/d\tau)\partial_{U}\)
\(\partial L/\partial R = (d/d\tau)\partial L/\partial U\)

Classical limit, spatial component
\(\partial L/\partial r = (d/dt)\partial L/\partial u\)
\(\partial L/\partial x = (d/dt)\partial L/\partial u\)

4-PositionGradient: 4-Gradient
\(\partial_{R} = \partial_{R} = (\partial [c, \nabla])\)

\(\rightarrow (\partial [ct, \nabla], \partial [d\tau, \nabla], \partial [d\tau, \nabla], \partial [d\tau, \nabla])\)

Relativistic Particle Dynamics Eqn (4-Vector)

\(U = (d/d\tau)R\)

\(\partial_{R} = (d/d\tau)\partial_{U}\)

\(\rightarrow U\cdot \partial_{R}[\ldots] \gamma d/d\tau [\ldots] d/d\tau [\ldots] \ldots\)

\(\partial_{U} = 4\) SpaceTime Dimension

\(\partial U = 4\) SpaceTime Dimension

\(\partial_{U} = (\partial U/c, \nabla U)\)

\(\rightarrow (\partial U/c, \nabla U, \partial U/c, \nabla U)\)

SR 4-Tensor
- \((2,0)-\)Tensor \(T^{\mu\nu}\)
- \((1,1)-\)Tensor \(T^{\mu}_{\nu}\) or \(T^{\nu}_{\mu}\)
- \((0,2)-\)Tensor \(T_{\mu\nu}\)

SR 4-Vector
- \((1,0)-\)Tensor \(V^{\alpha}_{\mu} = (v^{\alpha}, v)\)

SR 4-CoVector: One-Form
- \((0,1)-\)Tensor \(V_{\alpha} = (v_{\alpha}, v)\)

SR 4-Scalar
- \((0,0)-\)Tensor \(S\) or \(s\) Lorentz Scalar

\(P_{\mu} = -\partial_{U}[L_{\mu}] = -\partial_{U}[\gamma L] = -\partial_{U}[\gamma \{-P_{T}(U)\}] = P_{T}
(E = (c, p)) = (\partial_{\mu}U)[L] = (\partial_{\mu}U)[L]\)

\(P_{T} = \partial_{\mu}L = (\partial \partial U)[L]\)

Trace\(\{T^{\mu\nu}\} = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu}_{\mu} = T\)

\(V \cdot V = V^{\alpha}V^{\beta} = (v^{\alpha})^{2} - (v^{\beta})^{2}\)

Lorentz Scalar Invariant

http://scirealm.org/SRQM.pdf
SRQM Diagram: Relativistic Euler-Lagrange Equation

The Easy Derivation \((U=(d/d\tau)R) \rightarrow (\partial_R=(d/d\tau)\partial_U)\)

Relativistic 4D Euler-Lagrange Eqn
\[ \partial_R = (d/d\tau)\partial_U \]
\[ \partial_t \partial_R = (d/d\tau)\partial_t \partial_U \]

Classical limit, spatial component
\[ \partial_t \partial_R = (d/d\tau)\partial_t \partial_U \]

\[ L = - [(m_0c^2 + V_0)\gamma] \]
\[ \partial_t \partial_t L/\partial U_0 = -(m_0c^2 + V_0)\partial_1/\partial U_0 \]
\[ \partial_t \partial_t L/\partial x = -(1/\gamma)\partial [V_0]/\partial x \]
\[ (d/d\tau)\partial_t \partial_t L/\partial u = (d/d\tau)[(m_0c^2 + V_0)\gamma u/c^2] \]
\[ (d/d\tau)\partial_t \partial_t L/\partial x = - (1/\gamma)\partial [V_0]/\partial x \]

4-Position
\[ R=(ct,r)=(R\cdot\partial_U)U = R^0=(\partial_U)U \]

4-Vector SRQM Interpretation of QM

http://scirealm.org/SRQM.pdf

SciRealm.org
John B. Wilson
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http://scirealm.org/SRQM.pdf

SR 4-Tensor
(2,0)-Tensor \( T^{ix} \)
(1,1)-Tensor \( T^{iv} \) or \( T^{iv}_i \)
(0,2)-Tensor \( T^{iv}_x \)

SR 4-Vector
(1,0)-Tensor \( V^i = V = (v^0,v) \)
(0,1)-Tensor \( V = V_0 \)

SR 4-Scalar
(0,0)-Tensor \( S = S_{action} \)

SR 4-CofVector: One-Form
(0,1)-Tensor \( V_0 = (V_0 - V) \)

Lorentz Scalar Invariant
\[ \nabla^2 \nabla f = \nabla \nabla \cdot f \]

4-Total-Momentum
\[ P_T = (E_i/c = H/c, p_T) \]
\[ = \partial U[S_{action}] = \partial U[\nabla S_{action}] \]
\[ p_T = \partial U[L_x] = \partial U[L] \]

Trace[\( T^{iv}_v \)] = \( \eta_{iv} T^{iv}_v = T^{iv}_v = T \)

\[ V \cdot V = V^i V^i = (v^0)^2 - V \cdot V = (v^0)^2 \]

\[ \eta_{iv} = \text{Lorentz Scalar Invariant} \]
4-Velocity \( \mathbf{U} \) is ProperTime Derivative of 4-Position \( \mathbf{R} \). The Euler-Lagrange Eqn can be generated by taking the differential form of the same equation.

Relativistic 4-Vector Kinematical Eqn
\[
\mathbf{U} = \left( \frac{d}{d\tau} \right) \mathbf{R} \\
\mathbf{U} \cdot \mathbf{K} = \left( \frac{d}{d\tau} \right) \left[ \mathbf{R} \cdot \mathbf{K} \right]
\]

Relativistic Euler-Lagrange Eqns
{uses gradient-type 4-Vectors}
\[
\partial_\mathbf{R} = \left( \frac{d}{d\tau} \right) \partial_\mathbf{U}; \text{ (particle format)} \\
\partial_\mathbf{R} = \left( \frac{d}{d\tau} \right) \partial_\mathbf{U} \\
\partial_\mathbf{U} = \left( \mathbf{U} \cdot \partial_\mathbf{R} \right) \partial_\mathbf{U} \\
\partial_\mathbf{U} = \left( \mathbf{U} \cdot \partial_\mathbf{R} \right) \partial_\mathbf{U} \\
\partial_\Phi = \left( \frac{d}{d\tau} \right) \partial_\mathbf{U} \\
\partial_\Phi = \left( \frac{d}{d\tau} \right) \partial_\mathbf{U} \\
\partial_\Phi = \left( \mathbf{U} \cdot \partial_\mathbf{R} \right) \partial_\mathbf{U} \\
\partial_\Phi = \left( \mathbf{U} \cdot \partial_\mathbf{R} \right) \partial_\mathbf{U} \\
\partial_\mathbf{R} = \left( \mathbf{U} \cdot \partial_\mathbf{R} \right) \partial_\mathbf{U}
\]

\( \mathbf{L} = \frac{1}{2} \left( \partial_\mathbf{\Phi} \cdot \partial_\mathbf{\Phi} \right) - \left( m_v c / h \right)^2 \Phi^2 \): KG Lagrangian Density
\[
\partial_\mathbf{\Phi} = \partial_\mathbf{\Phi} \cdot \partial_\mathbf{\Phi} \cdot \partial_\mathbf{\Phi} \cdot \partial_\mathbf{\Phi} \\
\partial_\mathbf{\Phi} = \left( \frac{d}{d\tau} \right) \partial_\mathbf{\Phi} \\
\partial_\mathbf{\Phi} = \left( \frac{d}{d\tau} \right) \partial_\mathbf{\Phi} \\
\partial_\mathbf{\Phi} = \left( \frac{d}{d\tau} \right) \partial_\mathbf{\Phi} \\
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\partial_\mathbf{\Phi} = \left( \frac{d}{d\tau} \right) \partial_\mathbf{\Phi}
\]

\( \partial_\mathbf{\Phi} \cdot \partial_\mathbf{\Phi} \cdot \partial_\mathbf{\Phi} \cdot \partial_\mathbf{\Phi} \\
\partial_\mathbf{\Phi} = \left( \frac{d}{d\tau} \right) \partial_\mathbf{\Phi} \\
\partial_\mathbf{\Phi} = \left( \frac{d}{d\tau} \right) \partial_\mathbf{\Phi} \\
\partial_\mathbf{\Phi} = \left( \frac{d}{d\tau} \right) \partial_\mathbf{\Phi} \\
\partial_\mathbf{\Phi} = \left( \frac{d}{d\tau} \right) \partial_\mathbf{\Phi}
\]

\( \partial_\mathbf{\Phi} \cdot \partial_\mathbf{\Phi} \cdot \partial_\mathbf{\Phi} \cdot \partial_\mathbf{\Phi} \\
\partial_\mathbf{\Phi} = \left( \frac{d}{d\tau} \right) \partial_\mathbf{\Phi} \\
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\partial_\mathbf{\Phi} = \left( \frac{d}{d\tau} \right) \partial_\mathbf{\Phi} \\
\partial_\mathbf{\Phi} = \left( \frac{d}{d\tau} \right) \partial_\mathbf{\Phi}
\]

\( \mathbf{L} = \frac{1}{2} \left( \partial_\mathbf{\Phi} \cdot \partial_\mathbf{\Phi} \right) - \left( m_v c / h \right)^2 \Phi^2 \): KG Lagrangian Density

\( \partial_\mathbf{\Phi} \cdot \partial_\mathbf{\Phi} \cdot \partial_\mathbf{\Phi} \cdot \partial_\mathbf{\Phi} \\
\partial_\mathbf{\Phi} = \left( \frac{d}{d\tau} \right) \partial_\mathbf{\Phi} \\
\partial_\mathbf{\Phi} = \left( \frac{d}{d\tau} \right) \partial_\mathbf{\Phi} \\
\partial_\mathbf{\Phi} = \left( \frac{d}{d\tau} \right) \partial_\mathbf{\Phi} \\
\partial_\mathbf{\Phi} = \left( \frac{d}{d\tau} \right) \partial_\mathbf{\Phi}
\]

Klein-Gordon Relativistic Quantum Wave Eqn
A Tensor Study of Physical 4-Vectors

**SRQM Diagram:**

**Relativistic Euler-Lagrange Equation**

**Equation of Motion (EoM) for EM particle**

\[ \delta \mathbf{U} = \frac{\partial}{\partial t} \mathbf{U} = (\mathbf{v} - \mathbf{u}) \]

**Proper Time**

\[ \mathbf{U} \cdot \partial = \gamma \mathbf{U} \cdot \partial = (\mathbf{v} - \mathbf{u}) \]

**4-Position**

\[ \mathbf{R} = (ct, \mathbf{r}) \]

**4-Displacement**

\[ \Delta \mathbf{R} = (c \Delta t, \Delta \mathbf{r}) \]

**4-Velocity**

\[ \mathbf{U} = \gamma (\mathbf{c}, \mathbf{u}) \]

**4- EM Vector Potential**

\[ \mathbf{A} = (\phi / c^2) \]

**4-Momentum**

\[ \mathbf{p} = (mc, \mathbf{p}) = (E/c, \mathbf{p}) \]

**4-Force**

\[ \mathbf{F} = \gamma (E/c, f) \]

**4-Vector SRQM Interpretation of QM**

SciRealm.org
John B. Wilson
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http://scirealm.org/SRQM.pdf
SRQM Diagram:
Relativistic Euler-Lagrange Equation using $L_0$
Equation of Motion (EoM) for EM particle

4-TotalMomentum
$P_T = (E_T/c, p_T) = (H/c, p_T)$

= $P + qA$

4-Velocity
$U = \gamma(c, u)$

Relativistic Euler-Lagrange Equations of Motion

(d/d$\tau$)$\partial[U[L_0]] = (d/d$\tau$)$\partial[U[L_0]]$

= $-L_T$

= $-P_T$ - $qA$

= $-\nabla U$

Relativistic Rest Lagrangian
$Euler-Lagrange$ Equations

Rest Lagrangian $L_0$

= $-\gamma(pT - U)$

= $-(P + qA)$. $U$

= $-P . U - qA . U$

4-Velocity Gradient
$\partial_U = (\partial_U / \nabla U)$

= $\omega_\parallel = (\partial U / \partial \tau)$

4-Position Gradient
$\partial_U = (\partial_U / \partial \tau)$

= $\omega_\perp = (\partial U / \partial \tau)$

4-Vector SRQM

EM Faraday
$F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha$

SR 4-Vector

$T^{\mu\nu}$ - (0,1)-Tensor $V^{\nu} = (v^\nu, 0)

SR 4-Scalar

A = $\phi / c, a$

SR 4-CoVector OneForm

$V = (\nu, 0)$ - Tensor $V_\nu = (v_\nu, 0)$

SR 4-Tensor

$(2,0)$-Tensor $T^{\mu\nu}$

$(1,1)$-Tensor $T^{\nu} = T^\nu$

$(2,0)$-Tensor $T_{\mu\nu}$
SRQM Diagram:

Relativistic Euler-Lagrange Equation using $L_0'$

Equation of Motion (EoM) for EM particle

$L_0' = aL_0 + b$: Lagrangian-with-affine-transform $L_0'$ gives same physics, when using the Euler-Lagrange Equation

Let $(a = \text{const} = -1)$ and $(b = \text{const} = -\frac{1}{2} m c^2 = -\frac{1}{2} m U \cdot U)$

Then $L_0' = L_0 - \frac{1}{2} m c^2 = \frac{1}{2} m U \cdot U + q A \cdot U$

Note, however, that some other definitions do *not* work with $L_0'$. The 4-TotalMomentum relation no longer works.

4-Velocity is ProperTime

Derivative of 4-Position

$U = \frac{d}{dt} R \quad [\text{m/s}] = [1/\text{s}] [\text{m}]

Relativistic Euler-Lagrange Eqn

$\dot{x}_0 = \frac{d}{dz} \dot{u}_0 \quad [1/\text{m}] = [1/\text{s}] [\text{m}]

\frac{d}{d\tau} \dot{R} = \frac{d}{dz} \dot{u} \quad \frac{d}{d\tau} \dot{L} = \frac{d}{dz} \dot{u} 

Classical limit, spatial component

$\dot{u}_0 = \frac{d}{dz} \dot{u}_0 \quad \frac{d}{d\tau} \dot{x}_0 = \frac{d}{dz} \dot{u}

F_{\text{EM}} = \gamma q \{ (u \times e)/c, (c + (u \times b)) \}

e = (-\nabla \phi - \dot{a})$ and $b = [\nabla \times a]$

If $a = 0$, then $f = -q \nabla \phi = -\nabla U$, the force is neg grad of a potential

4-TotalMomentum

$P_T = (E/c, p_T) = (H/c, p_T)$

$= P + q A

4-VelocityGradient

$\dot{u}_0 = \frac{\dot{u}_0}{c, -\nabla U}

4-VelocityGradient part

$\frac{d}{d\tau} \dot{u}_0 [L_0 '] = \frac{d}{d\tau} \dot{u}_0 [L_0]$

$d/d\tau [L_0] = \frac{d}{dt} \dot{x}_0 = \dot{r} + \frac{\dot{\gamma}}{\gamma}[\gamma^2 R - 2 \gamma R \dot{\gamma} + \dot{\gamma}^2]$

Relativistic Rest Lagrangian Euler-Lagrange Equations of Motion

$(d/d\tau) \dot{u}_0 [L_0 '] = \dot{\gamma} [L_0 ']

4-(EM)VectorPotential

$A = A^\mu = (\phi/c, a)$

Relativistic 4-Vector

$V = (c, \vec{v})$

SR 4-Vector

$T^{\mu\nu} = T_{\mu\nu}$

EM Faraday

$F^{\mu\nu} = \partial_\mu \partial_\alpha \phi - \partial_\alpha \partial_\mu \phi$

Trace $[T^{\mu\nu}] = \eta_{\mu\nu} T^{\mu\nu} = T^{\mu\nu} = T$

$V \cdot V = V^2 = V^\mu V^\nu = (V^\mu)^2 = (V^\nu)^2$

Lorentz Scalar Invariant

4-PositionGradient part

$\delta R [L_0 '] = \delta R [L_0]$

Assumes:

$\delta [U] = \delta [U] = 0^\mu$

$\delta [P] = \delta [P] = 0^\mu

4-PositionGradient

$\dot{R} = \frac{d}{d\tau} \dot{R} [L_0 ']

\dot{R} [L_0 '] = \dot{R} [L_0]$

$= 0, q \delta R [A \cdot U]$

$= q R \{ U_0 \partial^\mu [A^\mu]$

$= q R \{ U_0 \partial^\mu [A^\mu]$

$(d/d\tau) [L_0 '] = \dot{\gamma} [L_0 ']

4-Vector SRQM Interpretation of QM

SciRealm.org

John B. Wilson

http://scirealm.org/SRQM.pdf

A Tensor Study of Physical 4-Vectors
SRQM Diagram:
Relativistic Hamilton’s Equations of Motion (EoM) for EM particle

\[ (\partial/d\tau)[X] = (\partial/\partial P_T)[H_0] \]
\[ (\partial/d\tau)[P_T] = (\partial/\partial X)[H_0] \]

**4-Position**
\[ X = (ct, x) \]

**4-Velocity**
\[ U = γ(c, u) \]

**Relativistic Rest Hamilton’s Equations of Motion**

\[ \mathbf{F}^a = qU_\beta \beta^a[A^\beta] = q^2A^0U_\beta \]
\[ \mathbf{F}^0 = qA^\alpha U_\alpha - qU_\beta \beta^a[A^\beta] \]
\[ \mathbf{F}^0 = qU_\beta (\partial^a[A^\beta] - \partial^a[A^\beta]) \]
\[ \mathbf{F}^0 = qU_\beta (\partial^a[A^\beta] - \partial^a[A^\beta]) \]

**Lorentz Force Equation**

\[ \mathbf{F}^0 = qU_\beta (\partial^a[A^\beta] - \partial^a[A^\beta]) \]

**Trace**
\[ \text{Trace}[T^\alpha_\beta] = \eta_\alpha_\beta T^\alpha_\beta = T^{\alpha}_\alpha = T \]
\[ \mathbf{V} \cdot \mathbf{V} = \mathbf{V}^\alpha \eta_\alpha_\beta \mathbf{V}^\beta \]
\[ = (\mathbf{V}^\alpha \eta_\alpha_\beta \mathbf{V}^\beta) \]

**Lorentz Scalar Invariant**
SRQM Diagram: Relativistic Hamilton’s Equations of Motion (EoM) for EM particle

SRQM Diagram:

Relativistic Hamilton’s Equations of Motion using Rest Hamiltonian

\[ (d/d\tau)[X^a] = (\partial_{P^a})[H_0] = (\partial/P_{\tau a})[H_0] \]

\[ (d/d\tau)[P_{\tau a}] = (d/d\tau)[\partial_{\tau a}X^\beta] = \partial_{\tau a}[X^\beta] \]

\[ = [F^a + q(U\cdot\partial)c^a - q\partial^a[U^\beta] - q\partial^a[U^\beta]U_\beta] \]

Lorentz Force Equation

\[ \text{EM Faraday} \]

\[ A = (\phi/c, \mathbf{a}) \]

4-(EM) Vector Potential

\[ 4-\text{Vector Potential} \]

\[ \text{SR 4-Tensor} \]

\[ (2,0)\text{-Tensor} T^{\mu\nu} \]

\[ \text{SR 4-Vector}(1,0)\text{-Tensor} V = (v^0, \mathbf{v}) \]

\[ \text{SR 4-Vector}(0,1)\text{-Tensor} V = (v_0, \mathbf{v}) \]

\[ \text{EM Faraday} \]

\[ F^\alpha{}_{\beta} = \partial_\alpha A^\beta - \partial_\beta A^\alpha \]

\[ = \begin{bmatrix} 0 & -e/c & \epsilon_\alpha b & \epsilon_\beta b \\ e/c & 0 & -\epsilon_\alpha a & \epsilon_\beta a \\ -\epsilon_\alpha b & \epsilon_\beta b & 0 & -e/c \\ -\epsilon_\alpha a & \epsilon_\beta a & e/c & 0 \end{bmatrix} \]

\[ \text{SR 4-Scalar} \]

\[ (0,0)\text{-Tensor} S_\text{SR} \]

\[ \text{SR 4-Scalar} \]

\[ S_\text{SR} \]

\[ \text{Trace}[T^{\mu\nu}] = \eta_{\mu\nu}T^{\mu\nu} = T^{\mu\nu} = T \]

\[ V\cdot V = V^0 \cdot V^0 = (v^0)^2 = (-v^0)^2 = (v_0)^2 \]

\[ \text{Lorentz Scalar Invariant} \]
Lorentz EM Force Equation:
\[ dP^\beta/d\tau = F^\alpha = q(F^{\alpha\beta})U_\beta \]
\[ dP^\alpha/d\tau = F^\alpha = q(\partial^\alpha A^\beta - \partial^\beta A^\alpha)U_\beta \]

Examine just the spatial components \( F^i \) of 4-Force \( F = F^\alpha \):
\[ F^i = q(\partial^\alpha A^\beta - \partial^\beta A^\alpha)U_\beta \]
\[ F^i = q(\partial^\alpha A^0 - \partial^0 A^\alpha)U_0 + q(\partial^\alpha A^i - \partial^i A^\alpha)U_i \]
\[ \gamma f = q(-\nabla[\phi/c] - (\partial^\alpha/c)\alpha(c) + q(-\nabla[a\cdot u] - u\cdot\nabla[a])\gamma \]
\[ f = q(-\nabla[\phi/c] - (\partial^\alpha/c)\alpha(c) + q(u\cdot\nabla[a]-\nabla[a\cdot u]) \]
\[ f = q(-\nabla[\phi/c] - \partial^\alpha + u\cdot u) \]
\[ f = q(e + u\cdot u) \]

The Classical Force = -Grad[Potential]
\[ \text{Correct} \quad U = kx^2/2, \quad f = -\nabla[kx^2/2] = -kx \]
Hooke's Law

SR 4-Tensor
(2,0)-Tensor \( T_{\alpha\beta} \)
(1,0)-Tensor \( V^\alpha = V = (\sqrt{\gamma}, \gamma) \)
SR 4-CoVector:OneForm
(0,1)-Tensor \( V_\alpha = (\gamma, -\sqrt{\gamma}) \)
SR 4-Scalar
(0,0)-Tensor \( S \) or \( S_0 \)
Lorentz Scalar

Maxwell EM Wave Eqn
\[ \partial \times A = \mu_0 J \]
Conservation of EM Field
\[ \partial A = 0 \]
Lorentz Gauge
\[ \text{Trace}[T^{\alpha\beta}] = \eta^{\alpha\beta} T^{\alpha\beta} = T^{\alpha\beta} = T \]
\[ V\cdot V = V_\alpha V^\alpha = (|V|^2) - V_\alpha V^\alpha = (|V|^2) \]
Lorentz Scalar Invariant

SRQM Study:
EM Lorentz Force Eqn
\[ \text{Classical Force} = - \nabla [U] \]
SRQM Diagram:

Relativistic Hamilton’s Equations
Equation of Motion (EoM) for Harmonic Oscillator

\[ A \cdot U = \phi_o : \text{RestScalar-Potential} \]
\[ qA \cdot U = q\phi_o = V_o : \text{RestVoltage} = \text{Electrical PotentialEnergy} \]

Let \( \{ qA \cdot U = V_o = -kX \cdot X/2 \} \) then \( A \cdot U = \phi_o = -(k/q)X \cdot X/2 \)

RestHamiltonian \( H_o = (P\cdot U) = P \cdot U + qA \cdot U = P \cdot U - kX \cdot X/2 \)

\[ \partial(A \cdot U) = \partial(A) \cdot U + A \cdot \partial(U) \]
\[ \partial^2(A^\mu U_\mu) = \partial^\mu[A^\mu][U_\mu] + A^\mu \partial[U_\mu] \]
\[ \partial^2(A^\mu U_\mu) = \partial^\mu[A^\mu]U_\mu + 0^\mu : \text{assuming conservative field} \partial^\mu[U_\mu] = 0^\mu \]
\[ \partial[-(k/q)X \cdot X/2] = -(k/q)X \]

\[ F^\mu = qU_\mu \partial^\mu[A^\mu] - \partial[A^\mu]U_\mu = -(k/q)X^\mu \]

Spring Potential (\( U = kx^2/2 \)), then \( \{ f = -\nabla[kx^2/2] = -kx \} \) Hooke’s Law

RestLagrangian
\[ L_o = -(P\cdot U) = -P \cdot U + qA \cdot U = -P \cdot U + kX \cdot X/2 \]
\[ (d/dt)[L_o] = \delta_{x}[\text{Classical}] \]

RestLagrangian
\[ L_o = -(P\cdot U) = -P \cdot U + qA \cdot U = -P \cdot U + kX \cdot X/2 \]
\[ (d/dt)[L_o] = \delta_{x}[\text{Classical}] \]

Classical limit:
\[ \gamma \rightarrow 1 : |v| \ll c \]
\[ d/dt[a] \rightarrow 0 : \text{3-vector-potential changes very slowly} \]

Spring Potential (\( U = kx^2/2 \)), then \( \{ f = -\nabla[kx^2/2] = -kx \} \) Hooke’s Law

RestLagrangian
\[ L_o = -(P\cdot U) = -P \cdot U + qA \cdot U = -P \cdot U + kX \cdot X/2 \]
\[ (d/dt)[L_o] = \delta_{x}[\text{Classical}] \]

RestLagrangian
\[ L_o = -(P\cdot U) = -P \cdot U + qA \cdot U = -P \cdot U + kX \cdot X/2 \]
\[ (d/dt)[L_o] = \delta_{x}[\text{Classical}] \]

RestLagrangian
\[ L_o = -(P\cdot U) = -P \cdot U + qA \cdot U = -P \cdot U + kX \cdot X/2 \]
\[ (d/dt)[L_o] = \delta_{x}[\text{Classical}] \]

RestLagrangian
\[ L_o = -(P\cdot U) = -P \cdot U + qA \cdot U = -P \cdot U + kX \cdot X/2 \]
\[ (d/dt)[L_o] = \delta_{x}[\text{Classical}] \]
SRQM: The Speed-of-Light (c) c² Invariant Relations (part 1)

The Speed-of-Light (c) is THE connection between Time and Space: dR = (cdt, dR)

This physical constant appears in several seemingly unrelated equations. Don’t notice these cool relations when you set c→1. Also notice that the set of all these relations definitely rules out a variable speed-of-light. (c) is an Invariant Lorentz Scalar constant.

\[ \frac{\partial \cdot \mathbf{R}}{\partial t} = \frac{\partial}{\partial c} \cdot \nabla \]

Invariant 4-Gradient Magnitude
\[ \partial \cdot \mathbf{R} = 4 \] SpaceTime Dimension

\[ \partial \mathbf{R} = 4 \] Minkowski Metric

\[ \partial \mathbf{R} = 4 \]

4-Position \( \mathbf{R} = (ct, \mathbf{r}) \)

4-Velocity \( \mathbf{U} = \gamma(c, \mathbf{u}) \)

EM Faraday 4-Tensor

4-Vector SRQM Interpretation of QM

http://scirealm.org/SRQM.pdf

SciRealm.org
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4-Gradient SR 4-Vector Analysis

\[ \mathbf{T}^{uv} = \frac{\partial^2}{\partial u^2} \mathbf{V} = \gamma \mathbf{V} = \gamma \mathbf{V} = \gamma \mathbf{V} \]

SR 4-Vector

(1,0)-Tensor \( V^{(1,0)} \) or \( T^{(1,0)} \)

(0,1)-Tensor \( V_{(0,1)} \)

Lorentz Scalar

SR 4-Scalar

(0,0)-Tensor \( S \) or \( \Theta \)

Lorentz Scalar

\[ \text{Lorentz Gauge} \quad \mathbf{a} = c^2 \]

̂complex 4-WaveVector

\[ \mathbf{K} = (\omega/c, \mathbf{k}) = (\omega/c, \omega/t) \]

Invarient 4-Wavevector Magnitude

\[ (c^2 \omega/c)^2 \]

Electric: Magnetic

\[ \varepsilon (\varepsilon_0 c^2) \]

Every Physical 4-Vector has a (c) factor to maintain equivalent dimensional units across the whole 4-Vector

\[ T^{\mu\nu} = \eta^{\mu\nu} T^{\mu\nu} = \mathbf{T}^{\mu\nu} = \mathbf{T}^{\mu\nu} \]

Trace \( T^{\mu\nu} = 0 \)

\[ \mathbf{V} \cdot \mathbf{V} = \mathbf{V} \cdot \mathbf{V} = \mathbf{V} \cdot \mathbf{V} = \mathbf{V} \cdot \mathbf{V} \]

Lorentz Scalar Invariant

\[ \mathbf{V} \cdot \mathbf{V} = \gamma \mathbf{V} \cdot \mathbf{V} = \gamma \mathbf{V} \cdot \mathbf{V} = \gamma \mathbf{V} \cdot \mathbf{V} \]
The Speed-of-Light \( c \) is THE connection between Time and Space: \( \Delta t = \frac{\Delta x}{c} \).

This physical constant appears in several seemingly unrelated places. You don’t notice these cool relations when you set \( c \rightarrow 1 \).

Also notice that the set of all these relations definitely rules out a variable speed-of-light. \( c \) is an Invariant Lorentz Scalar constant.

\( U \cdot U = (c^2 u \cdot u) = c^2 \) Speed of all things into the Future

\( (E/m_0) = (\gamma E/m) = c^2 \) Mass is concentrated Energy, \( E = mc^2 \)

\( |u \times v_{\text{phase}}| = |v_{\text{group}} \times v_{\text{phase}}| = c^2 \) Particle-Wave “Duality” Correlation

\( \lambda (\omega^2 - \omega_0^2) = \lambda^2 (\gamma^2 - 1) = c^2 \) Wavelength-Frequency Relation: \( \lambda f = c \) for photons

\( \frac{1}{\epsilon_\mu} \mu_\nu = c^2 \) Electric (\( \epsilon_\mu \)) and Magnetic (\( \mu_\nu \)) EM Field Constants

\( -\hbar (\frac{m_0}{c^2}) (\partial - \partial) = c^2 \) Relativistic Quantum Wave Equation

\( \hbar \lambda (m_0/c^2) = c^2 \) Reduced Compton Wavelength: \( \lambda_c = (\hbar/m_c) \)

\( 2GM/R_s = c^2 \) GR Black Hole Equation

\( R_s = \text{Schwarzschild Radius} \)

\( G = \text{GR GravitationalConst} \), \( M = \text{BH Mass} \)

\( 8\pi G/\kappa = c^2 \) GR Einstein Curvature Constant\([\text{mass density form}]\): \( \kappa = 8\pi G/c^2 \)

\( c^{ct^2} \) Every Physical 4-Vector has a (\( c \)) factor to maintain equivalent dimensional units across the whole 4-Vector

**SRQM: The Speed-of-Light (c)**

**c^2 Invariant Relations (part 2)**

\((c^2-u\cdot u) = c^2\)

\(E/m_0 = (E/m) = c^2\)

\(|u \times v_{\text{phase}}| = |v_{\text{group}} \times v_{\text{phase}}| = c^2\)

\(\lambda (\omega^2 - \omega_0^2) = \lambda^2 (\gamma^2 - 1) = c^2\)

\(\frac{1}{\epsilon_\mu} \mu_\nu = c^2\)

\( -\hbar (\frac{m_0}{c^2}) (\partial - \partial) = c^2\)

\(\hbar \lambda (m_0/c^2) = c^2\)

\(2GM/R_s = c^2\)

\(8\pi G/\kappa = c^2\)

\(c^{ct^2}\)

\(\frac{1}{\epsilon_\mu} \mu_\nu = c^2\)

\(\frac{E_0}{m_0} = (\hbar/\lambda c m_0)^2\)

\(-\partial \phi/\nabla \cdot a\in\text{Lorenz Gauge}\)

\(\frac{E_0}{m_0} = (\hbar/\lambda c m_0)^2\)

\(\frac{E_0}{m_0} = (\hbar/\lambda c m_0)^2\)

\(\frac{E_0}{m_0} = (\hbar/\lambda c m_0)^2\)

\(\omega_0^2/\mathbf{K} \cdot \mathbf{K}\)

\((-\hbar m_0^2)^2 \partial - \partial\)

**SRQM**

\(-S_{\text{action,free}}/(m_0 c^2)\)

\(8\pi G/\kappa\)

\(2GM/R_s\)

\(\mathbf{U} \cdot \mathbf{U}\)

\(\mathbf{P} \cdot \mathbf{P}/m_0^2\)

\(\mathbf{E}_0^2/\mathbf{P} \cdot \mathbf{P}\)

\(\omega_0^2/\mathbf{K} \cdot \mathbf{K}\)

\((\hbar/m_0)^2 \mathbf{K} \cdot \mathbf{K}\)

\((-\hbar m_0^2)^2 \partial - \partial\)

**SRQM**
4-Vector SRQM Interpretation of QM

SRQM 4-Vector Study:

4-ThermalVector
Relativistic Thermodynamics

The 4-ThermalVector is used in Relativistic Thermodynamics.

My prime motivation for the form of this 4-Vector is that the probability distributions calculated by statistical mechanics ought to be covariant functions since they are based on counting arguments.

F(\text{state}) \sim e^{\lambda^\text{state}/k_\text{B} T} = e^{\lambda^\text{state}/\beta E}, with this \beta = 1/k_\text{B} T, (not v/c)

A covariant way to get this is the Lorentz Scalar Product of the 4-Momentum P with the 4-ThermalVector \Theta.

F(\text{state}) \sim e^{\lambda^\text{state}/(P \Theta)} = e^{\lambda^\text{state}/(E_o/k_\text{B} T_o)}

This also gets Boltzmann's constant (k_o) out there with the other Lorentz Scalars like (c) and (h)

see (Relativistic) Maxwell-Jüttner distribution

\[ f(P) = N_o/(2c(m_o c) K_{(\theta^o/2\pi)})(m_o c \omega_o/2\pi)^{(d-1)/2} * e^{-(P \Theta)} \]

\[ f(P) = N_o/(2c(m_o c) K_{\theta^o/2\pi})(m_o c \omega_o/2\pi)^{(d-1)/2} * e^{-(P \Theta)} \]

\[ f(P) = N_o/(4\pi\omega_o K_{\theta^o/2\pi})(m_o c \omega_o/2\pi)^{(d-1)/2} * e^{-(P \Theta)} \]

\[ f(P) = N_o/(4\pi\omega_o K_{\theta^o/2\pi})(m_o c \omega_o/2\pi)^{(d-1)/2} * e^{-(P \Theta)} \]

It is possible to find this distribution written in multiple ways because many authors don’t show constants, which is quite annoying. Show the damn constants people! (k_o), (c), (h) deserve at least that much respect.

SR 4-Tensor

\[(2,0)\text{-Tensor} \quad T^{\mu \nu} = \varepsilon^{\mu \nu} \varepsilon_{\lambda \sigma} \lambda \sigma \]

SR 4-Vector

\[(1,1)\text{-Tensor} \quad V^\mu = V^\mu = (\nu^\mu , \nu^\mu) \]

SR 4-CoVector: OneForm

\[(0,1)\text{-Tensor} \quad V_\mu = (\nu, \nu) \]

SR 4-Scalar

\[(0,0)\text{-Tensor} \quad S_o \quad \text{Lorentz Scalar} \]

Be careful not to confuse (unfortunate symbol clash):

Thermal \beta = 1/k_\text{B} T

Relativistic \beta = v/c

These are totally separate uses of (\beta)
The 4-ThermalVector is used in Relativistic Thermodynamics. It can be used in a partial derivation of Unruh-Hawking Radiation (up to a numerical constant).

Let a “Unruh-DeWitt thermal detector” be in the Momentarily-Comoving-Rest-Frame (MCRF) of a constant spatial acceleration (a), in which |u|→0, γ→1, γ′→0.

4-AccelerationMCRF = A_MCRF = A_MCRF^μ = (0, a)_MCRF

Take the Lorentz Scalar Product with the 4-ThermalVector
A_MCRF^μ Θ = (0, a)_MCRF (c/k_BT, u/k_BT) = (-a·u/k_BT) = Lorentz Scalar Invariant

A_MCRF^μ Θ = (ac/k_BT) = Invariant Lorentz Scalar

Use Dimensional Analysis to find appropriate Lorentz Scalar Invariant with same units: Invariant Units = [m/s] / [kg·m²/s] = [1/kg·s] ~ c²/ℏ = [m²] / [kg·m²/s]

Temperature T ~ a/k_BT, (from EM radiation, only from the dir. of acceleration)

Further methods give the constant of proportionality (1/2ℏ):

See (Imaginary Time, Euclideanization, Wick Rotation, Matsubara Frequency)

See (Thermal QFT, Bogoliubov transformation)

Trace[T^μν] = η_μν T^μν = T_μν = T

Note that the temperature here is relativistically direction-specific, unlike in the classical use of temperature.
SRQM 4-Vector Study: 4-ThermalVector
Unruh-Hawking Radiation

Temperature $T \sim \hbar k_B$, (from EM radiation, only from the dir. of acceleration)

Further methods give the constant of proportionality $(1/2\pi)$:
See (Imaginary Time, Euclideanization, Wick Rotation, Matsubara Frequency)
See (Thermal QFT, Bogoliubov transformation)

$T_{\text{Unruh}} = \hbar g/2\pi k_B c$ (due to constant Minkowski-hyperbolic acceleration)
$T_{\text{Hawking}} = g/2\pi k_B c$ (due to gravitational acceleration $a=g$)
$T_{\text{Schwarzchild BH}} = \hbar c^2/8\pi G M k_B$ (Temp at BH Event Horizon, $g=GM/R^2$, $R_s=2GM/c^2$)
$T_{\text{SR}} = \hbar (a\cdot u)/2\pi k_B c^2$ (correct version from 4-Vector derivation $A_{\text{MCRF}}\Theta_{\text{Radiation}} = 2\pi c^3/\hbar$)

Alternate forms:
$A_{\text{MCRF}}\Theta_{\text{Radiation}} = 2\pi c^3/\hbar$
$(1/k_B)A_{\text{MCRF}} U = 2\pi c^3/\hbar$
$(1/k_B)A_{\text{MCRF}} U = 2\pi \omega_c c^3/\hbar$
$A_{\text{MCRF}} U = 2\pi \omega_c c^2$
$A_{\text{MCRF}} U = 2\pi (K-U)c^2$
$A_{\text{MCRF}} = 2\pi (K)c^2$
$A_{\text{MCRF}} = (2\pi c^3)K = (2\pi c^3/\hbar)P$
$(dP/d\tau)_{\text{MCRF}}\Theta_{\text{Radiation}} = 2\pi \omega_c$
$F_{\text{MCRF}}\Theta_{\text{Radiation}} = 2\pi \omega_c : \{ \text{for } m_c = \text{constant} \}$

The $2\pi$ factor is interesting:
There are cases when the dimensional units must match. see 4-Momentum related to 4-WaveVector:
$P = \hbar K \rightarrow [J/\text{s/m}] = [J/\text{rad}] [\text{rad/m}]$
$h = \hbar/2\pi \rightarrow [J/\text{s/\pi rad}] = [J/\text{s}][2\pi \text{ rad}]$

And other where the $2\pi$ factor doesn’t seem to use [rad] units.
see Circles & Spheres:
$C = 2\pi r \rightarrow [\text{m}] = [2\pi \text{m}]
 A = \pi r^2 \rightarrow [\text{m}^2] = [\pi \text{m}^2]
 V = (4/3)\pi r^3 \rightarrow [\text{m}^3] = [(4/3)\pi \text{m}^3]$

The 4-Vector SRQM Study:
4-Vector SRQM Interpretation of QM

4-Velocity
$U = \gamma(c,u)$
$U \cdot \theta = (c)^2$
$\gamma_d/dt [\ldots]$
$\theta/c$
$1/k_B T_0$

4-InverseTempMomentum
$\Theta = (\Theta/\Theta)(c/k_B T, u/k_B T) = (\Theta/c) U = (1/k_B T_0) U$

P-Theta
$= (E/c, p) \cdot (c/k_B T, \Theta) = (E/k_B T - p \cdot \Theta) = (E/k_B T_0) = (\text{Invaraint}_{\text{dimensionless}})$
Just a number

4-Acceleration
$A = A_{\mu} = \gamma(c', \gamma' u + \gamma a)$
$= dU/d\tau - d^2R/d\tau^2$

4-Acceleration
$A_{\text{MCRF}} = A_{\text{MCRF}}_{\mu} = (0, a)_{\text{MCRF}}$

4-Momentum
$P = (mc, p) = (E/c, p) = m_c U$
$P-P = (mc, c) = (E/c, c) = (E/c, c)$

Invariant Distribution Function
$N_i = 1/[e^{\Theta(E/k_B T) \pm 1}]$
$= 1/[e^{\Theta(P, \Theta) \pm 1}]$
$\rightarrow \text{Bose-Einstein}$
$\rightarrow \text{Fermi-Dirac}$

Note that the temperature here is relativistically direction-specific, unlike in the classical use of temperature.

SR 4-Tensor
$\text{(2,0)-Tensor} T^{iv}$
$\text{(1,1)-Tensor} T^{iv}, \text{ or } T^{v}$
$\text{(0,2)-Tensor} T_{iiv}$

SR 4-Vector
$\text{SR 4-Vector} V = V = (\nu, \nu^*)$
$\text{SR 4-CoVector: OneForm}$
$(0,1)-\text{Tensor} V_i = (\nu_i, -\nu^*)$

SR 4-Scalar
$\text{(0,0)-Tensor} S_0, \text{ or } S_0$
Lorentz Scalar

\[ \theta U = (c)^2 \]
\[ \gamma U = (c)^2 \]
\[ \theta/c \]
\[ 1/k_B T_0 \]
\[ (0, a)_{\text{MCRF}} (c/k_B T, u/k_B T) \]
\[ (0, a)_{\text{MCRF}} (c/k_B T, u/k_B T) \]
\[ (-a \cdot u/k_B T) \]
\[ \Theta = \Theta(c/k_B T, u/k_B T) \]
\[ \Theta = \Theta(c/k_B T, u/k_B T) \]
\[ \Theta = \Theta(c/k_B T, u/k_B T) \]
\[ \Theta = \Theta(c/k_B T, u/k_B T) \]
\[ \Theta = \Theta(c/k_B T, u/k_B T) \]
\[ \Theta = \Theta(c/k_B T, u/k_B T) \]
SRQM 4-Vector Study: 4-Thermal Vector

Wick Rotations, Matsubara Freqs

The operator which governs how a quantum system evolves in time, the time evolution operator, and the density operator, a time-independent object which describes the statistical state of a many-particle system in an equilibrium state (with temperature $T$) can be related via arithmetic substitutions:

In the Matsubara Formalism, the basic idea (due to Felix Bloch) is that the expectation values of operators in a canonical ensemble:

$$\langle A \rangle = \frac{\text{Tr} \left[ \exp (-\beta H) A \right]}{\text{Tr} \left[ \exp (-\beta H) \right]}$$

may be written as expectation values in ordinary quantum field theory (QFT), where the configuration is evolved by an imaginary time $\tau = -i t \ (0 \leq \tau \leq \beta)$. One can therefore switch to a spacetime with Euclidean signature, where the above trace (Tr) leads to the requirement that all bosonic and fermionic fields be periodic and antiperiodic, respectively, with respect to the Euclidean time direction with periodicity $\beta = \hbar / (k_B T)$.

This allows one to perform calculations with the same tools as in ordinary quantum field theory, such as functional integrals and Feynman diagrams, but with compact Euclidean time.

Note that the definition of normal ordering has to be altered. In momentum space, this leads to the replacement of continuous frequencies by discrete imaginary (Matsubara) frequencies:

Bosonic $\omega_n = (n) (2\pi/\beta)$

Fermionic $\omega_n = (n+1/2) (2\pi/\beta)$

and, through the de Broglie relation $E = \hbar \omega$, to a discretized EM thermal energy spectrum $E_n = \hbar \omega_n = n(2\pi k_B T)$.

In SRQM Physics, the operator that governs the motion of a quantum system evolves in time, the time evolution operator, and the density operator, a time-independent object which describes the statistical state of the many-particle system in an equilibrium state (with temperature $T$) can be related via arithmetic substitutions:

$$e^{-(i H t / \hbar)} \rightarrow e^{i(\mathbf{P}_T \cdot \mathbf{X})/\hbar}$$

where $\beta$, called Euclidean Time (Imaginary Time) is cyclic with period $\beta$, $0 \leq \tau \leq +\beta$.

In Quantum Mechanics (or Quantum Field Theory), the Hamiltonian $H$ acts as the generator of the Lie group of time translations while in Statistical Mechanics the role of the same Hamiltonian $H$ is as the Boltzmann weight in an ensemble.

**Time Evolution Operator**

$$U(t) = \sum_{n=0,=} \left[ e^{-(i E_n t / \hbar)} \right] | n \rangle \langle n | = e^{-(i H t / \hbar)}$$

**Partition Function** (time-independent function of state)

$$Z = \sum_{n=0,=} \left[ e^{-(i E_n / k_B T)} \right] = \text{Trace}[ e^{-(i H t / \hbar)} ]$$

Note that the temperature here is relativistically direction-specific, unlike in the classical use of temperature.

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$$e^{\frac{-i(P \cdot R)}{\hbar}} = e^{\frac{-i[H_0 t]}{\hbar}}$$

where $\tau$, called Euclidean Time (Imaginary Time) is cyclic with period $\beta$, $(0 \leq \tau \leq +\beta)$.

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The QM/QFT $\leftrightarrow$ SM Correspondence

Quantum Mechanics (QM)

Wick Rotation $t \rightarrow i \tau$

$e^{\frac{-i(P \cdot R)}{\hbar}} = e^{\frac{-i[H_0 t]}{\hbar}}$

Imaginary Time $\leftrightarrow$ Inv Temp

$e^{\frac{-i[H_0 t]}{\hbar}} = e^{\frac{+i[H_0 t]}{\hbar}}$

Euclidean Time $\sim$ Inv Temp

$\frac{t}{\hbar} \rightarrow \frac{\beta}{1}$

Statistical Mechanics (SM)

$e^{\frac{-i[H_0 t]}{\hbar}} = e^{\frac{+i[H_0 t]}{\hbar}}$

S$_{\text{action}} = -(P \cdot R)$

$-\frac{1}{\hbar}[P \cdot dR]$

$-\frac{1}{\hbar}[P \cdot U]d\tau = [Ldt$

$-\frac{1}{\hbar}[(H/c, p \cdot r) \cdot \gamma(c, u)]d\tau$

$e^{\frac{-i[H_0 t]}{\hbar}}$

where $\tau$, called Euclidean Time (Imaginary Time) is cyclic with period $\beta$, $(0 \leq \tau \leq +\beta)$.

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In Quantum Mechanics (or Quantum Field Theory), the Hamiltonian $H$ acts as the generator of the Lie group of time translations while in Statistical Mechanics the role of the same Hamiltonian $H$ is as the Boltzmann weight in an ensemble.
SRQM 4-Vector Study: Deep Symmetries: Schrödinger Relations & Cyclic Imaginary Time \leftrightarrow \text{Inv Temp}

**SRQM 4-Vector Study:**

1. **4-Momentum**
   \[
   P = P^\mu = (mc, p) = (mc, mu) = mU = (E/c, p) = (E/c^2)U
   \]

2. **Einstein de Broglie**
   \[
   P = hK
   \]
   \[
   [kgs/m/s] = [N \cdot s]
   \]

3. **4-Position**
   \[
   \mathbf{R} = \mathbf{r} = (ct, \mathbf{r}) \in \text{Event}
   \]
   \[
   \text{alt. notation} \quad \mathbf{X} = X^\mu
   \]
   \[
   -i = 1/i
   \]

4. **4-WaveVector**
   \[
   \mathbf{K} = K^\mu = (\omega/c, \mathbf{k}) = (\omega/c^2)U
   \]
   \[
   = (\omega/c, \omega/\nu_{\text{phase}}) = (1/c\mathbf{T}, \mathbf{n}/\Lambda)
   \]
   \[
   \text{Complex Plane-Waves} \quad K = i\mathbf{\Theta}
   \]

5. **4-Gradient**
   \[
   \partial = \partial_{\mathbf{R}} = \partial_{\mathbf{r}} \mathbf{R}^\mu = \partial^\mu = (\partial/c, -\mathbf{V})
   \]
   \[
   \rightarrow (\partial/c, -\partial_x, -\partial_y, -\partial_z)
   \]
   \[
   = (\partial/c\partial_t, \partial/\partial\mathbf{x}, \partial/\partial\mathbf{y}, \partial/\partial\mathbf{z})
   \]

6. **4-Position Temperature**
   \[
   \mathbf{O} = \mathbf{\Theta} = (\Theta_0, \Theta) = (ct, \mathbf{r}) = (\Theta/c)U
   \]
   \[
   = (1/c\mathbf{T})(c, \mathbf{u}) = (1/c\mathbf{T} \mathbf{u})(1/k\mathbf{B} \mathbf{T})U
   \]
   \[
   \text{SR 4-Vector\text{\,}\,} (0,1)-\text{Tensor S or S}_0 \text{\,}\, \text{Lorentz Scalar}
   \]

7. **4-Velocity**
   \[
   \mathbf{V} = \mathbf{v} = (v^0, \mathbf{v}) = \frac{(v^0, \mathbf{v})}{c} \cdot \mathbf{v}
   \]
   \[
   \text{Lorentz Scalar Invariant}
   \]

8. **SpaceTime Dimension**
   \[
   \partial \mathbf{R} = \partial_{\mathbf{r}} \mathbf{R}^\mu = 4
   \]

9. **Boltzmann Distribution**
   \[
   P = \mathbf{\Theta} = (E/c, \mathbf{p}) = (c/k\mathbf{T}, \mathbf{u}) = (E/c\mathbf{T} \mathbf{u})(1/k\mathbf{B} \mathbf{T})U
   \]
   \[
   \text{Note that the temperature here is relativistically direction-specific, unlike in the classical use of temperature.}
   \]
The 4-EntropyVector is used in Relativistic Thermodynamics.

Pure Entropy is a Lorentz Scalar in all frames.
Up to this point, we have mostly been exploring the SR aspects of 4-Vectors.

It is now time to show how RQM and QM fit into the works...

This is SRQM, [ SR → QM ]

RQM & QM are derivable from principles of SR

Let that sink in...

Quantum Mechanics is derivable from Special Relativity

GR → SR → RQM → QM → {CM & EM}
SRQM Diagram:

Special Relativity → Quantum Mechanics

RoadMap of SR → QM (w/ EM Potential)

4-Gradient=Alteration of SR <Events>
SR SpaceTime Dimension=4
SR SpaceTime "Flat" 4D Metric
SR Lorentz Transforms
SR Action → 4-Momentum
SR Phase → 4-WaveVector
SR Proper Time Derivative
SR & QM Invariant Waves

SR → QM Klein-Gordon Relativistic Quantum Particle in EM Potential
d'Alembertian Wave Equation

Limit: { v << c }

d[\eta_{\mu\nu}]=[\eta_{\mu\nu}] = -(\omega/c)^2 = -(m/c)^2 = (\partial/c)^2

4-Gradient \( \partial \mu = (partial)/(partial \epsilon) - \mathbf{V} \) = -iK

 ProperTime \( U \cdot \partial \frac{d}{dt} = \gamma \frac{d}{dt} \) Derivative

-4-WaveVector \( K^\mu = (\omega/c)^2 \mathbf{v} \)

-4-Momentum \( P^\mu = [(E/c)/p]^0 = m_0 \mathbf{u} \)

4-Vector SRQM Interpretation of QM

SciRealms.org
John B. Wilson
SciRealms@ao.com
http://scirealm.org/SRQM.pdf

SR 4-Tensor
(2,0)-Tensor \( T^{\mu}_{\nu} \)
(1,1)-Tensor \( T^{\mu}_{\nu} \), or \( T^{\nu}_{\mu} \)
(0,2)-Tensor \( T^{\mu}_{\nu} \)

SR 4-Vector
(1,0)-Tensor \( V^\nu = (v^\nu, v^0) \)
SR 4-CoVector:OneForm
(0,1)-Tensor \( V_\nu = (v_\nu, v^0) \)

SR 4-Scalar
(0,0)-Tensor \( S \) or \( S \)

Existing SR Rules
Quantum Principles

4-Vector SRQM Interpretation

John B. Wilson
SciRealms@ao.com
http://scirealm.org/SRQM.pdf

EM Faraday
\[ \partial^\mu A^\mu = \frac{F^\mu}{\epsilon_0 c^2} \]

4-Tensor

4-WaveVector Substitution of SR Wave <Events>
oscillations proportional to mass:energy & 3-momentum

4-Position Substitution of SR Particle <Events>
mass:energy & 3-momentum

SR Phase
\( \Phi_{\text{phase}} \)

SR Action
\( \Phi_{\text{action}} \)

SR Lorentz Scalar
\( \Phi_{\text{Lorentz}} \)

Einstein, de Broglie
\( \hbar \omega = E_0 \mathbf{E}/c_0 \)

SR Action
\( \hbar \frac{d}{dt} = \gamma \frac{d}{dt} \)

SR Lorentz Transform
\( \Lambda^\nu_{\mu} \)
SRQM Basic Idea (part 1)

SR → Relativistic Wave Eqn

The basic idea is to show that Special Relativity plus a few empirical facts lead to Relativistic Wave Equations, and thus RQM, without using any assumptions or axioms from Quantum Mechanics.

Start only with the concepts of SR, no concepts from QM:
(1) SR provides the ideas of Invariant Intervals and \( c \) as a Physical Constant, as well as: Poincaré Invariance, Minkowski 4D SpaceTime, ProperTime, ProperLength, Physical SR 4-Vectors.

Note empirical facts which can relate the SR 4-Vectors from the following:
(2a) Elementary matter particles each have RestMass, \( m_0 \), a physical constant which can be measured by experiment: eg. in collisions, cyclotrons, Compton Scattering, etc.

(2b) There is a physical constant, \( \hbar \), which can be measured by classical experiment – eg. the Photoelectric Effect, the inverse Photoelectric Effect, LED's=Injection Electroluminescence, Duane-Hunt Law in Bremsstralung, the Watt/Kibble-Balance, Electron Diffraction, Incandescence, Stern-Gerlach, Compton Scattering, Atomic Line Spectra, etc. All known particles types obey this constant.

(2c) The use of imaginary:complex numbers \( i \) and differential operators \( \{ \partial_t, \nabla \rightarrow (\partial_x, \partial_y, \partial_z) \} \) in wave-type equations comes from pure mathematics: not necessary to assume any QM Axioms

These few things are enough to derive the RQM Klein-Gordon equation, the most basic of the relativistic wave equations. Taking the low-velocity limit \( \{ |v| \ll c \} \) (a standard SR technique) leads to the Schrödinger Equation, the basic QM equation.
If one has a Relativistic Wave Equation, such as the Klein-Gordon equation, then one has RQM, and thence QM via the low-velocity limit \( |v| \ll c \).

The physical and mathematical properties of QM, usually regarded as axiomatic, are inherent in the Klein-Gordon RWE itself (i.e. derivable from it).

QM Principles emerge not from \{ QM Axioms + SR → RQM \}, but from \{ SR + Empirical Facts → RQM \}.

The result is a paradigm shift from the idea of \{ SR and QM as separate theories \} to \{ QM derived from SR \} – leading to a new interpretation of QM: *The SRQM or [SR→QM] Interpretation.*

\[ \text{GR} \rightarrow (\text{low-mass limit = \{curvature \sim 0\} limit}) \rightarrow \text{SR} \]
\[ \text{SR} \rightarrow (+ \text{a few empirical facts giving Lorentz Invariant Scalars}) \rightarrow \text{RQM} \]
\[ \text{RQM} \rightarrow (\text{low-velocity limit \{ |v| \ll c \}) \rightarrow \text{QM} \]

The results of this analysis will be facilitated by the use of SR 4-Vectors.
### SRQM 4-Vector Study: Basic 4-Vectors on the path to QM

<table>
<thead>
<tr>
<th>SR 4-Vector</th>
<th>Dimens. Units (SI)</th>
<th>Definition Component Notation</th>
<th>Properties united in SR: 4D SpaceTime</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-Position (\mathbf{R})</td>
<td>[m]</td>
<td>(R = R^\mu = (r^\mu) = (r^0, r^i) \in \langle \text{Event} \rangle = (ct, r) \rightarrow (ct, x, y, z))</td>
<td><strong>Time, Space</strong> ((c^* \text{when, where}) = \text{SR location of} \ &lt;\text{Event}&gt;)</td>
</tr>
<tr>
<td>4-Velocity (\mathbf{U})</td>
<td>[m/s]</td>
<td>(U = U^\mu = (u^\mu) = (u^0, u^i) = \gamma(c, u))</td>
<td><strong>Temporal velocity, Spatial velocity</strong> (\text{SR motion of} \ &lt;\text{Event}, \text{ max speed} = c)</td>
</tr>
<tr>
<td>4-Momentum (\mathbf{P})</td>
<td>[kg\cdot m/s] [N\cdot s]</td>
<td>(P = P^\mu = (p^\mu) = (p^0, p^i) = (E/c, p))</td>
<td><strong>Mass:Energy, Momentum</strong> (\text{used in} \ 4\text{-Momentum Conservation} ) (\Sigma P_{\text{final}} = \Sigma P_{\text{initial}})</td>
</tr>
<tr>
<td>4-WaveVector (\mathbf{K})</td>
<td>[{rad}/m]</td>
<td>(K = K^\mu = (k^\mu) = (k^0, k^i) = (\omega/c, \hat{n})/\nu_{\text{phase}} = (1/cT, \hat{n}/\lambda) = 2\pi (1/cT, \hat{n}/\lambda))</td>
<td><strong>Angular Frequency, WaveNumber</strong> (\text{used in Relativistic Doppler Shift} ) (\omega_{\text{obs}} = \omega_{\text{emit}} / [\gamma(1 - \beta \cos[\theta])], k = \omega/c \text{ for photons})</td>
</tr>
<tr>
<td>4-Gradient (\partial)</td>
<td>[1/m]</td>
<td>(\partial = \partial^\mu = (\partial^\mu) = (\partial^0, \partial^i) = (\partial/c, -\nabla) \rightarrow (\partial/c, -\partial_x, -\partial_y, -\partial_z) \rightarrow (\partial/\partial ct, -\partial/\partial x, -\partial/\partial y, -\partial/\partial z))</td>
<td><strong>Temporal Partial, Spatial Partial</strong> (\text{used in} \ \text{SR Continuity Eqns., ProperTime}) eg. (\partial \cdot A = 0 \text{ means} \ A \text{ is locally conserved})</td>
</tr>
</tbody>
</table>

All of these are standard SR 4-Vectors, which can be found and used in a totally relativistic context, with no mention or need of QM. I want to emphasize that these objects are ALL relativistic in origin.
SRQM 4-Vector Study:  
**SR Lorentz Scalar Invariants**

<table>
<thead>
<tr>
<th>SR 4-Vector</th>
<th>Lorentz Scalar Invariant</th>
<th>What it means in SR...</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-Position $\mathbf{R} = (ct, r)$</td>
<td>$\mathbf{R} \cdot \mathbf{R} = (ct)^2 - r \cdot r = (ct_o)^2 = (ct)^2$</td>
<td>SR Invariant Interval, ProperTime</td>
</tr>
<tr>
<td>4-Velocity $\mathbf{U} = \gamma(c, u)$</td>
<td>$\mathbf{U} \cdot \mathbf{U} = \gamma^2(c^2 - u \cdot u) = c^2$</td>
<td>$&lt;\text{Event}&gt;$ Motion Invariant Magnitude (c)</td>
</tr>
<tr>
<td>4-Momentum $\mathbf{P} = (E/c, p)$</td>
<td>$\mathbf{P} \cdot \mathbf{P} = (E/c)^2 - p \cdot p = (E_o/c)^2 = (m_o c)^2$</td>
<td>Einstein Invariant Mass:Energy Relation</td>
</tr>
<tr>
<td>4-WaveVector $\mathbf{K} = (\omega/c, k)$</td>
<td>$\mathbf{K} \cdot \mathbf{K} = (\omega/c)^2 - k \cdot k = (\omega_o/c)^2$</td>
<td>Wave/Dispersion Invariance Relation</td>
</tr>
<tr>
<td>4-Gradient $\partial = (\partial/c, -\nabla)$</td>
<td>$\partial \cdot \partial = (\partial/c)^2 - \nabla \cdot \nabla = (\partial/c)^2$</td>
<td>The d'Alembert Invariant Operator</td>
</tr>
</tbody>
</table>

All 4-Vectors have invariant magnitudes, found by taking the scalar product of the 4-Vector with itself. Quite often a simple expression can be found by examining the case when the spatial part is zero. This is usually found when the 3-velocity is zero. The temporal part is then specified by its “rest” value.

For example:  
$\mathbf{P} \cdot \mathbf{P} = (E/c)^2 - p \cdot p = (E_o/c)^2 = (m_o c)^2$  
$E = \sqrt{[ (E_o)^2 + p \cdot p \ c^2 ]}$, from above relation  
$E = \gamma E_o$, using $\{ \gamma = 1/\sqrt{[1-\beta^2]} = \sqrt{[1+\gamma^2 \beta^2]} \}$ and $\{ \beta = v/c \}$  
meaning the relativistic energy $E$ is equal to the relative gamma factor $\gamma$ * the rest energy $E_o$
SR + A few empirical facts:  
**SRQM Overview**

<table>
<thead>
<tr>
<th>SR 4-Vector</th>
<th>Empirical Fact</th>
<th>What it means in SR...</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-Position ( \mathbf{R} = (ct,r) ); alt. ( \mathbf{X} = (ct,x) )</td>
<td>( \mathbf{R} \in &lt;\text{Event}&gt; ); alt. ( \mathbf{X} )</td>
<td>Location of 4D SpaceTime &lt;Event&gt;</td>
</tr>
<tr>
<td>4-Velocity ( \mathbf{U} = \gamma(c,u) )</td>
<td>( \mathbf{U} = d\mathbf{R}/d\tau )</td>
<td>Motion of 4D SpaceTime &lt;Event&gt;</td>
</tr>
<tr>
<td>4-Momentum ( \mathbf{P} = (E/c,p) = (mc,p) )</td>
<td>( \mathbf{P} = m_o \mathbf{U} )</td>
<td>&lt;Events&gt; described as Particles</td>
</tr>
<tr>
<td>4-WaveVector ( \mathbf{K} = (\omega/c,k) )</td>
<td>( \mathbf{K} = \mathbf{P}/\hbar )</td>
<td>&lt;Events&gt; described as Waves</td>
</tr>
<tr>
<td>4-Gradient ( \partial = (\partial t/c, -\nabla) )</td>
<td>( \partial = -i\mathbf{K} )</td>
<td>Alteration of 4D SpaceTime &lt;Event&gt;</td>
</tr>
</tbody>
</table>

The Axioms of SR, which is actually a GR limiting-case, lead us to the use of Minkowski SpaceTime and Physical 4-Vectors, which are elements of Minkowski Space = (4D **Time**·**Space**).

Empirical Observation leads us to the transformation relations between the components of these SR 4-Vectors, and to the chain of relations between the 4-Vectors themselves. These relations all use Lorentz Scalar Invariant Constants, whose values are measured empirically. They are manifestly invariant relations, true in all reference frames...

The combination of these SR objects and their relations is enough to derive RQM.
SRQM Chart: Special Relativity → Quantum Mechanics

SRQM: The [ SR→QM ] Interpretation of Quantum Mechanics

Special Relativity (SR) Axioms: Invariant Interval + LightSpeed (c) as Universal Physical Constant lead to SR, although technically SR is itself the Minkowski-SpaceTime low-curvature:"flat" limiting-case of GR.

\{c, \tau, m_0, \hbar, i\} = \{c: SpeedOfLight, \tau: ProperTime, m_0: RestMass, \hbar: Dirac/PlanckReducedConstant(\hbar=h/2\pi), i: ImaginaryNumber\}:
are all Empirically-Measured SR Lorentz-Invariant Physical and/or Mathematical Constants

\(i = +\sqrt{-1} = (0,1)\) complex#

Standard SR 4-Vectors:

- 4-Position: \(R = (ct, r)\)
- 4-Velocity: \(U = (c, u)\)
- 4-Momentum: \(P = (E/c, p)\)
- 4-WaveVector: \(K = (\omega/c, k)\)
- 4-Gradient: \(\partial = (\partial_t/c, -\nabla)\)

SR + Empirically Measured Physical Constants lead to RQM via the (KG) Klein-Gordon Quantum Eqn, and thence to QM via the low-velocity limit \(|v| << c\), giving the Schrödinger Eqn. This fundamental KG Relation also leads to the other Quantum Wave Equations:

spin=0 boson field = 4-Scalar:
- Free Scalar Wave (Higgs)

spin=1/2 fermion field = 4-Spinor:
- Weyl

spin=1 boson field = 4-Vector:
- Maxwell (EM photonic)
- Proca

SRQM: A treatise of SR→QM by John B. Wilson (SciRealm@aol.com)
SRQM Diagram:
RoadMap of SR (4-Vectors)

4-Position
\( R = (ct, r) \in \langle \text{Event} \rangle \)

4-Velocity
\( U = \gamma(c, u) \)

4-WaveVector
\( K = (\omega/c, k) \)

4-Momentum
\( P = (mc, p) = (E/c, p) \)

4-Gradient
\( \partial = (\partial/c, -\nabla) \)

4-Position
\( R = (ct, r) \in \langle \text{Event} \rangle \)

4-Vector SRQM Interpretation
of QM

SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf

Trace\( [T^{\mu \nu}] = \eta^{\mu \nu} T^{\mu \nu} = T^{\mu}_{\mu} = T \)

\( V \cdot V = V^{\mu} \eta_{\mu \nu} V^{\nu} = ((v^0)^2 - \mathbf{v} \cdot \mathbf{v}) = (v^0)^2 \)

= Lorentz Scalar Invariant
SRQM Diagram: RoadMap of SR (Connections)

4-Velocity
\[ U = \gamma(c, u) \]

4-Momentum
\[ P = (mc, p) = \left( \frac{E}{c}, p \right) \]

4-WaveVector
\[ K = (\omega/c, k) \]

4-Gradient
\[ \partial = \left( \frac{\partial}{\gamma c}, -\nabla \right) \]

Minkowski Metric
\[ \delta_{\mu\nu} = \eta_{\mu\nu} \]

Lorentz Transform
\[ \partial[R^\nu] = \Lambda_{\mu}{}^\nu \]

ProperTime
\[ U \cdot \partial = \gamma \frac{dt}{d\tau} = d/d\tau \]

Space Time Dim
\[ \partial \cdot R = 4 \]

Derivative
\[ \partial_{\nu} [R_{\mu'}] = \Lambda_{\nu}{}^{\mu'} \]

SR 4-Tensor
\[ T_{\mu\nu} \]

SR 4-Vector
\[ V^\mu = \mathbf{V} = (v^0, \mathbf{v}) \]

SR 4-CoVector: OneForm
\[ V_{\mu} = (v_0, -\mathbf{v}) \]

SR 4-Scalar
\[ S \]

SR Phase
\[ K \cdot R = \Phi_{\text{phase,free}} \]

SR Action
\[ P \cdot R = S_{\text{action,free}} \]

SR → QM

Physics

A Tensor Study of Physical 4-Vectors

SciRealm.org
John B. Wilson
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http://scirealm.org/SRQM.pdf

\[ \begin{align*}
\text{Trace}[T_{\mu\nu}] &= \eta_{\mu\nu} T^{\mu\nu} = T_{\mu}{}^{\mu} = T \\
V \cdot V &= V^\mu V^\nu \left( (v^0)^2 - \mathbf{v} \cdot \mathbf{v} \right) = (v^0)^2
\end{align*} \]

Lorentz Scalar Invariant
SRQM Diagram: RoadMap of SR (Free Particle) with Magnitudes
SR Invariant Waves
SR Proper Time Derivative
SR Phase → 4-WaveVector
SR Lorentz Transforms
SR SpaceTime Dimension=4
SR SpaceTime “Flat” Minkowski 4D Metric

4-Gradient=

mass:energy & 3-momentum
oscillations proportional to

SR Wave

SR 4-Tensor

SR 4-Vector

SR 4-Scalar

A Tensor Study of Physical 4-Vectors

SRQM Diagram:
RoadMap of SR (EM Potential)

4-Gradient=Alteration of SR <Events>
SR SpaceTime “Flat” Minkowski 4D Metric
SR SpaceTime Dimension=4
SR Lorentz Transforms
SR Action → 4-Momentum
SR Phase → 4-WaveVector
SR Proper Time Derivative
SR Invariant Waves

4-Gradient=

\[ \partial = (\partial/c)^2 - \nabla \cdot \nabla \]

\[ = (\partial/c)^2 \]
d’Alembertian
Particle Wave Equation
in EM Potential

4-WaveVector=Substantiation
of SR Wave <Events>
oscillations proportional to
mass:energy & 3-momentum

\[ K = (\omega/c, k) \]

\[ K \cdot K = (\omega/c)^2 - k \cdot k \]

SR Particle <Events> have 4-Momentum=Substantiation
mass:energy & 3-momentum

\[ P = (mc, p) = (E/c, p) \]

4-PotentialMomentum

\[ Q = (V/c, q) = q(\varphi/c, a) \]

4-TotalMomentum

\[ P_T = (E/c, p_t) \]

Trace[T^\nu_\nu] = \eta_{\mu\nu}T^\mu_\nu = T^\nu_\nu = T

\[ V \cdot V = V^\mu V_\mu = \left(\frac{V^\mu}{c}\right)^2 - V \cdot V = \left(\frac{V^\mu}{c}\right)^2 \]

Lorentz Scalar Invariant

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Interpretation
John B. Wilson
SRQM Diagram:
Special Relativity → Quantum Mechanics
RoadMap of SR → QM (w/ EM Potential)

SR 4-Tensor
(2,0)-Tensor $T^{\mu\nu}$
(1,1)-Tensor $V^\nu = (\vec{v}, v)$
(1,0)-Tensor $V_\mu = (v_0, \vec{v})$
(0,0)-Tensor $S$ or $S_\mu = (\omega_0, \vec{0})$

SR 4-CoVector:OneForm
(0,1)-Tensor $V_\mu = (v_0, \vec{v})$

SR 4-Vector
(0,2)-Tensor $V^\nu = (\vec{v}, v)$

SR 4-Scalar
Mass:Energy & 3-momentum

4-Gradient $\partial^\mu$ $\partial_\mu = (\partial/c, -\vec{V}) = -i\mathbf{k}$

4-Position $R^\mu = (ct, \vec{r})$

4-Velocity $U^\mu = (c, \vec{v})$

EM Faraday $\partial R^\mu A^\mu = F_{\mu\nu}$

SR Action $S[R] = \int \mathcal{L} dt$

SR Phase $\Phi^\mu_{\text{phase}}$

SR 4-Vector SRQM

SR SpaceTime "Flat" Minkowski 4D Metric

SR Lorentz Transforms

SR Proper Time Derivative

SR & QM Invariant Waves

SR → QM Klein-Gordon

Relativistic Quantum

SR → RQM

Klein-Gordon

SCIENTIFIC ROADMAP

SR Action $\mathcal{L} = -\frac{1}{2} \gamma_{\mu\nu} \partial^\mu \partial_\nu \Phi^\mu - \frac{1}{2} \gamma_{\mu\nu} \partial^\mu \Phi^\mu \partial_\nu \Phi^\mu - \gamma_{\mu\nu} \partial^\mu \Phi^\mu \partial_\nu \Phi^\mu$
### SRQM Study:
The Empirical 4-Vector Facts

<table>
<thead>
<tr>
<th>SR 4-Vector</th>
<th>Empirical Fact</th>
<th>Discoverer</th>
<th>Physics</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-Position</td>
<td>( \mathbf{R} \in \langle \text{Event} \rangle )</td>
<td>Newton+Einstein</td>
<td>([ t \ &amp; \mathbf{r}] ) Time &amp; Space (&lt;\text{time}&gt; ) &amp; (&lt;\text{location}&gt; ) [ \mathbf{R}=(ct,r) ] SpaceTime as 4D=(1+3)D</td>
</tr>
<tr>
<td>4-Velocity</td>
<td>( \mathbf{U} = \frac{d\mathbf{R}}{d\tau} )</td>
<td>Newton Einstein</td>
<td>([ \mathbf{v}=\dot{\mathbf{r}}=\frac{d\mathbf{r}}{dt} ] ) Calculus of motion [ \mathbf{U}=\gamma(c,u)=\frac{d\mathbf{R}}{d\tau} ] Gamma &amp; Proper Time</td>
</tr>
<tr>
<td>4-Momentum</td>
<td>( \mathbf{P} = m_o \mathbf{U} )</td>
<td>Newton Einstein</td>
<td>([ \mathbf{p}=mv ] ) Classical Mechanics [ \mathbf{P}=(E/c,p)=m_o \mathbf{U} ] SR Mechanics</td>
</tr>
<tr>
<td>4-WaveVector</td>
<td>( \mathbf{K} = \frac{\mathbf{P}}{\hbar} )</td>
<td>Planck Einstein</td>
<td>([ \hbar ] ) Photon Thermal Distribution [ E=\hbar \nu=\hbar \omega ] Photoelectric Effect (( \hbar=\hbar/2\pi )) [ \mathbf{p}=\hbar \mathbf{k} ] Matter Waves [ \mathbf{P}=(E/c,p)=\hbar \mathbf{K}=\hbar(\omega/c,k) ] as 4-Vector Math</td>
</tr>
<tr>
<td>4-Gradient</td>
<td>( \partial = -i \mathbf{K} )</td>
<td>Schrödinger</td>
<td>([ \omega=i\partial, \mathbf{k}=-i\nabla ] ) (SR) Wave Mechanics [ \mathbf{P}=(E/c,p)=i\hbar \partial=i\hbar(\partial/c,-\nabla) ] (QM) 4-Vecctor</td>
</tr>
</tbody>
</table>

1. The SR 4-Vectors and their components are related to each other via constants.
2. We have not taken any 4-vector relation as axiomatic, the constants come from experiment.
3. \( c, \tau, m_o, \hbar \) come from physical experiments, \((-i)\) comes from the general mathematics of waves.
# SRQM Study:
## 4-Vector Relations Explained

<table>
<thead>
<tr>
<th>SR 4-Vector</th>
<th>Empirical Fact</th>
<th>What it means in SRQM...</th>
<th>Lorentz Invariant</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-Position $\mathbf{R} = (ct, \mathbf{r})$</td>
<td>$\mathbf{R} \in \langle \text{Event} \rangle$</td>
<td>SpaceTime:LightCone = Unified Concepts</td>
<td>$c = \text{LightSpeed}$</td>
</tr>
<tr>
<td>4-Velocity $\mathbf{U} = \gamma(c, \mathbf{u})$</td>
<td>$\mathbf{U} = \frac{d\mathbf{R}}{d\tau}$</td>
<td>Velocity is ProperTime Derivative of $\mathbf{R}$</td>
<td>$\tau = t_o = \text{ProperTime}$</td>
</tr>
<tr>
<td>4-Momentum $\mathbf{P} = (E/c, \mathbf{p})$</td>
<td>$\mathbf{P} = m_o \mathbf{U}$</td>
<td>Mass:Energy-Momentum Equivalence</td>
<td>$m_o = \text{RestMass}$</td>
</tr>
<tr>
<td>4-WaveVector $\mathbf{K} = (\omega/c, \mathbf{k})$</td>
<td>$\mathbf{K} = \mathbf{P}/\hbar$</td>
<td>Wave-Particle Duality</td>
<td>$\hbar = \text{UniversalAction}$</td>
</tr>
<tr>
<td>4-Gradient $\partial = \left( \frac{\partial}{c}, -\nabla \right)$</td>
<td>$\partial = -i\mathbf{K}$</td>
<td>Unitary Evolution, Operator Formalism</td>
<td>$i = \text{ComplexSpace}$</td>
</tr>
</tbody>
</table>

Three old-paradigm QM Axioms:
Particle-Wave Duality $[(\mathbf{P})=\hbar(\mathbf{K})], \text{Unitary Evolution }[\partial=(-i)\mathbf{K}], \text{Operator Formalism }[(\partial)=-i\mathbf{K}]$ are actually just empirically-found constant relations between known SR 4-Vectors.
Note that these constants are in fact all Lorentz Scalar Invariants.

Minkowski Space and 4-Vectors also lead to idea of Lorentz Invariance. A Lorentz Invariant is a quantity that always has the same value, independent of the motion or orientation of inertial observers. Lorentz Invariants can typically be derived using the scalar product relation. $\mathbf{U} \cdot \mathbf{U} = c^2, \quad \mathbf{U} \cdot \partial = d/d\tau, \quad \mathbf{P} \cdot \mathbf{U} = E_o = m_o c^2, \text{ etc.}$

A very important Lorentz invariant is the Proper Time $\tau$, which is defined as the time displacement between two points on a worldline that is at-rest wrt. an observer. It is used in the relations between 4-Position $\mathbf{R}$, 4-Velocity $\mathbf{U} = \frac{d\mathbf{R}}{d\tau}$, and 4-Acceleration $\mathbf{A} = \frac{d\mathbf{U}}{d\tau}$. 
SRQM: The SR Path to RQM
Follow the Invariants...

<table>
<thead>
<tr>
<th>SR 4-Vector</th>
<th>Lorentz Invariant</th>
<th>What it means in SRQM...</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-Position ( \mathbf{R} = (ct, \mathbf{r}) \in \langle \text{Event} \rangle )</td>
<td>( \mathbf{R} \cdot \mathbf{R} = (ct)^2 - \mathbf{r} \cdot \mathbf{r} = (c\tau)^2 )</td>
<td>SR Invariant Interval</td>
</tr>
<tr>
<td>4-Velocity ( \mathbf{U} = \gamma(c, \mathbf{u}) = \frac{d\mathbf{R}}{d\tau} )</td>
<td>( \mathbf{U} \cdot \mathbf{U} = \gamma^2(c^2 - \mathbf{u} \cdot \mathbf{u}) = c^2 )</td>
<td>Events move into future at magnitude ( c )</td>
</tr>
<tr>
<td>4-Momentum ( \mathbf{P} = (E/c, \mathbf{p}) = m_o \mathbf{U} )</td>
<td>( \mathbf{P} \cdot \mathbf{P} = (m_o c)^2 = (E_o/c)^2 )</td>
<td>Einstein Mass:Energy Relation</td>
</tr>
<tr>
<td>4-WaveVector ( \mathbf{K} = (\omega/c, \mathbf{k}) = \frac{\mathbf{P}}{\hbar} )</td>
<td>( \mathbf{K} \cdot \mathbf{K} = (m_o c/\hbar)^2 = (\omega_o/c)^2 )</td>
<td>Matter-Wave Dispersion Relation</td>
</tr>
<tr>
<td>4-Gradient ( \partial = (\partial/c, -\nabla) = -i\mathbf{K} )</td>
<td>( \partial \cdot \partial = (-im_o c/\hbar)^2 = -(m_o c/\hbar)^2 )</td>
<td>The Klein-Gordon Equation (\rightarrow) RQM!</td>
</tr>
</tbody>
</table>

\[ \mathbf{U} = \frac{d\mathbf{R}}{d\tau} \]
Remember, everything after 4-Velocity was just a constant times the last 4-vector, and the Invariant Magnitude of the 4-Velocity is itself a constant \( \mathbf{P} = m_o \mathbf{U}, \mathbf{K} = \frac{\mathbf{P}}{\hbar}, \partial = -i\mathbf{K} \), so e.g. \( \mathbf{P} \cdot \mathbf{P} = m_o \mathbf{U} \cdot m_o \mathbf{U} = m_o^2 \mathbf{U} \cdot \mathbf{U} = (m_o c)^2 \)

The last equation is the Klein-Gordon RQM Equation, which we have just derived without invoking any QM axioms, only SR plus a few empirical facts
SRQM: Some Basic 4-Vectors

4-Momentum, 4-WaveVector, 4-Position, 4-Velocity, 4-Gradient, Wave-Particle

- **4-Momentum** $P = (mc, p) = (E/c, p)$
  - Hamilton-Jacobi $P = -\partial[S_{action,free}]/\partial T$
  - Treat moving particles like a particle

- **4-WaveVector** $K = (\omega/c, k) = (\omega/c, \omega n/\nu_{phase})$
  - de Broglie $P = hK$
  - Treat moving waves as plane-waves

- **4-Position** $R = (ct, r)$
  - $\dot{R} = -\Phi_{phase,plane}$
  - $\dot{R} = -\partial[S_{action,free}]/\partial T$

- **4-Velocity** $U = \gamma(c, u)$
  - $U \cdot d[..] = d/dt[..]$
  - $\gamma = (v/c)$

**See Hamilton-Jacobi Formulation of Mechanics**

for info on the Lorentz Scalar Invariant SR Action.

- $[P] = (E/c, p) = -\partial[S] = -\partial[\gamma(c, u)]$
- **temporal component** $E = -\partial[\gamma(c, u)] = -\partial[S]$
- **spatial component** $p = \nu_{phase}/c$

**Note** This is the Action $S_{action}$ for a free particle.
Generally Action is for the 4-TotalMomentum $P_T$ of a system.

**SR 4-Tensor**

(2,0)-Tensor $T^\nu_\mu$

(1,1)-Tensor $T^\nu_\nu$, or $T$

(0,2)-Tensor $T^{\nu\rho}_\mu$

**SR 4-Vector**

(1,0)-Tensor $V^\nu = V = (v^\nu)$

SR 4-Vector One Form (0,1)-Tensor $V_\mu = (v_\mu)$

**SR 4-Scalar**

(0,0)-Tensor $S$ or $S_0$

Lorentz Scalar

**Existing SR Rules**

Quantum Principles
The 4-Vector Wave-Particle relation is inherent in all particle types: Einstein-de Broglie \( \textbf{P} = (E/c, p) = \hbar \textbf{K} = \hbar (\omega/c, k) \).

All waves can superpose, interfere, diffract: Water waves, gravitational waves, photonic waves of all frequencies, etc. In all cases: experiments using single particles build the diffraction/interference pattern over the course many iterations.

**Photon/light Diffraction:** Photonic particles diffracted by matter particles. Photons of any frequency encounter a translucent “solid=matter” object, grating, or edge. Most often encountered are diffraction gratings and the famous double-slit experiment.

**Matter Diffraction:** Matter particles diffracted by matter particles. Electrons, neutrons, atoms, small molecules, buckyballs (fullerenes), macromolecules, etc. have been shown to diffract through crystals. Crystals may be solid single pieces or in powder form.

**Kapitsa-Dirac Diffraction:** Matter particles diffracted by photonic standing waves. Electrons, atoms, super-sonic atom beams have been diffracted from resonant standing waves of light.

SRQM:
Hold on, aren't you getting the “ℏ” from a QM Axiom?

<table>
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<tr>
<th>SR 4-Vector</th>
<th>SR Empirical Fact</th>
<th>What it means...</th>
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</thead>
<tbody>
<tr>
<td>4-WaveVector</td>
<td>K = (ω/c,k) = (ω/c,ωn/ν_{phase}) = (ωo/c^2)U</td>
<td>Wave-Particle Duality</td>
</tr>
</tbody>
</table>

(ℏ) is actually an empirically measurable quantity, just like (e) or (c). It can be measured classically from the photoelectric effect, from the inverse photoelectric effect, from LED’s (injection electroluminescence), from Atomic Line Spectra (Bohr Atom), from the Duane-Hunt Law in Bremsstrahlung, from Electron Diffraction in crystals, from the Watt/Kibble-Balance, from Incandescent Blackbody Intensity-Temperature Relations, from Compton Scattering, from the Stern-Gerlach experiment, etc. See [http://scirealm.org/Physics-PlanckConstantViaLEDs.html](http://scirealm.org/Physics-PlanckConstantViaLEDs.html)

For the LED experiment, one uses several different LED’s, each with its own characteristic wavelength. One then makes a chart of each LED’s wavelength (λ) vs the threshold or activation voltage (V) needed to make that individual LED emit. One finds that: { λ = ℏc / (eV) } or { eV = ℏc / λ }, where (e)=ElectronCharge, (c)=LightSpeed, and (ℏ) is simply a slope that can be measured. Consider this as a [black-box] where no assumption about QM is made. However, we know the classical SR relations { E = eV }, and { λT = c }.

Due to the nature of 4-Vector (Tensor) mathematics, this means that 4-Momentum P = (E/c,p) = ℏK = ℏ(ωc,k) = ℏ4-WaveVector K.

The spatial component (due to De Broglie) follows naturally from the temporal component (due to Einstein) via to the nature of 4-Vector (Tensor) mathematics.

This is also derivable from pure SR 4-Vector (Tensor) arguments: P = m,U = (E/c)U and K = (ω/c)U
Since P and K are both Lorentz Scalar proportional to U, then by the rules of tensor mathematics, P must also be Lorentz Scalar proportional to K i.e. Tensors obey certain mathematical structures:
Transitivity{if a~b and b~c, then a~c} & Euclideaness: {if a~c and b~c, then a~b}

This invariant proportional constant is empirically measured to be (ℏ), for each known particle type, whether massive (m_o>0) or massless (m_o=0): P = m_o,U = (E_o/c^2)U = (E_o/c^2)/(ω_o/c)K = (E_o/ω_o)K = (γE_o/γω_o)K = (E/ω)K = (ℏ)K

also from standard SR Lorentz 4-Vector Scalar Products:
(P•U)/(K•U) = E_o/ω_o → |P||/|K| = E_o/ω_o = (ℏ)
(P•K)/(K•K) = m_oω_o/(ω_o/c)^2 → |P||/|K| = E_o/ω_o = (ℏ)
(P•P)/(K•P) = (m_o/c)^2/(m_oω_o) → |P||/|K| = E_o/ω_o = (ℏ)
(P•R)/(K•R) = (-S_{action,free}/(c^2Φ_{phase,plane}) → |P||/|K| = E_o/ω_o = (ℏ)
SRQM:
Hold on, aren't you getting the “K” from a QM Axiom?

<table>
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<th>SR 4-Vector</th>
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<tbody>
<tr>
<td>4-WaveVector</td>
<td>$K = (\omega/c, k) = (\omega/c, \omega \hat{n}/v_{\text{phase}}) = (\omega/c^2)U$</td>
<td>Wave-Particle Duality</td>
</tr>
</tbody>
</table>

$K$ is a standard SR 4-Vector, used in generating the SR formulae:

**Relativistic Doppler Effect:**

$$\omega_{\text{obs}} = \frac{\omega_{\text{emit}}}{\gamma (1 - \beta \cos[\theta])}, \quad |k| = k = \frac{\omega}{c} \text{ for photons}$$

**Relativistic Aberration Effect:**

$$\cos[\theta_{\text{obs}}] = \frac{(\cos[\theta_{\text{emit}}] + |\beta|)}{(1 + |\beta|\cos[\theta_{\text{emit}}])}$$

The 4-WaveVector $K$ can be derived in terms of periodic motion, where families of surfaces move through space as time increases, or alternately, as families of hypersurfaces in SpaceTime, formed by all events passed by the wave surface. The 4-WaveVector is everywhere in the direction of propagation of the wave surfaces.

$$K = -\partial[\Phi_{\text{phase, planewave}}]$$

From this structure, one obtains relativistic/wave optics without ever mentioning QM.
SRQM: Hold on, aren't you getting the "-i" from a QM Axiom?

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<th>SR 4-Vector</th>
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</thead>
<tbody>
<tr>
<td>4-Gradient</td>
<td>( \partial = (\partial_t/c, -\nabla) = -iK )</td>
<td>Unitary Evolution of States Operator Formalism</td>
</tr>
</tbody>
</table>

\[ \partial = -iK \] gives the sub-equations \( \partial_t = -i\omega \) and \( \nabla = i\mathbf{k} \), and is certainly the main equation that relates QM and SR by allowing Operator Formalism. But, this is a basic equation regarding the general mathematics of plane-waves; not just quantum-waves, but anything that can be mathematically described by plane-waves and superpositions of plane-waves… This includes purely SR waves, an example of which would be EM plane-waves (i.e. photons),

\[ \psi(t, r) = ae^{i(k \cdot r - \omega t)} \]: Standard mathematical plane-wave equation

\[ \partial[\psi(t, r)] = \partial[ae^{i(k \cdot r - \omega t)}] = (-i\omega)[ae^{i(k \cdot r - \omega t)}] = (-i\omega)\psi(t, r), \text{ or } [\partial_t = -i\omega] \]
\[ \nabla[\psi(t, r)] = \nabla[ae^{i(k \cdot r - \omega t)}] = (i\mathbf{k})[ae^{i(k \cdot r - \omega t)}] = (i\mathbf{k})\psi(t, r), \text{ or } [\nabla = i\mathbf{k}] \]

In the more economical SR notation:
\[ \partial[\psi(R)] = \partial[ae^{-i\mathbf{K} \cdot R}] = (-i\mathbf{K})[ae^{-i\mathbf{K} \cdot R}] = (-i\mathbf{K})\psi(R), \text{ or in 4-Vector shorthand } [\partial = -i\mathbf{K}] \]

This one is more of a mathematical empirical fact, but regardless, it is not axiomatic. It can describe purely SR waves, again without any mention of QM.
SRQM: Hold on, aren't you getting the “∂” from a QM Axiom?

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<tr>
<td>4-Gradient</td>
<td>$\partial = (\partial/c, -\nabla) = -iK$</td>
<td>4D Gradient Operator</td>
</tr>
</tbody>
</table>

$[\partial = (\partial/c, -\nabla)]$ is the SR 4-Vector Gradient Operator. It occurs in a purely relativistic context without ever mentioning QM.

$\partial \cdot \mathbf{X} = \partial^\mu \eta_{\mu\nu} X^\nu = (\partial/c, -\nabla) \cdot (ct, \mathbf{x}) = (\partial/c[ct] - (-\nabla \cdot \mathbf{x})) = (\partial[t] + \nabla \cdot \mathbf{x})$ (1)+(3) = 4

The 4-Divergence of the 4-Position gives the dimensionality of SpaceTime.

$\partial[\mathbf{X}] = \partial^\mu [X^\nu] = (\partial/c, -\nabla)[(ct, \mathbf{x})] = (\partial/c[ct], -\nabla[\mathbf{x}]) = \text{Diag}[1, -\mathbf{I}_3] = \eta^{\mu\nu}$

The 4-Gradient acting on the 4-Position gives the Minkowski Metric Tensor.

$\partial \cdot \mathbf{J} = \partial^\mu \eta_{\mu\nu} J^\nu = (\partial/c, -\nabla) \cdot (pc, \mathbf{j}) = (\partial/c[pc] - (-\nabla \cdot \mathbf{j})) = (\partial[p] + \nabla \cdot \mathbf{j}) = 0$

The 4-Divergence of the 4-CurrentDensity is equal to 0 for a conserved current. It can be rewritten as $(\partial[p] = -\nabla \cdot \mathbf{j})$, which means that the time change of ChargeDensity is balanced by the space change or divergence of CurrentDensity. It is a Continuity Equation, giving local conservation of ChargeDensity. It is related to Noether's Theorem.
SRQM:

Hold on, doesn’t using “∂” in an Equation of Motion presume a QM Axiom?

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<td>4-(Position)Gradient</td>
<td>∂ₚ = ∂ = (∂/c, -∇) = -iK</td>
<td>4D Gradient Operator</td>
</tr>
</tbody>
</table>

Klein-Gordon Relativistic Quantum Wave Equation
\[ \partial \cdot \partial [\Psi] = -(m_0c/\hbar)^2[\Psi] = - (\omega_0/c)^2 [\Psi] \]

Relativistic Euler-Lagrange Equations
\[ \partial_{\mathcal{R}}[L] = (d/d\tau)\partial_{\mathcal{U}}[L]: \{\text{particle format}\} \]
\[ \partial_{\mathcal{L}}[\Phi] = (\partial_{\mathcal{R}}) \partial_{[\partial_{\mathcal{R}}(\Phi)]}[L]: \{\text{density format}\} \]

\([\partial = (∂/c, -∇)]\) is the SR 4-Vector (Position)Gradient Operator.
It occurs in a purely relativistic context without ever mentioning QM.
There is a long history of using the gradient operator on classical physics functions, in this case the Lagrangian. And, in fact, it is another area where the same mathematics is used in both classical and quantum contexts.
The QM Schrödinger Relation

\[ P = \imath \hbar \frac{\partial}{\partial t} \]

This is derived from the combination of:

- The Einstein-de Broglie Relation
  \[ P = \hbar K \]

- Complex Plane-Waves
  \[ K = i \partial / \partial t = -i \partial \]

These are the standard QM Schrödinger Relations.

It is this Lorentz Scalar Invariant relation \( \imath \hbar \) which connects the 4-Momentum to the 4-Gradient, making it into a QM operator.

Note that these 4-Vectors are already connected in multiple ways in standard SR.
SRQM: Review of SR 4-Vector Mathematics

4-Gradient \( \partial \equiv (\partial_t/c, -\nabla) \)

4-Position \( X = (ct, x) \)

4-Velocity \( U = \gamma(c, u) \)

4-Momentum \( P = (E/c, p) = (E_0/c^2)U \)

4-WaveVector \( K = (\omega/c, k) = (\omega_0/c^2)U \)

\( \partial \cdot X = \left( (\partial_t/c - \nabla) \right) (ct, x) = \left( (\partial_t/c) - (\nabla \cdot x) \right) = 1 - (-3) = 4: \)

Dimensionality of SpaceTime

Derivative wrt. ProperTime is Lorentz Scalar

The Minkowski Metric

Phase of SR Wave

Neg 4-Gradient of Phase gives 4-WaveVector

Wave Continuity Equation, No sources or sinks

Standard mathematical plane-waves if \( \{ b = -i \} \)

Unitary Evolution, Operator Formalism

The Klein-Gordon Equation → RQM

Note that no QM Axioms are assumed: This is all just pure SR 4-vector (tensor) manipulation
Klein-Gordon Equation: \( \partial \cdot \partial = (\partial / c)^2 - \nabla \cdot \nabla = -(m_o c / \hbar)^2 = -(\omega_o / c)^2 = -(1 / \lambda_C)^2 \)

Let \( X_T = (ct + c \Delta t, x) \), then \( \partial [X_T] = (\partial / c, -\nabla) (ct + c \Delta t, x) = \text{Diag}[1, -I_{(3)}] = \partial [X] = \eta^{\mu \nu} \)

so \( \partial [X_T] = \partial [X] \) and \( \partial [K] = [0] \)

let \( f = a e^{-i(K \cdot X_T)} \), the time translated version

\[
(\partial \cdot \partial)[f] \\
\partial \cdot (\partial[f]) \\
\partial \cdot (e^{-i(K \cdot X_T)} \partial[-i(K \cdot X_T)]) \\
-\partial \cdot (f \partial[K \cdot X_T]) \\
-\partial[f] \partial[K \cdot X_T] + \Psi(\partial \cdot \partial)[K \cdot X_T] \\
(-i)^2 f(\partial[K \cdot X_T])^2 + 0 \\
(-i)^2 f(\partial[K] \cdot X_T + K \cdot \partial[X_T])^2 \\
(-i)^2 f(0 + K \cdot \partial[X])^2 \\
(-i)^2 f(K)^2 \\
-(K \cdot K)f \\
-(\omega_o / c)^2 f
\]
What does the Klein-Gordon Equation give us?

A lot of RQM!

Relativistic Quantum Wave Equation: \( \mathbf{\alpha} \cdot \mathbf{\alpha} = (\mathbf{\alpha} / c)^2 - \nabla \cdot \nabla = -(m_o c / \hbar)^2 = (im_o c / \hbar)^2 = -(\omega_o / c)^2 \)

The Klein-Gordon Eqn is itself the Relativistic Quantum Equation for spin=0 particles (4-Scalars)

Factoring the KG Eqn leads to the RQM Dirac Equation for spin=1/2 particles (4-Spinors)

Applying the KG Eqn to a 4-Vector field leads to the RQM Proca Equation for spin=1 particles (4-Vectors)

Taking the low-velocity-limit of the KG leads to the standard QM non-relativistic Schrödinger Eqn, for spin=0

Taking the low-velocity-limit of the Dirac leads to the standard QM non-relativistic Pauli Eqn, for spin=1/2

Setting RestMass \( \{m_o \rightarrow 0\} \) leads to the RQM Free Wave Eqn., Weyl Eqn., and Free Maxwell (Standard EM) Eqn.

In all of these cases, the equations can be modified to work with various potentials by using more SR 4-Vectors, and more empirically found relations between them, e.g. the Minimal Coupling Relations:

4-TotalMomentum \( P_T = P + qA \), where \( P \) is the particle 4-Momentum, \( q \) is a charge, and \( A \) is a 4-VectorPotential, typically the 4-EMVectorPotential.

Also note that generating QM from RQM (via a low-energy limit) is much more natural than attempting to “relativize or generalize” a given NRQM equation. Facts assumed from a non-relativistic equation may or may not be applicable to a relativistic one, whereas the relativistic facts are still true in the low-velocity limiting-cases. This leads to the idea that QM is an approximation only of a more general RQM, just as SR is an approximation only of GR.
## SRQM: Relativistic Quantum Wave Eqns.

| Spin-(Statistics) | Relativistic Light-like Mass = 0 | Relativistic Matter-like Mass > 0 | Non-Relativistic Limit (|v|<<c) Mass >0 | Field Representation |
|-------------------|----------------------------------|-----------------------------------|--------------------------------------|----------------------|
| Bose-Einstein=n   | Free Wave N-G Bosons             | Klein-Gordon Higgs Bosons, maybe Axions | Schrödinger Common NRQM Systems | Scalar (0-Tensor) \( \Psi = \Psi[K_{\mu}X^\mu]\) = \( \Psi[\Phi]\) |
| Fermi-Dirac=n/2   | \((\partial \cdot \partial)\Psi = 0\) | \((\partial \cdot \partial + (m_{\text{c}}/c)^2 - c^2(-i\hbar \nabla - qa)^2)\Psi = 0\) with minimal coupling | \((i\hbar \delta - \varphi + (p-q\mathbf{a})^2/2m_{\text{c}})\Psi = 0\) with minimal coupling \((i\hbar \delta + m_{\text{c}}c/\hbar)[\partial^\mu + im_{\text{c}}/\hbar \sigma \cdot \mathbf{A}]\Psi = 0\) | Spinor \( \Psi = \Psi[K_{\mu}X^\mu]\) = \( \Psi[\Phi]\) |

### 0-(Boson)
- **Maxwell Photons/Gluons**
  - \((\partial \cdot \partial)A = 0\) free
  - \((\partial \cdot \partial)A = \mu c \mathbf{J}\) w current src where \(\partial \cdot \mathbf{A} = 0\)
  - \((\partial \cdot \partial)A = \mu_e \mathbf{F} \cdot \Psi\) QED

### 1/2-(Fermion)
- **Weyl Idealized Matter Neutinos**
  - \((\sigma \cdot \partial)\Psi = 0\)
  - factored to Right & Left Spinors
    - \((\sigma \cdot \partial)\Psi_R = 0, (\bar{\sigma} \cdot \partial)\Psi_L = 0\)
  - \(L = i\Psi_{\text{R}}^\dagger \sigma^\mu \partial^\mu \Psi_R, L = i\Psi_{\text{L}}^\dagger \bar{\sigma}^\mu \partial^\mu \Psi_L\)
- **Dirac Matter Leptons/Quarks**
  - \((i\gamma \cdot \partial - m_{\text{c}}/\hbar)\Psi = 0\)
  - \((\gamma \cdot \partial + im_{\text{c}}/\hbar)\Psi = 0\)
  - with minimal coupling
    - \((i\gamma \cdot (\partial + iqA) - m_{\text{c}}/\hbar)\Psi = 0\)
  - \(L = ic \Psi \gamma^\nu \partial^\nu \Psi - m_{\text{c}}c^2 \Psi \Psi\)

### 1-(Boson)
- **Maxwell Photons/Gluons**
  - \((\partial \cdot \partial)A = 0\) free
  - \((\partial \cdot \partial)A = \mu c \mathbf{J}\) w current src where \(\partial \cdot \mathbf{A} = 0\)
  - \((\partial \cdot \partial)A = \mu_e \mathbf{F} \cdot \Psi\) QED
Klein-Gordon Equation: $\partial\cdot\partial = (\partial t/c)^2 - \nabla\cdot\nabla = -(m_0c/\hbar)^2$

Since the 4-vectors are related by constants, we can go back to the 4-Momentum description/representation:

\[(\partial t/c)^2 - \nabla\cdot\nabla = -(m_0c/\hbar)^2\]
\[(E/c)^2 - p\cdot p = (m_0c)^2\]
\[E^2 - c^2p\cdot p - (m_0c^2)^2 = 0\]

Factoring: $[E - c\alpha\cdot p - \beta(m_0c^2)] [E + c\alpha\cdot p + \beta(m_0c^2)] = 0$

$E$ & $p$ are quantum operators, $\alpha$ & $\beta$ are matrices which must obey $\alpha\beta = -\beta\alpha$, $\alpha\alpha = -\alpha\alpha$, $\alpha^2 = \beta^2 = I$

The left hand term can be set to 0 by itself, giving...

$[E - c\alpha\cdot p - \beta(m_0c^2)] = 0$, which is the momentum-representation form of the Dirac equation

Remember: $P^\mu = (p^0, p)$ and $\alpha^\mu = (\alpha^0, \alpha)$ where $\alpha^0 = I_{(2)}$

$[E - c\alpha\cdot p - \beta(m_0c^2)] = [c\alpha^0 p^0 - c\alpha\cdot p - \beta(m_0c^2)] = [c\alpha^0 P^0 - \beta(m_0c^2)] = 0$
\[\alpha^\mu P^\mu - \beta(m_0c) = [\text{i\hbar} \alpha^\mu \partial_\mu - \beta(m_0c)] = 0\]
\[\alpha^\mu \partial_\mu = -\beta(\text{i\hbar}c/\hbar)\]

Transforming from Pauli Spinor (2 component) to Dirac Spinor (4 component) form:

Dirac Equation: $(\gamma^\mu \partial_\mu)[\psi] = -\text{i\hbar}c \psi$

Thus, the Dirac Eqn is guaranteed by construction to be one solution of the KG Eqn

The KG Equation is at the heart of all the various relativistic wave equations, which differ based on mass and spin values, but all of them respect $E^2 - c^2p\cdot p - (m_0c^2)^2 = 0$
SRQM Study:
Lots of Relativistic Quantum Wave Equations
A lot of RQM!

Relativistic Quantum Wave Equation: \( \partial \cdot \partial = (\partial_t/c)^2 - \nabla \cdot \nabla = -(m_o c/\hbar)^2 = (m_o c/\hbar)^2 = -(\omega_o/c)^2 \)

The Klein-Gordon Eqn is itself the Relativistic Quantum Equation for spin=0 particles \{Higgs\} (4-Scalars)
Factoring the KG Eqn ("square root method") leads to the RQM Dirac Equation for spin=1/2 particles (4-Spinors)
Applying the KG Eqn to a SR 4-Vector field leads to the RQM Proca Equation for spin=1 particles (4-Vectors)

Setting RestMass \(m_o \rightarrow 0\) leads to the:
RQM Free Wave (4-Scalar massless)
RQM Weyl (4-Spinor massless)
Free Maxwell Eqns (4-Vector massless) = Standard EM

So, the same Relativistic Quantum Wave Equation is simply applied to different SR Tensorial Quantum Fields
See Mathematical_formulation_of_the_Standard_Model at Wikipedia:

4-Scalar (massive) Higgs Field \(\varphi\) \[\partial \cdot \partial = -(m_o c/\hbar)^2 \varphi\] Free Field Eqn \rightarrow\ Klein-Gordon Eqn \[\partial \cdot [\varphi] = -(m_o c/\hbar)^2 \varphi\]
4-Vector (massive) Weak Field \(Z^\mu, W^\mu\) \[\partial \cdot \partial = -(m_o c/\hbar)^2 Z^\mu\] Free Field Eqn \rightarrow\ Proca Eqn \[\partial \cdot [Z^\mu] = -(m_o c/\hbar)^2 Z^\mu\]
4-Vector (massless \(m_o=0\)) Photon Field \(A^\mu\) \[\partial \cdot \partial = 0\] Free Field Eqn \rightarrow\ EM Wave Eqn \[\partial \cdot [A^\mu] = 0\]
4-Spinor (massive) Fermion Field \(\psi\) \[\gamma \cdot \partial = -i(m_o c/\hbar) \psi\] Free Field Eqn \rightarrow\ Dirac Eqn \[\gamma \cdot [\psi] = -(i m_o c/\hbar) \psi\]

*The Fermion Field is a special case, the Dirac Gamma Matrices \(\gamma^\mu\) and 4-Spinor field \(\psi\) work together to preserve Lorentz Invariance.
Relativistic Quantum Wave Equation:

\[ \partial \cdot \partial = \frac{(\partial t/c)^2 - \nabla \cdot \nabla}{(m_0 c/\hbar)^2} = -(\omega_0/c)^2 \]

\[ \partial \cdot \partial = -(m_0 c/\hbar)^2 \]

\[ (\partial \cdot \partial) A \nu = 0 \]

\[ \nu: \text{The Free Classical Maxwell EM Equation} \{\text{no source, no spin effects}\} \]

\[ (\partial \cdot \partial) A \nu = \mu_0 J \nu: \text{The Classical Maxwell EM Equation} \{\text{with 4-Current J source, no spin effects}\} \]

\[ (\partial \cdot \partial) A \nu = q(\bar{\psi} \gamma \nu \psi): \text{The QED Maxwell EM Spin-1 Equation} \{\text{with QED source, including spin effects}\} \]

So, the same Relativistic Quantum Wave Equation is simply applied to different SR Tensorial Quantum Fields

See Mathematical_formulation_of_the_Standard_Model at Wikipedia:

4-Scalar (massive)

Higgs Field \( \Phi \)

\[ [\partial \cdot \partial = -(m_0 c/\hbar)^2] \Phi \]

Free Field Eqn \( \rightarrow \) Klein-Gordon Eqn

\[ \partial \cdot \partial \Phi = -(m_0 c/\hbar)^2 \Phi \]

4-Vector (massive)

Weak Field \( Z^\mu, W^\mu \)

\[ [\partial \cdot \partial = -(m_0 c/\hbar)^2] Z^\mu \]

Free Field Eqn \( \rightarrow \) Proca Eqn

\[ \partial \cdot \partial Z^\mu = -(m_0 c/\hbar)^2 Z^\mu \]

4-Vector (massless \( m_0 = 0 \))

Photon Field \( A^\mu \)

\[ [\partial \cdot \partial = 0] A^\mu \]

Free Field Eqn \( \rightarrow \) EM Wave Eqn

\[ \partial \cdot \partial A^\mu = 0 \]

4-Spinor (massive)

Fermion Field \( \Psi \)

\[ [\gamma \cdot \partial = -i(m_0 c/\hbar)] \Psi \]

Free Field Eqn \( \rightarrow \) Dirac Eqn

\[ \gamma \cdot \partial \Psi = -(i(m_0 c/\hbar)) \Psi \]

*The Fermion Field is a special case, the Dirac Gamma Matrices \( \gamma^\mu \) and 4-Spinor field \( \Psi \) work together to preserve Lorentz Invariance.*

We can also do the same physics using Lagrangian Densities.

Proca Lagrangian Density \( L = -(1/2)(\partial \Phi^* \cdot \partial \Phi) + (\partial^\mu \Phi \cdot \partial_{\mu} \Phi^*) + (m_0 c/\hbar)^2 \Phi \cdot \Phi^* \) with \( B^\mu = (\phi/a)(ct, r) \) is a generalized complex 4-(Vector)Potential

KG Lagrangian Density \( L = -\eta^{\mu\nu}(\partial \psi^* \cdot \partial \psi) - (m_0 c/\hbar)^2 \psi^* \psi \) with \( \psi = \psi[R] = \psi[[ct, r]] \)

Dirac Lagrangian Density \( L = \bar{\psi}(i\gamma^\mu P^\mu - m_0 c) \psi : \) with \( \psi = \) a spinor \( \psi[[ct, r]] \)

QED Lagrangian Density \( \bar{\psi}(i\hbar \gamma^\mu D^\mu - m_0 c) \psi - (1/4)F_{\mu\nu}F^{\mu\nu} : \) with \( D^\mu = \partial^\mu + iqA^\mu + iqB^\mu \) and \( A^\mu = \) EM field of the e; \( B^\mu = \) external source EM field
In relativistic quantum mechanics and quantum field theory, the Bargmann–Wigner equations describe free particles of arbitrary spin $j$, an integer for bosons ($j = 1, 2, 3 ...$) or half-integer for fermions ($j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} ...$). The solutions to the equations are wavefunctions, mathematically in the form of multi-component spinor fields.

Bargmann–Wigner equations: 

$$(-\gamma^\mu P_\mu + mc)^{\alpha r \alpha' r' \alpha_{1}...\alpha_{r}...\alpha_{2j}} \psi = 0$$

In relativistic quantum mechanics and quantum field theory, the Joos–Weinberg equation is a relativistic wave equations applicable to free particles of arbitrary spin $j$, an integer for bosons ($j = 1, 2, 3 ...$) or half-integer for fermions ($j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} ...$). The solutions to the equations are wavefunctions, mathematically in the form of multi-component spinor fields. The spin quantum number is usually denoted by $s$ in quantum mechanics, however in this context $j$ is more typical in the literature.

Joos–Weinberg equation: 

$$\begin{bmatrix} \gamma^{\mu_1 \mu_2 ... \mu_{2j}} P_{\mu_1} P_{\mu_2} ... P_{\mu_{2j}} + (mc)^3 \end{bmatrix} \Psi = 0$$

The primary difference appears to be the expansion in either the wavefunctions for (BW) or the Dirac Gamma's for (JW).

For both of these: A state or quantum field in such a representation would satisfy no field equation except the Klein-Gordon equation.

Yet another form is the Duffin-Kemmer-Petiau Equation vs Dirac Equation

DKP Eqn {spin 0 or 1}: \((i\hbar \beta^a \partial_a - m_0 c)\Psi = 0\), with $\beta^a$ as the DKP matrices

Dirac Eqn (spin $\frac{1}{2}$): \((i\gamma^a \partial_a - m_0 c)\Psi = 0\), with $\gamma^a$ as the Dirac Gamma matrices
### SRQM: A few more SR 4-Vectors

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<tr>
<td>4-TotalWaveVector</td>
<td>( K_{\text{tot}} = (\omega/c+(q/\hbar)\varphi/c, k+(q/\hbar)a) )</td>
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<td>QM Probability (Density, Current Density)</td>
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</table>

**SR 4-Tensor**
- \((2,0)\)-Tensor \( T^{\mu \nu} \)
- \((1,1)\)-Tensor \( T^\mu_\nu \) or \( T^\nu_\mu \)

**SR 4-CoVector**
- \((0,2)\)-Tensor \( V_{\mu \nu} \)

**SR 4-Scalar**
- \((0,0)\)-Tensor \( S \) or \( S_\alpha \)
- Lorentz Scalar

---

**SRQM**
A treatise of SR→QM by John B. Wilson (SciRealm@aol.com)
# SRQM: More SR 4-Vectors Explained

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<td>CurrentDensity is conserved</td>
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<td>( J_{prob} = (c\rho_{prob}, j_{prob}) ) ( \partial \cdot J_{prob} = 0 )</td>
<td>QM Probability from SR Probability Worldlines are conserved</td>
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</table>
Minimal Coupling = Potential Interaction

Klein-Gordon Eqn $\rightarrow$ Schrödinger Eqn

$P_T = P + Q = P + qA$

Minimal Coupling: Total = Dynamic + Charge_Coupled to 4-(EM)VectorPotential

$K = i\partial$

Complex Plane-Waves

$P = \hbar K$

Einstein-de Broglie QM Relations

$P = i\hbar \partial$

Schrödinger Relations

$P = (E/c, p) = (E_T - q\varphi/c, p_T - qa)$

$hK = i\hbar \partial$

The Klein-Gordon RQM Wave Equation (relativistic QM)

$\delta \cdot \delta = (\partial/c)^2 - \nabla^2 = -(m_0c/h)^2$

$P \cdot P = (E/c)^2 - p^2 = (m_0c)^2$

Einstein Mass:Energy:Momentum Equivalence

$E^2 = (m_0c^2)^2 + c^2p^2$

Relativistic

$E \sim \left[ (m_0c^2) + \frac{p^2}{2m_0} \right]$

Low velocity limit $\{ |v| << c \}$ from $(1+x)^n \sim [1 + nx + O(x^2)]$ for $|x|<<1$

$(E_T - q\varphi)^2 = (m_0c^2)^2 + c^2(p_T - qa)^2$

Relativistic with Minimal Coupling

$(E_T - q\varphi) \sim \left[ (m_0c^2) + \frac{(p_T - qa)^2}{2m_0} \right]$

Low velocity with Minimal Coupling

$(i\hbar \partial_T - q\varphi)^2 = (m_0c^2)^2 + c^2(-i\hbar\nabla_T - qa)^2$

Relativistic with Minimal Coupling

$(i\hbar \partial_T - q\varphi) \sim \left[ (m_0c^2) + \frac{(-i\hbar\nabla_T - qa)^2}{2m_0} \right]$

Low velocity with Minimal Coupling

$(i\hbar \partial_T) \sim [ q\varphi + (m_0c^2) + i\hbar\nabla_T + qa]/2m_0$

Low velocity with Minimal Coupling

$V = q\varphi + (m_0c^2)$

Typically the 3-vector-potential $a \sim 0$ in many situations

$V \cdot V = V^2 = \left( [v^0]^2 - \mathbf{v} \cdot \mathbf{v} \right)$

Lorentz Scalar Invariant

The better statement is that the Schrödinger Eqn is the limiting low-velocity case of the more general KG Eqn, not that the KG Eqn is the relativistic generalization of the Schrödinger Eqn
SRQM: Once one has a Relativistic Wave Eqn...

Klein-Gordon Equation: \( \partial \cdot \partial = (\partial_t/c)^2 - \nabla \cdot \nabla = (-im_0c/\hbar)^2 = -(m_0c/\hbar)^2 = -(\omega_0/c)^2 \)

Once we have derived a RWE, what does it imply?

The KG Eqn. was derived from the physics of SR plus a few empirical facts. It is a 2nd order, linear, wave PDE that pertains to physical objects of reality from SR.

Just being a linear wave PDE implies all the mathematical techniques that have been discovered to solve such equations generally: Hilbert Space, Superpositions, \(<\text{Bra}|,|\text{Ket}>\) notation, wavevectors, wavefunctions, etc. These things are from mathematics in general, not only and specifically from an Axiom of QM.

Therefore, if one has a physical RWE, it implies the mathematics of waves, the formalism of the mathematics, and thus the mathematical Principles and Formalism of QM. Again, QM Axioms are not required – they emerge from the physics and math...
Once one has a Relativistic Wave Eqn…

Examine Photon Polarization

From the Wikipedia page on [Photon Polarization]

Photon polarization is the quantum mechanical description of the classical polarized sinusoidal plane electromagnetic wave. An individual photon can be described as having right or left circular polarization, or a superposition of the two. Equivalently, a photon can be described as having horizontal or vertical linear polarization, or a superposition of the two.

The description of photon polarization contains many of the physical concepts and much of the mathematical machinery of more involved quantum descriptions and forms a fundamental basis for an understanding of more complicated quantum phenomena. Much of the mathematical machinery of quantum mechanics, such as state vectors, probability amplitudes, unitary operators, and Hermitian operators, emerge naturally from the classical Maxwell's equations in the description. The quantum polarization state vector for the photon, for instance, is identical with the Jones vector, usually used to describe the polarization of a classical wave. Unitary operators emerge from the classical requirement of the conservation of energy of a classical wave propagating through lossless media that alter the polarization state of the wave. Hermitian operators then follow for infinitesimal transformations of a classical polarization state.

Many of the implications of the mathematical machinery are easily verified experimentally. In fact, many of the experiments can be performed with pairs of polarized sunglass-lenses.

The connection with quantum mechanics is made through the identification of a minimum packet size, called a photon, for energy in the electromagnetic field. The identification is based on the theories of Planck and the interpretation of those theories by Einstein. The correspondence principle then allows the identification of momentum and angular momentum (called spin), as well as energy, with the photon.
SRQM: Principle of Superposition: From the mathematics of waves

Klein-Gordon Equation: \( \partial \cdot \partial = (\partial / c)^2 - \nabla \cdot \nabla = (-im_0c/\hbar)^2 = -(m_0c/\hbar)^2 = -(\omega_0/c)^2 \)

The Extended Superposition Principle for Linear Equations

Suppose that the non-homogeneous equation, where \( L \) is linear, is solved by some particular \( u_p \)
Suppose that the associated homogeneous problem is solved by a sequence of \( u_i \). \( i=\{0,1,2,...\} \)
\( L(u_p) = C \); \( L(u_0) = 0 \), \( L(u_1) = 0 \), \( L(u_2) = 0 \) ...
Then \( u_p \) plus any linear combination of the \( u_n \) satisfies the original non-homogeneous equation:
\( L(u_p + \Sigma a_n u_n) = C \),
where \( a_n \) is a sequence of (possibly complex) constants and the sum is arbitrary.

Note that there is no mention of partial differentiation. Indeed, it's true for any linear equation, algebraic or integro-partial differential-whatever.

QM superposition is not axiomatic, it emerges from the mathematics of the Linear PDE
The Klein-Gordon Equation is a 2\(^{nd}\)-order LINEAR Equation.
This is the origin of superposition in QM.
Klein-Gordon obeys Principle of Superposition

Klein-Gordon Equation: \( \partial \cdot \partial = (\partial_t/c)^2 - \nabla \cdot \nabla = (-i m_0 c/\hbar)^2 = -(m_0 c/\hbar)^2 = -\left( \omega_0/c \right)^2 \)

\( \mathbf{K} \cdot \mathbf{K} = (\omega/c)^2 - \mathbf{k} \cdot \mathbf{k} = (\omega_0/c)^2 \): The particular solution (w/ rest mass)

\( \mathbf{K}_n \cdot \mathbf{K}_n = (\omega_n/c)^2 - \mathbf{k}_n \cdot \mathbf{k}_n = 0 \): The homogenous solution for a (virtual photon?) microstate \( n \)

Note that \( \mathbf{K}_n \cdot \mathbf{K}_n = 0 \) is a null 4-vector (photonic)

Let \( \Psi_p = Ae^{-i(\mathbf{K} \cdot \mathbf{X})} \), then \( \partial \cdot \partial [\Psi_p] = (-i)^2(\mathbf{K} \cdot \mathbf{K})\Psi_p = -\left( \omega_0/c \right)^2\Psi_p \)

which is the Klein-Gordon Equation, the particular solution...

Let \( \Psi_n = A_ne^{-i(\mathbf{K}_n \cdot \mathbf{X})} \), then \( \partial \cdot \partial [\Psi_n] = (-i)^2(\mathbf{K}_n \cdot \mathbf{K}_n)\Psi_n = (0)\Psi_n \)

which is the Klein-Gordon Equation homogeneous solution for a microstate \( n \)

We may take \( \Psi = \Psi_p + \sum \Psi_n \)

Hence, the Principle of Superposition is not required as an QM Axiom, it follows from SR and our empirical facts which lead to the Klein-Gordon Equation. The Klein-Gordon equation is a linear wave PDE, which has overall solutions which can be the complex linear sums of individual solutions – i.e. it obeys the Principle of Superposition.

This is not an axiom – it is a general mathematical property of linear PDE's.

This property continues over as well to the limiting case \{ |v|<<c \} of the Schrödinger Equation.
Klein-Gordon Equation: $\partial^2 - (\partial_t/c)^2 - \nabla^2 = (-i m_0 c/\hbar)^2 = -(m_0 c/\hbar)^2 = -(\omega_0/c)^2$

Hilbert Space (HS) representation:
if $|\Psi\rangle \in$ HS, then $c|\Psi\rangle \in$ HS, where $c$ is complex number
if $|\Psi_1\rangle$ and $|\Psi_2\rangle \in$ HS, then $|\Psi_1\rangle + |\Psi_2\rangle \in$ HS
if $|\Psi\rangle = c_1|\Psi_1\rangle + c_2|\Psi_2\rangle$, then $<\Phi|\Psi\rangle = c_1<\Phi|\Psi_1\rangle + c_2<\Phi|\Psi_2\rangle$ and $<\Psi| = c_1^*<\Psi_1| + c_2^*<\Psi_2|$
$<\Phi|\Psi\rangle = <\Psi|\Phi>$
$<\Psi|\Psi\rangle \geq 0$
if $<\Psi|\Psi\rangle = 0$, then $|\Psi\rangle = 0$
eq

Hilbert spaces arise naturally and frequently in mathematics, physics, and engineering, typically as infinite-dimensional function spaces. They are indispensable tools in the theories of partial differential equations, Fourier analysis, signal processing, heat transfer, ergodic theory, and Quantum Mechanics.

The QM Hilbert Space emerges from the fact that the KG Equation is a linear wave PDE – Hilbert spaces as solutions to PDE's are a purely mathematical phenomenon – no QM Axiom is required.

Likewise, this introduces the $<\text{bra}|\text{ket}>$ notation, wavevectors, wavefunctions, etc.

**Note:**

One can use Hilbert Space descriptions of Classical Mechanics using the Koopman-von Neumann formulation. One can not use Hilbert Space descriptions of Quantum Mechanics by using the Phase Space formulation of QM.
The Standard QM Canonical Commutation Relation is simply an axiom in standard QM. It is just given, with no explanation. You just had to accept it.

I always found that unsatisfactory.

There are at least 4 parts to it:

Where does the commutation ([ , ]) come from?
Where does the imaginary constant (i) come from?
Where does the Dirac:reduced-Planck constant (ћ) come from?
Where does the Kronecker Delta (δ^jk) come from?

See the next page for SR enlightenment...
The SR Metric is the source of “quantization”.
SRQM Diagram:

Canonical QM Commutation Relation
Derived from standard SR

Let \( f \) be an arbitrary SR function
\[ X[t] = x \text{f, } \partial X[t] = \partial X \]
\( X \), function or not, has no effect on \( f \)
\( \partial = \partial X \) is definitely an SR function:operator

\[ X[\partial X] = x \partial [X] \]
\[ \partial X \text{[]} = \partial X \text{f} + x \partial [X] \]
\[ \partial X - x \partial X \text{[]} = \partial X \text{f} \]

Recognize this as a commutation relation
\[ [ \partial , X ] = \partial X \] Manifestly Invariant

\[ = \partial X \text{[]} \]
\[ = (\partial/c, \cdot \nabla) \text{[(ct,x)]} \]
\[ = (\partial/c, -\partial/c, -\partial/c, -\partial/c) \text{[(ct,x,y,z)]} \]
\[ = \text{Diag} [1,1,1,1] = \text{Diag}[1,\delta_{ij}] \]
\[ = \eta^{\text{ij}} = \text{Minkowski Metric} \]

\[ [\partial, X] = \eta^{\text{ij}} \]
\[ [X^{\text{ij}}, X^{\text{kl}}] = i \eta^{\text{ik}} \]
\[ \text{Tensor form: true for all observers} \]
\[ \text{Also true from empirical constants (i),(h)} \]
\[ P^{\mu} = i \eta^{\text{ij}} \]
\[ \eta^{\text{ij}} (\text{E}/c,ct) = [E,t] = i \hbar \]

\[ [X, P] = i \hbar \delta^{\text{ij}} \]

\[ \text{Position: Momentum} \]
\[ \text{QM Commutation Relation} \]

\[ \text{4-Discipation} \]
\[ \Delta X = (c \Delta t, \Delta x) \]
\[ 4-Position \]
\[ X = (ct,x) \]

\[ 4-Velocity \]
\[ U = \gamma (c,u) \]

\[ 4-Gradient \]
\[ \partial = (\partial/c,-\nabla) \]

\[ \partial X = \partial X = 4 \text{ SpaceTime Dimension} \]

\[ \text{Lorentz} \]
\[ \partial x = \gamma (\partial c,-\nabla) \text{U} \]

\[ \text{Complex Plane-waves} \]
\[ K = i \theta \]

\[ i[\partial X] = [i \partial X] = [\mathbf{K}, X] = i \eta^{\text{ij}} \]

\[ \text{Non-Zero Commutation Relation via natural SR 4-Gradient} \]

\[ [i \partial X] = [i \partial X] = [\mathbf{K}, X] = i \eta^{\text{ij}} \]

\[ \text{Non-Zero Commutation Relation via SR 4-WaveVector} \]

\[ \text{Einstein de Broglie} \]
\[ P = \hbar \mathbf{K} \]

\[ \text{Non-Zero Commutation Relation via SR 4-Momentum} \]

\[ \{\mathbf{P} = \hbar \mathbf{K}\} \text{ and } \{\mathbf{K} = i \theta\} \text{ are empirical SR relations} \]

\[ \text{Trace}(T^{\mu \nu}) = \eta_{\mu \nu} T^{\mu \nu} = T^{\mu \nu} = T \]
\[ V V = V^{\nu} V^{\mu} = [(V^{\mu})^{2}] = (V^{\nu})^{2} = (V^{\nu})^{2} \]

\[ \text{Lorentz Scalar Invariant} \]

\[ \text{SR 4-Tensor} \]
\[ T^{\mu \nu} = V^{\nu} V^{\mu} \]

\[ \text{SR 4-Vector} \]
\[ (0,1)-T^{\nu} = (v^{0},v^{i}) \]

\[ \text{SR 4-Scalar} \]
\[ (0,0)-S = \text{Lorentz Scalar Invariant} \]
Standard QM Canonical Commutation Relation:

\[ [x^i, p^k] = i\hbar \delta^{ik} \]

As we have seen, this relation is generated from simple SR math.

\[
[\partial_x, X^\mu] = \partial_x [X^\mu] = (\partial_{\mu/c} - \nabla) \left[ (ct, x, y, z) \right] = \text{Diag}(1, -1, -1, -1) = \text{Diag}[1, -\delta^{ij}] = \eta_{\mu\nu} = \text{Minkowski Metric}
\]

\[
[\partial^\mu X^\nu] = \eta_{\mu\nu} : \text{This, Minkowski SpaceTime, is where the [ .. , .. ] commutation comes from.}
\]

\[
[P^\mu X^\nu] = i\hbar \eta_{\mu\nu} : \text{This is the more general 4D version, with the Standard QM version } [x^i, p^k] = i\hbar \delta^{ik} \text{ being just the spatial part.}
\]

The \{ i, \hbar \} are empirically found Lorentz Scalar Invariants and physical constants, relating these 4-Vectors.

One of the great misconceptions on modern physics is that since QM is about “tiny” things, that \textbf{ALL} things should be built up from it.

That paradigm of course works well for many things:

- Compounds are built-up from smaller molecules.
- Molecules are built-up from smaller elements: atoms.
- Elements: atoms are built-up from smaller protons, neutrons, and electrons.
- Protons and neutrons are built-up from smaller quarks.
- And all experiments to-date show that electrons and quarks appear to be point-like, with wave-type properties giving extent.

So, one can mistakenly think that “SpaceTime” must be made up of smaller “quantum” stuff as well. However, that is not what the math says. The “quantization” paradigm doesn’t apply to SpaceTime itself, just to properties of \textbf{events}.

All of the “quantum”-sized things above, electrons and quarks, are material things, \textbf{events}, which move around “within” SpaceTime.

Their “quantization” comes about from the properties of the math and metric of SR.

The math does *NOT* say that SpaceTime itself is “quantized”. It says that SR Minkowski SpaceTime is the source of “quantization”.

SRQM: A treatise of SR→QM by John B. Wilson (SciRealm@aol.com)
SRQM Study:  
Canonical Commutation Relation: Derived from SRQM

[Linear,Linear] Momentum QM Canonical Commutation Relation:

\[ [\partial^\mu, \partial^\nu] f = (\partial^\nu \partial^\mu - \partial^\mu \partial^\nu) f = 0^{\mu\nu} f \]

\[ [\partial^\mu, \partial^\nu] = (\partial^\nu \partial^\mu - \partial^\mu \partial^\nu) = 0^{\mu\nu} : \text{Partials commute, or in this case, the 4-Gradients commute} \]

\[ [i \hbar \partial^\mu, i \hbar \partial^\nu] = (i \hbar)^2 (\partial^\nu \partial^\mu - \partial^\mu \partial^\nu) = (i \hbar)^2 0^{\mu\nu} = 0^{\mu\nu} \]

\[ [P^\mu, P^\nu] = (i \hbar)^2 (\partial^\mu \partial^\nu - \partial^\nu \partial^\mu) = (i \hbar)^2 0^{\mu\nu} = 0^{\mu\nu} \]

\[ [P^\mu, P^\nu] = 0^{\mu\nu} : \text{see Poincaré Group: Algebra} \]

\{ i, h \} do not “create” the actual [ .. , .. ] commutation bracket, 4-Vectors do.  
\{ i, h \} are empirically found Scalar Invariants relating these 4-Vectors.  
\{ i, h \} are physical constants which give correct dimensional units.
SRQM Study: Canonical Commutation Relation: Derived from SRQM

[Angular, Linear] Momentum QM Canonical Commutation Relation:

Let $A^{uv} = X^i \partial^u - X^u \partial^i$ = Dimensionless version of 4-AngularMomentum, based on exterior product of 4-Position & 4-Gradient

$$iA^{uv} = X^i \partial^u - X^u \partial^i = X^P \partial^u - X^u \partial^P = M^{uv} = 4-AngularMomentum,$$

with $\{ i, h \}$ as Lorentz Scalar Invariants

$$[A^{uv}, \partial^sf] = A^{uv} \partial^sf - \partial^s A^{uv} f = A^{uv} \partial^sf - \partial^s A^{uv} [f] = - \partial^s [A^{uv}] f = - \partial^s (X^u \partial^v - X^v \partial^u) f = \partial^u (X^v \partial^i - X^i \partial^v) f = (\eta^u \partial^v - \eta^v \partial^u) f$$

$$[A^{uv}, \partial^sf] = (\eta^u \partial^v - \eta^v \partial^u) f$$

$$[A^{uv}, \partial^sf] = (\eta^u \partial^v - \eta^v \partial^u)$$

$$[iA^{uv}, P^s] = ih(\eta^{uv} P^i - \eta^{iu} P^v)$$

$$[M^{uv}, P^s] = ih(\eta^{uv} P^i - \eta^{iu} P^v) : see Poincaré Group: Algebra$$

$\{ i, h \}$ do not “create” the actual [ .. , .. ] commutation bracket, 4-Vectors do.

$\{ i, h \}$ are empirically found Scalar Invariants relating these 4-Vectors.

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[Angular, Linear] Momentum QM Canonical Commutation Relation:

\[ [M^{\alpha\nu}, P^\mu] = i\hbar (\eta^{\alpha\nu} P^\mu - \eta^{\alpha\mu} P^\nu) \] : see Poincaré Group: Algebra

ex.

\[ [M_{ab}^c, P_d^b] = i\hbar (\eta^{db} P_a^c + \eta^{da} P_b^c) \] : Get the regular 3-angular-momenta via \( L^a = \frac{1}{2} \varepsilon_{ijk} M^{jk} \)

\[ \frac{1}{2} \varepsilon_{abM} M_{ab}^c, P_d^b = \frac{1}{2} \varepsilon_{ab} \hbar (\eta^{db} P_a^c + \eta^{da} P_b^c) \]

\[ [L_c^a, P_{d}^b] = \frac{1}{2} \varepsilon_{ab} \hbar (\eta_{db} P_a^c + \eta_{da} P_b^c) : \text{chose first Minkowski to be non-zero, } d \rightarrow b \]

\[ [L_c^a, P_b^a] = \frac{1}{2} \varepsilon_{ab} \hbar (P_a^c + \{0\} P_b^c) \]

\[ [P_b^a, L_c^a] = \frac{1}{2} \varepsilon_{ab} \hbar (P_a^c) \]

Complex Plane-waves

\( \vec{K} = i \partial / \partial X^\mu \)

\( \vec{X} = (ct, r) \in \text{<Event>} \)

\( \vec{P} = (mc_\gamma, \vec{p}) = (E/c, \vec{p}) \)

\( \vec{E} \big/ \omega = \hbar \vec{K} \)

4-Vector SRQM Interpretation of QM

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SRQM Study: Canonical Commutation Relation: Derived from SRQM

[A angular,angular] Momentum QM Canonical Commutation Relation:

Let $A^{\mu v} = X^\mu \partial^v - X^v \partial^\mu = $ Dimensionless version of 4-AngularMomentum, based on exterior product of 4-Position & 4-Gradient

\[ i\hbar A^{\mu v} = X^\mu \partial^v A - X^v \partial^\mu A = X^\mu P^v - X^v P^\mu = M^{\mu v} = 4-\text{AngularMomentum}, \text{ with } \{ i, h \} \text{ as Lorentz Scalar Invariants} \]

\[
A^{\mu v}, X^{\sigma \rho} = \left( X^\mu \partial^v - X^v \partial^\mu \right) X^{\sigma \rho} - X^{\sigma \rho} \left( X^\mu \partial^v - X^v \partial^\mu \right) = X^\mu \partial^v X^{\sigma \rho} - X^v \partial^\mu X^{\sigma \rho} - X^{\sigma \rho} X^v \partial^\mu + X^{\sigma \rho} X^\mu \partial^v = X^\mu \eta^{\alpha \beta} \partial^\alpha - X^\alpha \eta^{\mu \beta} \partial^\alpha - X^{\alpha \beta} \eta^{\mu \nu} + X^{\alpha \beta} \eta^{\nu \mu} \]

\[
\{ A^{\mu v}, X^{\sigma \rho} \} = \left( \eta^{\alpha \beta} \partial^\alpha + \eta^{\nu \mu} \partial^\nu + \eta^{\rho \sigma} \partial^\rho \right) X^{\sigma \rho} - \left( \eta^{\alpha \beta} \partial^\alpha + \eta^{\nu \mu} \partial^\nu + \eta^{\rho \sigma} \partial^\rho \right) X^{\sigma \rho} = 0 \]

\[
\{ A^{\mu v}, A^{\alpha \beta} \} = \left( \eta^{\alpha \beta} \partial^\alpha + \eta^{\nu \mu} \partial^\nu + \eta^{\rho \sigma} \partial^\rho \right) A^{\mu v} - \left( \eta^{\alpha \beta} \partial^\alpha + \eta^{\nu \mu} \partial^\nu + \eta^{\rho \sigma} \partial^\rho \right) A^{\mu v} = 0 \]

\[
A^{\mu v}, ih A^{\alpha \beta} = \left( \eta^{\alpha \beta} \partial^\alpha + \eta^{\nu \mu} \partial^\nu + \eta^{\rho \sigma} \partial^\rho \right) A^{\mu v} - \left( \eta^{\alpha \beta} \partial^\alpha + \eta^{\nu \mu} \partial^\nu + \eta^{\rho \sigma} \partial^\rho \right) A^{\mu v} = 0 \]

\[
A^{\mu v}, M^{\alpha \beta} = \left( \eta^{\alpha \beta} \partial^\alpha + \eta^{\nu \mu} \partial^\nu + \eta^{\rho \sigma} \partial^\rho \right) M^{\alpha \beta} - \left( \eta^{\alpha \beta} \partial^\alpha + \eta^{\nu \mu} \partial^\nu + \eta^{\rho \sigma} \partial^\rho \right) M^{\alpha \beta} = 0 \]

\[
[ih A^{\mu v}, M^{\alpha \beta}] = i\hbar \left( \eta^{\alpha \beta} \partial^\alpha + \eta^{\nu \mu} \partial^\nu + \eta^{\rho \sigma} \partial^\rho \right) M^{\alpha \beta} - \left( \eta^{\alpha \beta} \partial^\alpha + \eta^{\nu \mu} \partial^\nu + \eta^{\rho \sigma} \partial^\rho \right) M^{\alpha \beta} = 0 \]

\[
[M^{\mu v}, M^{\alpha \beta}] = i\hbar \left( \eta^{\alpha \beta} \partial^\alpha + \eta^{\nu \mu} \partial^\nu + \eta^{\rho \sigma} \partial^\rho \right) M^{\mu v} - \left( \eta^{\alpha \beta} \partial^\alpha + \eta^{\nu \mu} \partial^\nu + \eta^{\rho \sigma} \partial^\rho \right) M^{\mu v} = 0 \]

\[\{ i, h \}\text{ do not “create” the actual } \{ \ldots, \ldots \}\text{ commutation bracket, 4-Vectors do.} \]

\[\{ i, h \}\text{ are empirically found Scalar Invariants relating these 4-Vectors.} \]

\[\{ i, h \}\text{ are physical constants which give correct dimensional units.} \]
Angular: Angular Momentum QM Canonical Commutation Relation:

\[ X^\mu P^\nu - X^\nu P^\mu = M^{\mu \nu} = 4\text{-AngularMomentum}, \text{ with } \{ i, h \} \text{ as Lorentz Scalar Invariants} \]

\[ [M^{\mu \nu}, M^{\rho \sigma}] = i\hbar (\eta^{\nu \rho} M^{\mu \sigma} + \eta^{\mu \rho} M^{\nu \sigma} + \eta^{\mu \sigma} M^{\nu \rho} + \eta^{\nu \sigma} M^{\mu \rho}) \]

\[ [M^{i m}, M^{m j}] = i\hbar (\eta^{k m} M^{i j} + \eta^{i m} M^{k j} + \eta^{i j} M^{k m} + \eta^{k j} M^{i m}) \text{: Look at just the spatial components} \]

Get the regular 3-angular-momentum via \( L^a = \frac{1}{2} \varepsilon^{ab} M^{b k} \)

\[ \{ \frac{1}{2} \varepsilon^{ab} M^{b k}, \frac{1}{2} \varepsilon^{mn} M^{m n} \} = i\hbar (\frac{1}{2} \varepsilon^{jk} \frac{1}{2} \varepsilon^{mn} \eta^{km} M^{nk} + \frac{1}{2} \varepsilon^{ak} \frac{1}{2} \varepsilon^{mn} \eta^{km} M^{ak} + \frac{1}{2} \varepsilon^{ak} \frac{1}{2} \varepsilon^{mn} \eta^{km} M^{ak}) \]

\[ [L^a, L^b] = \frac{1}{2} i\hbar (\varepsilon^{a k} \varepsilon^{h m} \eta^{k m} M^{h} + \varepsilon^{a k} \varepsilon^{m h} \eta^{k m} M^{k} + \varepsilon^{a k} \varepsilon^{k m} \eta^{h m} M^{k}) \]

ex. \[ [M^{i m}, M^{m j}] = i\hbar (\eta^{k m} M^{i j} + \eta^{i m} M^{k j} + \eta^{i j} M^{k m} + \eta^{k j} M^{i m}) \]

Quantum Interpretation of Maxwell Equations: Einstein de Broglie

\[ E/c = \hbar k \]

\[ 4\text{-AngularMomentum} \]

\[ M^{\mu \nu} = X^\mu P^\nu - X^\nu P^\mu = X^\mu X^\nu \]

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SRQM Study: 4-Position and 4-Gradient

SR: Lorentz Transform
\[ \partial[R^\nu] = \partial R^\nu/\partial R^\nu = \Lambda^\nu_v \]
\[ \Lambda^\mu_\alpha(\Lambda^{-1})^\nu_\beta = \eta^\mu_\alpha \Lambda^\nu_\beta = \eta^\nu_\beta \]
\[ \eta^\nu_\nu = \delta^\nu_\nu \]

SRQM: Tensor Zero Exterior Product
\[ \partial^\nu R = \partial R^\nu \]
Total Derivative Chain Rule
\[ \partial \cdot \partial = (\partial_t/c)^2 \cdot \nabla \cdot \nabla = (\partial_t/c)^2 \]

Invariant Interval
\[ R \cdot R = (ct)^2 - r \cdot r \]

Particle Physics Convention
\[ \eta^\mu_\mu = 1/\eta^\mu_\mu \]
Trace[T^\nu_\nu] = \eta^\nu_\nu T^\nu_\nu = T^\nu_\nu = T
\[ V \cdot V = (v^0)^2 - (v^1)^2 - (v^2)^2 - (v^3)^2 \]

SR: Minkowski Metric
\[ \partial[R] = \partial^\mu R^\mu = \eta^\mu_\nu \]

→Diag[1,-1,-1,-1]
= Diag[1,\delta^1_3]
\{in Cartesian form\}
"Particle Physics" Convention
\{\eta^\mu_\nu\} = 1/\{\eta^\mu_\nu\}
Tr[\eta^\mu_\nu] = 4
\[ \eta^\mu_\nu = \delta^\mu_\nu \]

SpaceTime
\[ \partial \cdot R = \partial R^\mu = 4 \]
Dimension

SRQM: Non-Zero Commutation
\[ [\partial, R] = [\partial^\mu, R^\nu] \]
=0

Invariant Calculus
\[ dR \cdot \partial = (dt, dr/c, -\nabla) \]
\[ dR^\eta \eta^\nu_\nu(\partial^\nu) = dR^\nu(\partial^\mu) = dR^\nu(\partial^\mu \partial R^\nu) = (dt \partial_t + dr \partial R^\nu) \]

Total Derivative Chain Rule
\[ = dt(\partial_t / c) + dx(\partial x / c) + dy(\partial y / c) + dz(\partial z / c) \]

SR 4-Tensor
(2,0)-Tensor T^\nu_\mu
(1,1)-Tensor T^\nu_\nu, or T^\nu_\nu
(0,2)-Tensor T^\mu_\nu

SR 4-Vector
(1,0)-Tensor V^\nu = V = (v^0, v^1, v^2, v^3)
(0,1)-Tensor V_\nu = (v_0, v_1, v_2, v_3)

SR 4-Scalar
(0,0)-Tensor S or S_0

Lorentz Scalar

SR → QM

Physics

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SRQM:

Heisenberg Uncertainty Principle: Viewed from SRQM

Heisenberg Uncertainty { $\sigma_A^2, \sigma_B^2$ } $\geq (1/2)|\langle [A,B]\rangle|$ arises from the non-commuting nature of certain operators.

The commutator is $[A,B] = AB-BA$, where $A$ & $B$ are functional "measurement" operators. The Operator Formalism arose naturally from our SR → QM path: $[\partial = -i\mathcal{K}]$.

The Generalized Uncertainty Relation: $\sigma_f^2 \sigma_g^2 \geq (\Delta F) * (\Delta G) \geq (1/2)|\langle [F,G]\rangle|$

The uncertainty relation is a very general mathematical property, which applies to both classical or quantum systems. From Wikipedia: Photon Polarization: "This is a purely mathematical result. No reference to a physical quantity or principle is required."

The Cauchy–Schwarz inequality asserts that (for all vectors $f$ and $g$ of an inner product space, with either real or complex numbers):

$$\sigma_f^2 \sigma_g^2 = |\langle f | g \rangle|^2 \geq \frac{1}{2}|\langle [F,G]\rangle|$$

But first, let's back up a bit; Using standard complex number math, we have:

$$z = a + ib$$
$$z^* = a - ib$$

$\text{Re}(z) = a = (z + z^*)/(2)$
$$\text{Im}(z) = b = (z - z^*)/(2i)$$

$$z^2 = |z|^2 = a^2 + b^2 = |\text{Re}(z)|^2 + |\text{Im}(z)|^2 = [(z + z^*)/(2)]^2 + [(z - z^*)/(2i)]^2$$

or

$$|z|^2 = [(z + z^*)/(2)]^2 + [(z - z^*)/(2i)]^2$$

Now, generically, based on the rules of a complex inner product space we can arbitrarily assign:

$$z = \langle f | g \rangle, z^* = \langle g | f \rangle$$

Which allows us to write:

$$|\langle f | g \rangle|^2 = |\langle f | f \rangle + \langle g | f \rangle + \langle f | g \rangle + \langle g | g \rangle|(2)|^2 + |\langle f | g \rangle - \langle g | f \rangle|/(2i)|^2$$

We can also note that:

$$|\langle f | g \rangle|^2 = |\langle f | \langle f \rangle \rangle + \langle f | \langle g \rangle \rangle + \langle g | \langle f \rangle \rangle + \langle g | \langle g \rangle \rangle|/(2)|^2$$

Thus,

$$|\langle f | g \rangle|^2 = |\langle f | \langle f \rangle \rangle + \langle f | \langle g \rangle \rangle + \langle g | \langle f \rangle \rangle + \langle g | \langle g \rangle \rangle|/(2)|^2$$

For Hermetian Operators...

$$F^* = +F, G^* = +G$$

For Anti-Hermetian (Skew-Hermetian) Operators...

$$F^* = -F, G^* = -G$$

Assuming that $F$ and $G$ are either both Hermetian, or both anti-Hermetian...

$$|\langle f | g \rangle|^2 = |\langle f | \langle f \rangle \rangle + \langle f | \langle g \rangle \rangle + \langle g | \langle f \rangle \rangle + \langle g | \langle g \rangle \rangle|/(2)|^2$$

$$|\langle f | g \rangle|^2 = |\langle f | \langle f \rangle \rangle + \langle f | \langle g \rangle \rangle + \langle g | \langle f \rangle \rangle + \langle g | \langle g \rangle \rangle|/(2)|^2$$

We can write this in commutator and anti-commutator notation...

$$|\langle f | g \rangle|^2 = |\langle F \rangle |\langle G \rangle \rangle + \langle f | \langle g \rangle \rangle + \langle g | \langle f \rangle \rangle + \langle f | \langle f \rangle \rangle|/(2)|^2$$

Due to the squares, the $(\pm)$'s go away, and we can also multiply the commutator by an $(i\mathcal{K})$

$$|\langle f | g \rangle|^2 = |\langle F \rangle |\langle G \rangle \rangle + \langle f | \langle g \rangle \rangle + \langle g | \langle f \rangle \rangle + \langle f | \langle f \rangle \rangle|/(2)|^2$$

The Cauchy–Schwarz inequality again...

$$\sigma_f^2 \sigma_g^2 = |\langle f | g \rangle|^2 = |\langle f | \langle f \rangle \rangle + \langle f | \langle g \rangle \rangle + \langle g | \langle f \rangle \rangle + \langle g | \langle g \rangle \rangle|/(2)|^2$$

Taking the root:

$$\sigma_f^2 \sigma_g^2 \geq (1/2)|\langle f | G \rangle|$$

Which is what we had for the generalized Uncertainty Relation.

*Note* This is not a QM axiom - This is just pure math. At this stage we already see the hints of commutation and anti-commutation. It is true generally, whether applying to a physical or purely mathematical situation.
Heisenberg Uncertainty Principle: Simultaneous vs Sequential

Heisenberg Uncertainty \( \{ \sigma_A^2 \sigma_B^2 \geq (1/2)|<[A,B]>| \} \) arises from the non-commuting nature of certain operators.

\[
\begin{align*}
[\partial^\mu, X^\nu] &= \partial[X] = \eta^{\mu\nu} = \text{Minkowski Metric} \\
[P^\mu, X^\nu] &= [i\hbar \partial^\mu, X^\nu] = i\hbar [\partial^\mu, X^\nu] = i\hbar \eta^{\mu\nu}
\end{align*}
\]

Consider the following:
Operator A acts on System \(|\Psi\rangle\) at SR Event A: \(A|\Psi\rangle \rightarrow |\Psi'\rangle\)
Operator B acts on System \(|\Psi'\rangle\) at SR Event B: \(B|\Psi'\rangle \rightarrow |\Psi''\rangle\)
or \(BA|\Psi\rangle = B|\Psi'\rangle = |\Psi''\rangle\)

If measurement Events A & B are space-like separated, then there are observers who can see \{A before B, A simultaneous with B, A after B\}, which of course does not match the quantum description of how Operators act on Kets

If Events A & B are time-like separated, then all observers will always see A before B. This does match how the operators act on Kets, and also matches how \(|\Psi\rangle\) would be evolving along its worldline, starting out as \(|\Psi\rangle\), getting hit with operator A at Event A to become \(|\Psi'\rangle\), then getting hit with operator B at Event B to become \(|\Psi''\rangle\).

The Uncertainty Relation here does NOT refer to simultaneous (space-like separated) measurements, it refers to sequential (time-like separated) measurements. This removes the need for ideas about the particles not having simultaneous properties. There are simply no “simultaneous measurements” of non-zero commuting properties on an individual system, a single worldline – they are sequential, and the first measurement places the system in such a state that the outcome of the second measurement will be altered wrt. if the order of the operations had been reversed.
The Pauli Exclusion Principle is a result of the empirical fact that nature uses identical (indistinguishable) particles, and this combined with the Spin-Statistics theorem from SR, leads to an exclusion principle for fermions (anti-symmetric, Fermi-Dirac statistics) and an aggregation principle for bosons (symmetric, Bose-Einstein statistics). The Spin-Statistics Theorem is related as well to the CPT Theorem.

For large numbers and/or mixed states these both tend to the Maxwell-Boltzmann statistics. In the \( \{kT \gg (\varepsilon_i - \mu)\} \) limit, Bose-Einstein reduces to Rayleigh-Jeans. The commutation relations here are based on space-like separation particle exchanges. Exchange operator \( P, P^2 = +1 \), Since two exchanges bring one back to the original state. \( P \) thus has two eigenvalues (\( \pm 1 \)) and two eigenvectors \( \{|\text{Symm}\>, |\text{AntiSymm}\>\} \)

\[
P|\text{Symm}\> = +|\text{Symm}\>
\]

\[
P|\text{AntiSymm}\> = -|\text{AntiSymm}\>
\]

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\]

<table>
<thead>
<tr>
<th>Spin-Symmetry</th>
<th>Particle Type</th>
<th>Quantum Statistics</th>
<th>Classical ( {kT \gg (\varepsilon_i - \mu)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>spin:(0,1,...,N) bosons symmetric</td>
<td>Indistinguishable, Commutation relation ( [a,b] = ab-ba = [-b,a] = \text{constant} ) if commutes</td>
<td>Bose-Einstein: ( n_i = g_i / [e^{(\varepsilon_i - \mu)/kT} -1 ] ) aggregation principle (condensation)</td>
<td>Rayleigh-Jeans: from ( e^x \sim (1 + x +...) ) ( n_i = g_i / [ (\varepsilon_i - \mu)/kT ] )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>↓ Limit as ( e^{(\varepsilon_i - \mu)/kT} \gg 1 ) ↓</td>
<td></td>
</tr>
<tr>
<td>Multi-particle Mixed</td>
<td>Distinguishable, or high temp, or low density</td>
<td>Maxwell-Boltzmann: ( n_i = g_i / [e^{(\varepsilon_i - \mu)/kT} +0 ] )</td>
<td>Maxwell-Boltzmann: ( n_i = g_i / [e^{(\varepsilon_i - \mu)/kT} ] )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>↑ Limit as ( e^{(\varepsilon_i - \mu)/kT} \gg 1 ) ↑</td>
<td></td>
</tr>
<tr>
<td>spin:(1/2,3/2,...,N/2) fermions anti-symmetric</td>
<td>Indistinguishable, Anti-commutation relation ( {a,b} = ab+ba = +{b,a} = \text{constant} ) if anti-commutes</td>
<td>Fermi-Dirac: ( n_i = g_i / [e^{(\varepsilon_i - \mu)/kT} +1 ] ) exclusion principle</td>
<td></td>
</tr>
</tbody>
</table>
Complex 4-vectors are simply 4-Vectors where the components may be complex-valued

\[
\mathbf{A} = A^\mu = (a^0, a^1, a^2, a^3) \rightarrow (a^t, a^\gamma, a^\nu, a^\varphi)
\]

\[
\mathbf{B} = B^\mu = (b^0, b^1, b^2, b^3) \rightarrow (b^t, b^\gamma, b^\nu, b^\varphi)
\]

Examples of 4-Vectors with complex components are the 4-Polarization and the 4-ProbabilityCurrentDensity

Minkowski Metric \(g_{\mu\nu} \rightarrow \eta_{\mu\nu} \rightarrow \text{Diag}[1,-1,-1,-1] = \text{Diag}[1,-I_{(3)}]\), which is the \{curvature\~0 limit = low-mass limit\} of the GR metric \(g_{\mu\nu}\).

Applying the Metric to raise or lower an index also applies a complex-conjugation *

Scalar Product = Lorentz Invariant \(\rightarrow\) Same value for all inertial observers

\[
\mathbf{A} \cdot \mathbf{B} = \eta_{\mu\nu} A^\mu B^\nu = A_\nu^* B^\nu = A^\mu B^\mu_\ast = (a^0 b^0 - a^\ast \cdot b) \text{ using the Einstein summation convention}
\]

This reverts to the usual rules for real components
However, it does imply that \(\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}\)
The Phase is a Lorentz Scalar Invariant – all observers must agree on its value. 
\[ K \times X = (\omega \times k) \cdot (x, t) = (\omega t - k x) = -\delta P \cdot \text{Phase of SR Wave} \]

We take the point of view of an observer operating on a particle at 4-Position \( X \), which has an initial 4-Velocity Vector \( K \). The 4-Position \( X \) of the particle, the operation's event, will not change: we are applying the various operations only to the particle's 4-Momentum \( K \).

Note that for matter particles \( K = (\omega/c) N \), where \( N \) is the “Unit”-Null 4-Vector and \( n \) is a unit-spatial 3-vector. All operations listed below work similarly on the Null 4-Vector.

Do a Time Reversal Operation: \( T \) 
The particle's temporal direction is reversed & complex-conjugated: \( T^* \times T^* = \gamma(-1, \beta)^* \)

Do a Parity Operation (Space Reflection): \( P \) 
Only the spatial directions are reversed: \( T^* = \gamma(-1, \beta)^* \)

Do a Charge Conjugation Operation: \( C \) 
Charge Conjugation actually changes all internal quantum #s: charge, lepton #, etc. Feynman showed this is the equivalent of a world-line reversal & complex-conjugation: \( T^* = \gamma(-1, \beta)^* \)

Pairwise combinations: 
- \( T P = T P^* = T C = \gamma(-1, \beta)^* \)
- \( T S = T P^* = T T = \gamma(-1, \beta)^* \)

A Tensor Study of Physical 4-Vectors

The SR 4-Tensor is a 12x12 Matrix: 
- \( (1, 0) \)-Tensor \( V^\nu = (v^\nu) \)
- \( (0, 1) \)-Tensor \( V_\nu = (v_\nu) \)

SR 4-Vector of Physical Quantities \( k \)

SR 4-Dimensional Phase Connection Lorentz Invariance

CPT Theorem

Phase Connection, Lorentz Invariance

SRQM: 4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
http://scirealm.org/SRQM.pdf

\[ \text{SRQM: CPT Theorem} \]

\[ \text{Phase Connection, Lorentz Invariance} \]

\[ \text{SRQM: 4-Vector SRQM Interpretation of QM} \]

\[ \text{SciRealm.org} \]

\[ \text{John B. Wilson} \]

\[ \text{http://scirealm.org/SRQM.pdf} \]
SRQM: CPT Theorem
(Charge) vs (Parity) vs (Time)

Classical SR Time-Reversal neglects spin and charge. SRQM includes these effects. Then one gets (CC), (PP), (TT), & (CPT) transforms all leading back to the Identity (I_{4(4)}).

SR 4-Tensor:
- (2,0)-Tensor $T^{\mu\nu}$
- (1,1)-Tensor $V^{\mu} = V = (v^0, v)$
- (0,2)-Tensor $T_{\mu\nu}$

SR 4-Vector:
- (1,0)-Tensor $V^{\mu} = V = (v^0, v)$
- (0,1)-Tensor $V_{\mu} = (v_0, v)$

SR 4-CoVector: One Form
- (0,1)-Tensor $V_{\mu} = (v_0, v)$
- Lorentz Scalar $S$ or S^0

SR 4-Scalar
- (0,0)-Tensor $S$ or S^0
- Lorentz Scalar

SCIENTIFIC REALM

SRQM Interpretation of QM

John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf
SRQM Transforms: Venn Diagram

Poincaré = Lorentz + Translations

(10) (6) (4)

Transformations
(# of independent parameters = # continuous symmetries = # Lie Dimensions)

Poincaré Transformation Group aka. Inhomogeneous Lorentz Transformation
Lie group of all affine isometries of SR:Minkowski Time-Space (preserve quadratic form \( \eta_{\mu\nu} \))
General Linear, Affine Transform \( X'_{\mu} = \Lambda_{\mu\nu}X^\nu + \Delta X^\mu \) with \( \text{Det}[\Lambda_{\mu\nu}] = \pm 1 \)
\( (6+4=10) \)

Lorentz Transform \( \Lambda^\mu_{\nu} \)
\( 4\)-Tensor \{mixed type\-(1,1)\}

\[ \begin{align*}
&\text{Discrete} & &\text{Continuous} \\
&\text{Time-reversal} & &\text{Rotation} \\
&\Lambda^t_{\nu} \rightarrow T^t_{\nu} & &\Lambda^\nu_{\mu} \rightarrow R^\nu_{\mu} \\
&0 & &0 \\
&\text{SpatialFlipCombos} & &\text{Identity I}_4(\nu) \\
&\Lambda^x_{\nu} \rightarrow \bar{\eta}^x_{\nu} = \bar{\eta}^y_{\nu} \rightarrow -\{x|y|z\} & &x:y | x:z | y:z \\
&0 & &0 \\
&\text{Parity-Inversion} & &\text{Boost} \\
&\Lambda^r_{\nu} \rightarrow P^r_{\nu} & &\Lambda^\nu_{\mu} \rightarrow B^\nu_{\mu} \\
&r \rightarrow -r & &tx | ty | tz \\
&0 & &0 \\
&\text{Charge-Conjugation} & &\text{Isotropy} \\
&\Lambda^q_{\nu} \rightarrow C^q_{\nu} & &\{\text{Charge}\} \{\text{Time}\} \\
&0 \text{ Symmetry} & &\text{same all directions} \\
&\text{Charge-Parity} & &\text{Isotropy} \\
&R \rightarrow -R^* & &\{\text{Charge}\} \{\text{Time}\} \\
&q \rightarrow -q & &\text{same all points} \\
&0 & &\text{Isotropy} \\
\end{align*} \]

Translation Transform \( \Delta X^\mu \)
\( 4\)-Vector \{mixed type\-(1,1)\}

\[ \begin{align*}
&\text{Discrete} & &\text{Continuous} \\
&\text{Temporal} & &\text{Spatial} \\
&\Delta X^\mu \rightarrow (c\Delta t,0) & &\Delta X^\mu \rightarrow (0,\Delta x) \\
&1 & &3 \\
&\text{4-Zero} & &\text{4-Linear} \\
&\Delta X^\mu \rightarrow (0,0) & &\Delta X^\mu \rightarrow (c,0) \\
&0 & &0 \\
\end{align*} \]

SR: Lorentz Transform
\[\Lambda^{a\mu} = \Lambda^{a\nu} \Lambda^{\nu\mu} \]
\( \Lambda^{a\mu} = \Lambda^{a\nu} \Lambda^{\nu\mu} \)
\( \Lambda^{a\mu} = \Lambda^{a\nu} \Lambda^{\nu\mu} \)
\( \Lambda^{a\mu} = \Lambda^{a\nu} \Lambda^{\nu\mu} \)
\( \Lambda^{a\mu} = \Lambda^{a\nu} \Lambda^{\nu\mu} \)

Rotations \( J = -\varepsilon_{abc}M^{bc}/2 \), Boosts \( K_i = M_0 \)
\[ (R \rightarrow -R^*) \text{ or } [ (t \rightarrow -t^*) & (r \rightarrow -r) ] \text{ imply } q \rightarrow -q \]
Feynman-Stueckelberg Interpretation
Amusingly, Inhomogeneous Lorentz adds homogeneity.
The Hermitian Generators that lead to translations and rotations via unitary operators in QM...

These all ultimately come from the Poincaré Invariance → Lorentz Invariance that is at the heart of SR and Minkowski Space.

Infinitesimal Unitary Transformation

\[ \hat{U}_t(\hat{G}) = I + i\epsilon \hat{G} \]

Finite Unitary Transformation

\[ \hat{U}_0(\hat{G}) = e^{i\alpha \hat{G}} \]

let \( \hat{G} = P/\hbar = K \)

let \( \alpha = \Delta x \)

\[ \hat{U}_{\Delta x}(P/\hbar)\Psi(X) = e^{i(\Delta x \cdot P/\hbar)}\Psi(X) = e^{i(-\Delta x \cdot \partial)}\Psi(X) = \Psi(X - \Delta x) \]

Time component: \( \hat{U}_{\Delta t}(P/\hbar)\Psi(ct) = e^{i(\Delta t E/\hbar)}\Psi(ct) = e^{i(-\Delta t \partial_t)}\Psi(ct) = \Psi(ct - c\Delta t) = c\Psi(t - \Delta t) \)

Space component: \( \hat{U}_{\Delta x}(p/\hbar)\Psi(x) = e^{i(\Delta x \cdot p/\hbar)}\Psi(x) = e^{i(\Delta x \cdot \nabla)}\Psi(x) = \Psi(x + \Delta x) \)

By Noether's Theorem, this leads to \( \partial \cdot K = 0 \)

We had already calculated

\( (\partial \cdot \partial)[K \cdot X] = ((\partial \cdot /c)^2 - \nabla \cdot \nabla)(\omega t - k \cdot x) = 0 \)

\( (\partial \cdot \partial)[K \cdot X] = \partial \cdot (\partial[K \cdot X]) = \partial \cdot K = 0 \)

Poincaré Invariance also gives the Casimir invariants of mass and spin, and ultimately leads to the spin-statistics theorem of RQM.
SRQM: A treatise of SR→QM by John B. Wilson (SciRealm@aol.com)

QM Correspondence Principle: Analogous to the GR and SR limits

Basically, the old school QM Correspondence Principle says that QM should give the same results as classical physics in the realm of large quantum systems, i.e. where macroscopic behavior overwhelms quantum effects. Perhaps a better way to state it is when the change of system by a single quantum has a negligible effect on the overall state.

There is a way to derive this limit, by using Hamilton-Jacobi Theory:

(i\hbar\partial_t)\Psi \sim [ V - (\hbar\nabla_T)^2/2m_0 ]\Psi : The Schrödinger NRQM Equation for a point particle (non-relativistic QM)

Examine solutions of form \( \Psi = \Psi_o e^{i\Phi} = \Psi_o e^{iS/\hbar} \), where S is the QM Action

\[ \partial_t [\Psi] = (i/\hbar)\Psi \partial_t [S] \quad \text{and} \quad \partial_x [\Psi] = (i/\hbar)\Psi \partial_x [S] \quad \text{and} \quad \nabla^2 [\Psi] = (i/\hbar)\Psi \nabla^2 [S] - (\Psi/\hbar^2)(\nabla [S])^2 \]

\[ (i\hbar)(i/\hbar)\Psi \partial_t [S] = V\Psi - (\hbar^2/2m_0)((i\hbar/2m_0)\Psi \nabla^2 [S] - (\Psi/2m_0)(\nabla [S])^2) \]

\[ (i)(i)\Psi \partial_t [S] = V\Psi - ((i\hbar/2m_0)\Psi \nabla^2 [S] - (\Psi/2m_0)(\nabla [S])^2) \]

\[ \partial_t [S] = -V + (i\hbar/2m_0)\nabla^2 [S] - (1/2m_0)(\nabla [S])^2 \]

\[ \partial_t [S] + [V+(1/2m_0)(\nabla [S])^2] = (i\hbar/2m_0)\nabla^2 [S] : \text{Quantum Single Particle Hamilton-Jacobi} \]

\[ \partial_t [S] + [V+(1/2m_0)(\nabla [S])^2] = 0 : \text{Classical Single Particle Hamilton-Jacobi} \]

Thus, the classical limiting case is:

\[ \nabla^2 [\Phi] \ll (\nabla [\Phi])^2 \]

\[ \hbar \nabla^2 [S] \ll (\nabla [S])^2 \]

\[ \hbar \nabla \cdot \mathbf{p} \ll (\mathbf{p} \cdot \mathbf{p}) \quad \nabla \cdot \mathbf{k} \ll (\mathbf{k} \cdot \mathbf{k}) \]

\[ (p_\Lambda) \nabla \cdot \mathbf{p} \ll (\mathbf{p} \cdot \mathbf{p}) \]
QM Correspondence Principle:
Analogous to the GR and SR limits

\[ \partial_t [S] + [V + (1/2m_o)(\nabla[S])^2] = (i\hbar/2m_o)(\nabla[S])^2 \]

\[ \partial_t [S] + [V + (1/2m_o)(\nabla[S])^2] = 0 \]

Thus, the quantum→classical limiting-case is:
{all equivalent representations}

\[ \hbar^2 [S_{action}] << (\nabla[S_{action}])^2 \]

\[ \nabla^2 [\Phi_{phase}] << (\nabla[\Phi_{phase}])^2 \]

\[ \hbar \nabla \cdot [p \cdot p] << (p \cdot p) \]

\[ \hbar \nabla \cdot [k \cdot k] << (k \cdot k) \]

with

\[ P = (E/c, p) = -\partial[S_{action}] = -(\partial/c, \nabla)[S_{action}] = (-\partial/c, \nabla) \]

\[ K = (\omega/c, k) = -\partial[\Phi_{phase}] = -(\partial/c, \nabla)[\Phi_{phase}] = (-\partial/c, \nabla) \]

It is analogous to GR → SR in limit of low curvature (low mass), or SR → CM in limit of low velocity \( |v|<<c \).

It still applies, but is now understood as the same type of limiting-case as these others.

*Note* The commonly seen form of \( (c \rightarrow \infty, \hbar \rightarrow 0) \) as limits are incorrect!

\( c \) and \( \hbar \) are universal constants – they never change.

If \( c \rightarrow \infty \), then photons (light-waves) would have infinite energy \( E = pc \).

If \( \hbar \rightarrow 0 \), then photons (light-waves) would have zero energy \( E = \hbar \omega \).

Always better to write the SR Classical limit as \( |v|<<c \), the QM Classical limit as \( \nabla^2[\Phi_{phase}] << (\nabla[\Phi_{phase}])^2 \)

Again, it is more natural to find a limiting-case of a more general system than to try to unite two separate theories which may or may not ultimately be compatible. From logic, there is always the possibility to have a paradox result from combination of arbitrary axioms, whereas deductions from a single true axiom will always give true results.
SRQM: 4-Vector Quantum Probability

Conservation of Probability Density

Consider the following purely mathematical argument (based on Green's Vector Identity):
\[ \partial \cdot (f \partial g - \partial f \cdot g) = f \partial \cdot \partial g - \partial \cdot \partial (f \cdot g) \]
with \( f \) and \( g \) as SR Lorentz Scalar functions.

Proof:
\[ \partial \cdot (f \partial g - \partial f \cdot g) = (f \partial \cdot \partial g) + (\partial f \cdot \partial g) - (\partial \cdot \partial (f \cdot g)) \]

We can also multiply this by a Lorentz Invariant Scalar Constant \( s \):
\[ s (f \partial \cdot \partial g - \partial \cdot \partial (f \cdot g)) = s (f \partial \cdot \partial g) - s \partial \cdot \partial (f \cdot g) \]

Now, on to the physics... Start with the Klein-Gordon Eqn.
\[ \partial \cdot \partial g + \left( \frac{m_0 c}{\hbar} \right)^2 g = 0 \]
Let it act on SR Lorentz Invariant function \( g \):
\[ \partial \cdot \partial g + \left( \frac{m_0 c}{\hbar} \right)^2 g = 0 \]
Then pre-multiply by \( f \):
\[ [f] \partial \cdot \partial g + \left[ f \left( \frac{m_0 c}{\hbar} \right)^2 \right] [g] = [f] 0 [g] \]
\[ [f] \partial \cdot \partial g + \left( \frac{m_0 c}{\hbar} \right)^2 [f][g] = 0 \]
Now, subtract the two equations:
\[ [f] \partial \cdot \partial g - \partial \cdot \partial (f \cdot g) = 0 \]
As we noted from the mathematical Green's Vector identity at the start...
\[ [f] \partial \cdot \partial g - \partial \cdot \partial (f \cdot g) = \partial (f \partial g - \partial \cdot f \cdot g) = 0 \]
Therefore,
\[ s \partial (f \partial g - \partial \cdot f \cdot g) = 0 \]
\[ \partial \cdot s \partial (f \partial g - \partial \cdot f \cdot g) = 0 \]
Thus, there is a conserved current 4-Vector, \( J_{\text{prob}} = s (f \partial g - \partial \cdot f \cdot g) \), for which \( \partial \cdot J_{\text{prob}} = 0 \),
and which also solves the Klein-Gordon equation.

Let's choose as before (\( \partial = -i \mathbf{K} \)) with a plane wave function \( f = ae^{-i(K \cdot X)} = \psi \),
and choose \( g = f^* = ae^{i(K \cdot X)} = \psi^* \) as its complex conjugate.

At this point, I am going to choose \( s = (i\hbar/2m_0) \), which is Lorentz Scalar Invariant, in order to make the probability have dimensionless units and be normalized to unity in the rest case.
The reason for \( s = \frac{i\hbar}{2m} \) with 4-Divergence of Probability \{ \partial J = 0 \} by construction via Green’s Vector Identity and the Klein-Gordon RQM Eqn.

You immediately see where the \((i\hbar/2m_0)\) factor comes from.

The \( \rho_{\text{prob}} \) is then a function of the \( \psi \)’s divided by 2.

\[ \delta \cdot (f \partial [g] - \partial [f] g) = f \partial \partial [g] - \partial \partial [f] g \] Green’s Vector Identity

\[ \delta \partial \partial = \partial \partial \] 4-Vector SRQM Interpretation of QM

\[ \mathbf{J} = \rho \mathbf{c} \] 4-CurrentDensity

Examine the temporal component, the Relativistic Probability Density

\[ \rho_{\text{prob}} = \frac{(i\hbar/2m_0)^2}{\gamma} \mathbf{c} \rho \]

Assume wave solution in following general form:

\[ \{ \psi = A f \{ k \} e^{(i\omega t)} \} \]

Then

\[ \{ \delta \partial \partial \psi = (i\omega)A f \{ k \} e^{(i\omega t)} \} \]

Finally, multiply by charge \((q)\) to get standard SR EM

\[ 4-\text{CurrentDensity} = 4-\text{ChargeFlux} = J = (c\rho, j) = qJ_{\text{prob}} = q(c\rho_{\text{prob}}, j_{\text{prob}}) \]

\[ \mathbf{P} = (m_0, \mathbf{c}) = (E/c, \mathbf{p}) \] 4-Momentum
4-Vector Quantum Probability

4-Probability Flux, Klein-Gordon RQM Eqn with Minimal Coupling

4-Probability Current Density, a.k.a. 4-Probability Flux
\[ J_{\text{prob}} = (c \rho_{\text{prob}}, J_{\text{prob}}) = \left( \frac{\hbar}{2m}, \frac{\hbar}{2m} \right) ( \psi^*, \psi^* \psi ) + \left( \frac{q}{m}, \frac{q}{m} \right) ( \psi^* \psi ) \]

with 4-Divergence of Probability \[ \partial \cdot J_{\text{prob}} = 0 \] by construction via Green's Vector Identity and the Klein-Gordon RQM Eqn.

If we include minimal coupling:
\[ J_{\text{prob}} = (c \rho_{\text{prob}}, J_{\text{prob}}) = \left( \frac{\hbar}{2m}, \frac{\hbar}{2m} \right) ( \psi^*, \psi^* \psi ) + \left( \frac{q}{m}, \frac{q}{m} \right) ( \psi^* \psi ) \]

Follow past (q) factor to get to \( A \).

Start at \( A \).

Follow past Born Rule (\( \psi^* \psi \))

Now have the additional factor: \( \left( \frac{q}{m}, \frac{q}{m} \right) ( \psi^* \psi ) \)

Minimal Coupling allows passage back to \( P \) with no factors

Follow back past (1/m) to get to \( U \)

Follow past Born Rule (\( \psi^* \psi \))

Rest Number Density

4-Number Flux
\[ N = (nc, n) = (c, 0) \]

4-Probability Flux
\[ J_{\text{prob}} = (c \rho_{\text{prob}}, J_{\text{prob}}) = \left( \frac{\hbar}{2m}, \frac{\hbar}{2m} \right) ( \psi^*, \psi^* \psi ) + \left( \frac{q}{m}, \frac{q}{m} \right) ( \psi^* \psi ) \]

Complex
Complex Plane-waves
\[ K = (\omega/c, k) \]

An alternate way would be to take \( A \) to \( U \) via the direct route:
\[ + \left( \frac{c^2}{2m}, \frac{c^2}{2m} \right) ( \psi^* \psi ) \]

which would lead to a term like
\[ \rho_{\text{prob}} \to (\gamma)(\psi^* \psi) + (\gamma)(\phi/\phi_{\text{p}})(\psi^* \psi) = (\gamma)[1 + \frac{\phi_{\text{p}}}{\phi_{\text{p}}}] (\psi^* \psi) \]

with potential due to particle \( (\phi_{\text{p}}) \) typically much less than the potential due to the whole field \( (\phi_{\text{f}}) \)

\( (\phi_{\text{p}}) \ll (\phi_{\text{f}}) \)

Minimal Coupling

4-Momentum Field
\[ P_f = (E/c, P) \]

4-EM Potential Momentum
\[ Q = (\mu/c, q) = qA \]

4-Vector SRQM Interpretation of QM

SciRealm.org
John B. Wilson
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http://scirealm.org/SRQM.pdf
4-Vector Quantum Probability

Newtonian Limit

4-Probability Current Density: \( J_{\text{prob}} = (c p_{\text{prob}}, j_{\text{prob}}) = (i \hbar/2m_o)(\psi^* \partial[\psi]-\partial[\psi^*] \psi) + (q/m_o)(\psi^* \psi) \)

Examine the temporal component:
\( \rho_{\text{prob}} = (i \hbar/2m_o c^2)(\psi^* \partial_0[\psi]-\partial_0[\psi^*] \psi) + (q/m_o)(\psi^* \psi)(\phi/c^2) \)
\( \rho_{\text{prob}} \rightarrow (\gamma)(\psi^* \psi) + (\gamma)(q\phi_o/m_o c^2)(\psi^* \psi) = (\gamma)[1 + q\phi_o/E_o](\psi^* \psi) \)

Typically, the particle EM potential energy (q\( \phi_o \)) is much less than the particle rest energy (\( E_o \)), else it could generate new particles. So, take (q\( \phi_o \ll E_o \)), which gives the EM factor (q\( \phi_o/E_o \)) \( \ll 0 \)

Now, taking the low-velocity limit (\( \gamma \rightarrow 1 \)), \( \rho_{\text{prob}} = \gamma[1 + 0](\psi^* \psi) \), \( \rho_{\text{prob}} \rightarrow (\psi^* \psi) = (\rho_{\text{prob}}) \) for \( |v| \ll c \)

The Standard Born Probability Interpretation, \( \psi^* \psi = (\rho_{\text{prob}}) \), only applies in the low-potential-energy & low-velocity limit

This is why the \{non-positive-definite\} probabilities and \{|probabilities| > 1\} in the RQM Klein-Gordon equation gave physicists fits, and is the reason why one must regard the probabilities as charge conservation instead.

The original definition from SR is Continuity of Worldlines, \( \partial \cdot J_{\text{prob}} = 0 \), for which all is good and well in the RQM version.

The definition says there are no external sources or sinks of probability = conservation of probability.

The Born idea that (\( \rho_{\text{prob}} \rightarrow \sum[(\psi^* \psi)] = 1 \)) is just the Low-Velocity QM limit.

Only the non-EM rest version (\( \rho_{\text{prob}} = \sum[(\psi^* \psi)] = 1 \)) is true.

It is not a fundamental axiom, it is an emergent property which is valid only in the NRQM limit

We now multiply by charge (q) to instead get a
4-"Charge" Current Density: \( J = (c p, j) = qJ_{\text{prob}} = q(c p_{\text{prob}}, j_{\text{prob}}) \), which is the standard SR EM 4-Current Density
**SRQM 4-Vector Study: The QM Compton Effect Compton Scattering**

**Compton Scattering Derivation: Compton Effect**

\[ \mathbf{P} \cdot \mathbf{P} = (m_c)^2 \quad \text{generally} \rightarrow 0 \quad \text{for photons} \quad (m_e = 0) \]

\[ P_{\text{phot}} + P_{\text{mass}} = h^2 k \cdot k_0 = (h^2 \omega \omega' c' / c)^2 (1 - \hat{n} \cdot \hat{n}_0) = (h^2 \omega \omega' c' / c)^2 (1 - \cos[\phi]) \]

\[ P_{\text{phot}} + P_{\text{mass}} = h^2 k \cdot k_0 = (h^2 \omega \omega' c' / c)^2 \quad \text{rearrange} \]

\[ (P_{\text{phot}} + P_{\text{mass}} - P'_{\text{phot}})^2 = (P'_{\text{mass}})^2 : \text{square to get scalars} \]

\[ (P_{\text{phot}} + P_{\text{mass}} - P'_{\text{phot}})^2 = (P'_{\text{mass}})^2 \]

\[ (0 + 2 \mathbf{P}_{\text{phot}} \cdot \mathbf{P}_{\text{mass}} - 2 \mathbf{P}_{\text{phot}} \cdot \mathbf{P}_{\text{phot}} + \mathbf{P}_{\text{mass}} \cdot \mathbf{P}_{\text{mass}} + 2 \mathbf{P}_{\text{mass}} \cdot \mathbf{P}_{\text{phot}} + \mathbf{P}'_{\text{phot}} \cdot \mathbf{P}'_{\text{phot}}) = (\mathbf{m}_c)^2 \]

\[ \Delta \lambda = (\lambda' - \lambda) = (h/m_c) (1 - \cos[\phi]) = \lambda_c (1 - \cos[\phi]) \]

The Compton Effect: Compton Scattering

with \[ \lambda_c = \lambda_0 / 2 \pi = (h/m_c) = \text{Reduced Compton Wavelength} \]

\[ \lambda_c = (h/m_c) = \text{Compton Wavelength (not a rest-wavelength, but the wavelength of a photon with the energy equivalent to a massive particle of rest-mass m_0)} \]

Calculates the wavelength shift of a photon scattering from an electron (ignoring spin)

Proves that light does not have a “wave-only” description, photon 4-Momentum required

\[ E/\omega = \gamma E_0/\omega_0 = E_0/\omega_0 = h \]

\[ K_{\text{photon}} = (\omega/c)(1, \hat{n}) = \text{null} \quad \{\omega \lambda = \lambda \lambda = c\} \quad \text{for photons} \]
**SRQM 4-Vector Study:**

The QM Aharonov-Bohm Effect

**QM Potential**

\[ \Delta \Phi_{pot} = -\frac{q}{\hbar} \int_{path} A \cdot dX \]

**Aharonov-Bohm Effect**

The EM 4-Vector Potential gives the Aharonov-Bohm Effect.

\[ \Phi_{pot} = -\frac{q}{\hbar} A \cdot X = -K_{pot} \cdot X \]

or taking the differential...

\[ d\Phi_{pot} = -(q/\hbar)A \cdot dX \]

over a path...

\[ \Delta \Phi_{pot} = \int_{path} d\Phi_{pot} \]

\[ \Delta \Phi_{pot} = -(q/\hbar) \int_{path} A \cdot dX \]

\[ \Delta \Phi_{pot} = -(q/\hbar) \int_{path} [\vec{A}(\vec{V})] \cdot d\vec{X} \]

\[ \Delta \Phi_{pot} = -(q/\hbar) \int_{path} (\vec{A} \cdot \vec{dX}) \]

Note that both the Electric and Magnetic effects come out by using the 4-Vector notation.

**Electric AB effect:** \( \Delta \Phi_{pot, \text{Elec}} = -(q/\hbar) \int_{path} (\vec{A} \cdot d\vec{X}) \)

**Magnetic AB effect:** \( \Delta \Phi_{pot, \text{Mag}} = +(q/\hbar) \int_{path} (\vec{A} \cdot d\vec{X}) \)

Proves that the 4-Vector Potential \( A \) is more fundamental than \( \vec{E} \) and \( \vec{B} \) fields, which are just components of the Faraday EM Tensor.
Josephson Effect

The EM 4-VectorPotential gives the Aharonov-Bohm Effect. Phase $\Phi_{pot} = -(\hbar/q)A \cdot X = -K_{pot} \cdot X$

Rearrange the equation a bit:

$$-(\hbar/q)\Phi_{pot} = A \cdot \Delta X$$

$$A \cdot \Delta X = -(\hbar/q)\Phi_{pot}$$

$$d/dt[A \cdot \Delta X] = d/dt[-(\hbar/q)\Delta \Phi_{pot}] = d/dt[A \cdot \Delta X] + A \cdot d/dt[\Delta X] = d/dt[A \cdot \Delta X] + A \cdot U$$

Assume that ($d/dt[A \cdot \Delta X] \sim 0$)

$$[A \cdot U] = d/dt[-(\hbar/q)\Delta \Phi_{pot}]$$

Which explains Josephson Effect criteria:

$$\Delta X \sim 0$: small gap

$$d/dt[A \cdot \Delta X] \sim 0$: “critical current” & no voltage

$$d/dt[A \cdot \Delta X] \sim \text{orthogonal: ??}$$

$$A = (\hbar/q)K; K = (\omega/c,k) = (\hbar/q)A = (\hbar/q)(\phi/c,a)$$

Take the temporal part:

EM ScalarPotential $\phi = -(\hbar/q)(\partial/\partial t)[\Delta \Phi_{pot}]$; \hspace{1cm} $\omega = (q/\hbar)\phi$

If the charge (q) is a Cooper-electron-pair: \hspace{1cm} $q = -2e$

Voltage $V(t) = \phi(t) = (\hbar/2e)(\partial/\partial t)[\Delta \Phi_{pot}]$; \hspace{1cm} AngFreq $\omega = -2eV/h$

This is the superconducting phase evolution equation of the Josephson Effect

$(\hbar/2e)$ is defined to be the Magnetic Flux Quantum $\Phi_0$.

---

**SRQM 4-Vector Study:**

**The QM Josephson Junction Effect = SuperCurrent**

**EM 4-VectorPotential** $A = -(\hbar/q)\partial[\Delta \Phi_{pot}]$
SRQM Symmetries:
Hamilton-Jacobi vs Relativistic Action
Josephson vs Aharonov-Bohm
Differential (4-Vector) vs Integral (4-Scalar)

Differential Formats : 4-Vectors : HJ

SR Hamilton-Jacobi Equation

\[ P_T = P + qA = P + Q = -\partial[\Delta S_{act}] = -\partial[h\Delta \Phi_{phase}] \]

\[ = -\partial[h(\Delta \Phi_{phase,dyn} + \Delta \Phi_{phase,potential})] \]

4-Momentum (free part)

\[ P = -\partial[\Delta S_{act,dynamic}] \]

\[ -\partial[h\Delta \Phi_{phase,dynamic}] \]

4-WaveVector

\[ K = -\partial[\Delta S_{act,dyn}/h] \]

\[ -\partial[\Delta \Phi_{phase,dynamic}] \]

4-PotentialMomentum

\[ Q = qA = -\partial[\Delta S_{act,potential}] \]

\[ -\partial[h\Delta \Phi_{phase,potential}] \]

Josephson Junction Relation

\[ A = -(h/q)\partial[\Delta \Phi_{potential}] \]

\[ = -(1/q)\partial[\Delta S_{act,pot}]/h \]

\[ = Q/q \]

Technically, the standard Josephson Junction uses just the temporal part \( A = (\varphi/c, a) \) & Cooper-pair-electrons \( q = -2e \) giving \( V(t) = \varphi = (h/2e)\partial[\Delta \Phi_{pot}]. \)

There should be a spatial part as well.

Integral Formats : 4-Scalars : Action

SR Action Equation

\[ \Delta S_{act} = -\int_\text{path}(P_T \cdot dX) = -\int_\text{path}(P + qA) \cdot dX = -\int_\text{path}(P + Q) \cdot dX \]

\[ = h \Delta \Phi_{phase} = h(\Delta \Phi_{phase,dyn} + \Delta \Phi_{phase,potential}) \]

Action (free part)

\[ \Delta S_{act,dynamic} = h \Delta \Phi_{phase,dynamic} = -\int_\text{path}(P) \cdot dX \]

\[ \Delta \Phi_{phase,dyn} = \Delta S_{act,dyn}/h \]

\[ = -\int_\text{path}(K) \cdot dX \]

Action (potential part)

\[ \Delta S_{act,pot} = h \Delta \Phi_{phase,potential} = -\int_\text{path}(qA) \cdot dX = -\int_\text{path}(Q) \cdot dX \]

\[ = \Delta S_{act,pot}/h \]

Aharonov-Bohm Relation

\[ \Delta \Phi_{potential} = -(q/h)\int_\text{path} A \cdot dX \]

\[ = -(1/h)\int_\text{path} Q \cdot dX \]

\[ = \Delta S_{act,pot}/h \]
SRQM Symmetries:
Schrödinger Relations
Cyclic Imaginary Time ↔ Inv Temp

4-Momentum
\[ P = \frac{mc}{c^2} \] = \[ (E/c, p) = (E_{\text{cm}}, \mu) \]

Einstein-de Broglie
\[ P = hK \]

4-Position
\[ \mathbf{R} = \mathbf{r} = (c_t, r) \]

Covariant Wick Rotation
\[ \mathbf{R} = -i\mathbf{R}_m \]

4-ImaginaryPosition
\[ \mathbf{R}_m = \mathbf{R}_{im} = i(c_t, r) \]

Covariant Euclidean Time ~ Inv Temp
\[ \mathbf{R}_{em} = h\Theta \]

4-Gradient
\[ \partial = \partial_{R} = \partial R_{\mu} = \partial = (\partial/c, -\nabla) \]
\[ \rightarrow (\partial/c, -\partial_\mu - \partial_\nu - \partial_\gamma - \partial_\delta) \]
\[ = (\partial/c, \partial_\mu, -\partial_\nu, -\partial_\gamma, -\partial_\delta) \]

Complex Plane-Waves
\[ K = i\mathbf{a} \]

1/\hbar

\[ h \]

-\[ i \]

-\[ 1/i \]

\[ 1 \]

-\[ i = 1/i \]

4-Vector SRQM Interpretation of QM

Schrödinger Relations:
\[ \mathbf{P} = ih\Theta \rightarrow \{ \mathbf{E} = \hbar\omega : \mathbf{p} = \hbar\mathbf{k} \} \]

Wick Rotation:
\[ \mathbf{R} = -i\mathbf{R}_m \rightarrow \{ \mathbf{t} = -i\mathbf{r} : \mathbf{r} = -i(\mathbf{r}) \} \]

Cyclic Temp:
\[ \mathbf{R}_{em} = \hbar\Theta \rightarrow \{ \mathbf{t} = \hbar\mathbf{k}_B \mathbf{T} : \mathbf{r} = \hbar\mathbf{u}/\hbar\mathbf{k}_B \mathbf{T} \} \]

Time Temp:
\[ \mathbf{R} = -ih\Theta \rightarrow \{ \mathbf{t} = -i\hbar\mathbf{k}_B \mathbf{T} : \mathbf{r} = -i\hbar\mathbf{u}/\hbar\mathbf{k}_B \mathbf{T} \} \]

Boltzmann Distribution
\[ \mathbf{P} \cdot \mathbf{\Theta} = (E/c, \mathbf{p} \cdot \mathbf{c}_B \mathbf{T} \cdot \mathbf{\Theta}) = (E/k_B \mathbf{T} \cdot \mathbf{\Theta}) \]

Trace[T^\mu_\nu = 0]

Identifying the temperature here is relativistically direction-specific, unlike in the classical use of temperature.
SRQM Symmetries:
Relativistic Euler-Lagrange Equation
The Easy Derivation \((U=(d/d\tau)R)\rightarrow(\partial R=(d/d\tau)\partial u)\)

Note Similarity:
4-Velocity is ProperTime Derivative of 4-Position \(U = (d/d\tau)R\) \([m/s] = [1/s]^*[m]\)
Relativistic Euler-Lagrange Eqn \(\partial u = (d/d\tau)\partial \mu\) \([1/m] = [1/s]^*[s/m]\)
The differential form just inverts the dimensional units, so the placement of the \(R\) and \(U\) switch.

That is it: so simple! Much, much easier than how I was taught in Grad School.

To complete the process and create the Equations of Motion, one just applies the base form to a Lagrangian.

This can be: a classical Lagrangian a relativistic Lagrangian a Lorentz scalar Lagrangian a quantum Lagrangian

Trace\([T^\mu_\nu] = \eta^\mu_\nu\cdot T^\mu_\nu = T^\mu_\nu \cdot \eta_{\mu\nu}\) = Lorentz Scalar Invariant

\(P_T = -\partial[L] = -\partial_\mu[L] = -\partial_0[-(P_T\cdot U)] = P_T\)
\((E/c,P_r) = -(\partial_\mu,\partial_\nu)[L] = -(\partial_0,\partial_\mu)[L]\)
\(P_T = \partial_\mu[L] = (\partial \mu U)[L]\)

\(\partial[\gamma U] = \eta^{\mu\nu}\rightarrow\text{Diag}[1,-1,-1,1]\)
Relativistic Particle Dynamics Eqn (4-Vector)
\(\partial U = (d/d\tau)\partial R\)
\(\partial u = (d/d\tau)\partial \mu\)

\(\partial R = (d/d\tau)\partial u\)
\(\partial [L] = (d/d\tau)\partial u\)

4-Vector { Index-raised-from One-Form } = 4D (1,0)-Tensor

\(\partial_\mu \cdot \partial_\nu = \eta^{\mu\nu} = 4\cdot\text{Natural 4-Vector}\)

\(\rho_\mu = \partial_\mu \cdot \partial_\nu\)
\(\partial [L] = (d/d\tau)\partial u\)

\(\partial R = (d/d\tau)\partial_\mu\)

\(\partial R = (d/d\tau)\partial_\mu\)
\(\partial [L] = (d/d\tau)\partial u\)

\(\partial R = (d/d\tau)\partial_\mu\)
\(\partial [L] = (d/d\tau)\partial u\)

\(\partial R = (d/d\tau)\partial_\mu\)
\(\partial [L] = (d/d\tau)\partial u\)
SRQM Symmetries:

Lorentz Transform Connection Map – Trace Identification
CPT, Big-Bang, [Matter↔AntiMatter], Arrow(s)-of-Time

All Lorentz Transforms have Tensor Invariants: Determinant = ±1 and InnerProduct = 4. However, one can use the Tensor Invariant Trace to Identify CPT Symmetry & AntiMatter

<table>
<thead>
<tr>
<th>Transform Type</th>
<th>Trace Values</th>
<th>Determinant Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>NM-Rotate</td>
<td>{0...+4}</td>
<td>+4</td>
</tr>
<tr>
<td>NM-Identity</td>
<td></td>
<td>+4</td>
</tr>
<tr>
<td>NM-Boost</td>
<td>{+4...+∞}</td>
<td></td>
</tr>
<tr>
<td>AM-Rotate</td>
<td>{0...-4}</td>
<td>-4</td>
</tr>
<tr>
<td>AM-Identity</td>
<td></td>
<td>-4</td>
</tr>
<tr>
<td>AM-Boost</td>
<td>{-4...-∞}</td>
<td></td>
</tr>
<tr>
<td>Trace : Determinant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tr = +4 : Det = +1 Proper</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tr = +2 : Det = -1 Improper</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tr = 0 : Det = +1 Proper</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tr = -2 : Det = -1 Improper</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tr = -4 : Det = +1 Proper</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Two interesting properties of (1,1)-Tensors, of which the Lorentz Transform is an example:

- Trace = Sum (Σ) of EigenValues : Determinant = Product (Π) of EigenValues
- As 4D Tensors, each Lorentz Transform has 4 EigenValues (EV's).
- Create an Anti-Transform which has all EigenValue Tensor Invariants negated.
  \[\eta_{\mu\nu}\Lambda_{\mu\nu} = \delta_{\mu\nu}\Lambda_{\mu\nu} = \delta_{\mu\nu}\]
- The Trace Invariant identifies a “Dual” Negative-Side for all Lorentz Transforms.
The h Connection

\( P = \hbar K \): Basic Einstein-de Broglie

Sum over n particles: \( P_T = \sum_n(P+Q), K_T = \sum_n(K_{\text{dyn}}+K_{\text{pot}}) \)

\( P_T = \hbar K_T \)

\{SR Hamilton-Jacobi\} = \hbar\{QM Complex Plane-Waves\}

The SR Hamilton-Jacobi Equation, and the QM idea of Complex Plane-Waves, are related by a simple constant (h) relation.

---

**SR 4-Tensor**

(2,0)-Tensor \( T^\mu_\nu \)

(1,1)-Tensor \( V^\nu_\mu \) or \( T^\nu_\nu \)

(0,2)-Tensor \( T_{\mu \nu} \)

**SR 4-Vector**

(1,0)-Tensor \( V^{\mu} = (v^{\mu}, v) \)

(0,1)-Tensor \( V_{\nu} = (v_{\nu}, -v) \)

**SR 4-Scalar**

(0,0)-Tensor \( S \) or \( S_0 \)

**Lorentz Scalar**

**SR 4-Vector Study: Einstein-de Broglie The (h) Connection**

**SRQM 4-Vector Study:**

Einstein-de Broglie

The (h) Connection

4-Displacement

\( \Delta X = (c\Delta t, \Delta x) \)

4-Position

\( X = (ct, x) \)

4-WaveVector

\( K = (\omega/c, k) = (\omega/c, \omega \hbar /v_{\text{phase}}) \)

4-Momentum

\( P = (mc, p) = (E/c, p) \)

4-PotentialMomentum

\( Q = (U/c, q) = qA \)

4-MomentumIncField

\( P = (E/c, p) = P + Q = P + qA \)

4-Velocity

\( U = \gamma(c, u) \)

4-Gradient

\( \partial = (\partial /c, -\nabla) \)

4-Position

\( X = (ct, x) \)

4-WaveVector

\( K = (\omega/c, k) = (\omega/c, \omega \hbar /v_{\text{phase}}) \)

4-WaveVectorIncField

\( K_T = (\omega/c, k) = K + (q(\hbar))A \)

**SRQM 4-Vector Interpretation of QM**

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http://scirealm.org/QRQM.pdf
**SRQM 4-Vector Study:**

**Dimensionless Physical Objects**

There are a number of unit-dimensionless physical objects in SR that can be constructed from Physical 4-Vectors. There are examples with 4-Scalars, 4-Vectors, and 4-Tensors.

- **4-Displacement**
  \[ \Delta \mathbf{x} = (\Delta t, \Delta \mathbf{x}) \]
  \[ d \mathbf{x} = (cdt, dx) \]

- **K: Wave Vector**
  \[ \mathbf{K} = (\omega/c, \mathbf{k}) \]

- **4-UnitTemporal**
  \[ T = \gamma (1, \beta) \]

- **4-UnitSpatial**
  \[ S = \gamma (\mu, T) \]

- **SpaceTime Transform**
  \[ \delta \mathbf{X} = 4 \text{ Dimension} \]

- **SpaceTime Metric**
  \[ \mathbf{g} = \mathbf{g} \eta^\mu_\nu \]

- **Calculus dX = \frac{dX}{\partial \mathbf{x}} \partial_\mathbf{x} \]

- **Non-Zero Commutation**
  \[ [\Delta \mathbf{X}, \mathbf{X}] = [\gamma c, \mathbf{X}] = \gamma \eta^\mu_\nu \]

**SRQM 4-Vector Study**

- **4-Momentum**
  \[ \mathbf{P} = mc^2 \]

- **Rest Energy**
  \[ E = mc^2 \]

- **4-Gradient**
  \[ \frac{d}{dt} \frac{\partial}{\partial \mathbf{x}} = d \frac{\partial}{\partial t} \]

**SR 4-Vector**

- **(0,1)-Tensor**
  \[ V^\mu = (v^\mu, 0) \]

- **SR 4-Scalar**
  \[ S \text{ or } S_0 \]

**SR 4-Tensor**

- **(0,2)-Tensor**
  \[ T^\mu_\nu \]

**SR 4-CoVector:OneForm**

- **(1,0)-Tensor**
  \[ V_\nu = (v_\nu, 0) \]

**Quantum Principles**

- **Einstein de Broglie**
  \[ \frac{\hbar K}{2} \]

- **Stat Mech Particle Distribution Factor**
  \[ = \left( \frac{E}{k_B T} \right) \]

**Existing SR Rules**

- **SRQM Interpretation**
  \[ \text{of QM} \]

- **4-Vector SRQM Study**
  \[ \text{Phys cs} \]

**SciRealm.org**

- **John B. Wilson**
  \[ SciRealm@eol.com \]

- **http://scirealm.org/SRQM.pdf**

**Trace**

\[ T^\mu_\nu = \eta^\mu_\nu T^\nu_\mu = T \]

\[ V \cdot V = \sqrt{\eta^\mu_\nu (\mathbf{v})^\mu (\mathbf{v})^\nu} = (\mathbf{v}^\nu)^2 \]

\[ = \text{Lorentz Scalar Invariant} \]
SRQM: QM Axioms Unnecessary

QM Principles emerge from SR

QM is derivable from SR plus a few empirical facts – the “QM Axioms” aren't necessary. These properties are either empirically measured or are emergent from SR properties...

3 “QM Axioms” are really just empirical constant relations between purely SR 4-Vectors:
- Particle-Wave Duality \[ (P) = \hbar(K) \]
- Unitary Evolution \[ (\partial) = (-i)K \]
- Operator Formalism \[ (\partial) = -iK \]

2 “QM Axioms” are just the result of the Klein-Gordon Equation being a linear wave PDE:
- Hilbert Space Representation (<bra|,|ket>, wavefunctions, etc.) & The Principle of Superposition

3 “QM Axioms” are a property of the Minkowski Metric and the empirical fact of Operator Formalism:
- The Canonical Commutation Relation
- The Heisenberg Uncertainty Principle (time-like-separated measurement exchange)
- The Pauli Exclusion Principle (space-like-separated particle exchange)

1 “QM Axiom” only holds in the NRQM case
- The Born QM Probability Interpretation – Not applicable to RQM, use Conservation of Worldlines instead

1 “QM Axiom” is really just another level of limiting cases, just like SR → CM in limit of low velocity
- The QM Correspondence Principle (QM → CM in limit of \( \nabla^2[\phi] \ll (\nabla[\phi])^2 \))
SRQM Interpretation: Relational QM & EPR

The SRQM interpretation fits fairly well with Carlo Rovelli's Relational QM interpretation:

Relational QM treats the state of a quantum system as being observer-dependent, that is, the QM State is the relation between the observer and the system. This is inspired by the key idea behind Special Relativity, that the details of an observation depend on the reference frame of the observer.

All systems are quantum systems: no artificial Copenhagen dichotomy between classical/macroscopic/conscious objects and quantum objects.

The QM States reflect the observers' information about a quantum system.
Wave function "collapse" is informational – not physical. A particle always "knows" its complete properties – it "is" its properties. An observer has at best only partial information about the particle's properties.

No Spooky Action at a Distance. When a measurement is done locally on an entangled system, it is only the partial information about the distant entangled state that "changes/becomes-available-instantaneously". There is no superluminal signal. Measuring/physically-changing the local particle does not physically change the distant particle.

ex. Place two identical-except-for-color marbles into a box, close lid, and shake. Without looking, pick one marble at random and place it into another box. Send that box very far away. After receiving signal of the far box arrival at a distant point, open the near box and look at the marble. You now instantaneously know the far marble's color as well. The information did not come by signal. You already had the possibilities (partial knowledge). Looking at the near marble color simply reduced the partial knowledge of both marble's color to complete knowledge of both marbles' color. No signal was required, superluminal or otherwise.

ex. The quantum version of the same experiment uses the spin of entangled particles. When measured on the same axis, one will always be spin-up, the other will be spin-down. It is conceptually analogous. Entanglement is only about correlations of system that interacted in the past and are determined by conservation laws.
Einstein and Bohr can both be “right” about EPR:
Per Einstein: The QM State measured is not a “complete” description, just one observer's point-of-view.
Per Bohr: The QM State measured is a “complete” description, it's all that a single observer can get.

The point is that many observers can all see the “same” system, but see different facets of it. But a single measurement is the maximal information that a single observer can get without re-interacting with the system, which of course changes the system in general. Remember, the Heisenberg Uncertainty comes from non-zero commutation properties which *require separate measurement arrangements*. The properties of a particle are always there. Properties define particles. We as observers simply have only partial information about them.

Relativistic QM, being derived from SR, should be local – The low-velocity limit to QM may give unexpected anomalous results if taken out of context, or out of the applicable validity range, such as with velocity addition $v_3 = v_1 + v_2$, where the correct formula should be the relativistic velocity composition $v_3 = (v_1 + v_2)/(1 + v_1v_2/c^2)$

These ideas lead to the conclusion that the wavefunction is just one observer's state of information about a physical system, not the state of the physical system itself. The “collapse” of the wavefunction is simply the change in an observer's information about a system brought about by a measurement or, in the case of EPR, an inference about the physical state.

EPR doesn't break Heisenberg because measurements are made on different particles. The happy fact is that those particles interacted and became correlated in the causal past. The EPR-Bell experiments prove that it is possible to maintain those correlations over long distances. It does *not* prove superluminal (FTL) signaling
SRQM Interpretation:
Range-of-Validity Facts & Fallacies

We should not be surprised by the “quantum” probabilities being correct instead of “classical” in the EPR and Bell Inequalities experiments. Classical thinking (in both CM and QM) has a number of fallacies when it is mistakenly applied outside of its range-of-validity.

Examples

*Classical Physics as the limit of \(\hbar \to 0\) \{Fallacy\}: 
\(\hbar\) is a Lorentz Scalar Invariant and Fundamental Physical Constant. It never becomes 0. \{Fact\}

*The classical commutator being zero \([p^i, x^j] = 0\) \{Fallacy\}: 
\([P^\mu, X^\nu] = i\hbar\eta^{\mu\nu}; [p^i, x^j] = -i\hbar\delta^{ij}; [p^0, x^0] = [E/c, ct] = [E, t] = i\hbar(1);\) Again, it never becomes 0, it’s just really small. \{Fact\}

*Using Maxwell-Boltzmann (distinguishable) statistics for counting probabilities of (indistinguishable) quantum states \{Fallacy\}: 
One must use Fermi-Dirac (FD) statistics for Fermions: Spin=(n+1/2); Bose-Einstein (BE) statistics for Bosons: Spin=(n) \{Fact\}

*Using sums of classical probabilities on quantum states \{Fallacy\}: 
Must use sums of quantum probability-amplitudes \{Fact\}

*Ignoring phase cross-terms and interference effects in calculations \{Fallacy\}: 
Quantum systems and entanglement require phase cross-terms \{Fact\}

*Assuming that one can simultaneously “measure” non-commuting properties at a single spacetime event \{Fallacy\}: 
Particle properties always exist. However, non-commuting ones require separate measurement arrangements to get information about the properties. 
The required measurement arrangements on a single particle/worldline are at best sequential events, where the temporal order plays a role; \{Fact\}
However, EPR allows one to “infer (not measure)” the other property of a particle by the separate measurement of an entangled partner. \{Fact\}
This does not break Heisenberg Uncertainty, which is about the order of operations (measurement events) on a single worldline. \{Fact\}
In the entangled case, both/all of the entangled partners share common past-causal entanglement events, typically due to a conservation law. \{Fact\}
Information is not transmitted at FTL. The particles simply carried their normal respective “correlated” properties (no hidden variables) with them. \{Fact\}

*Assuming that QM is a generalization of CM, or that classical probabilities apply to QM \{Fallacy\}: 
CM is a limiting-case of QM for when changes in a system by a few quanta have a negligible effect on the whole/overall system. \{Fact\}
We should not be surprised by the "quantum" probabilities being correct instead of "classical" in the EPR and Bell Inequalities experiments. Classical thinking (in both CM and QM) has a number of fallacies when it is mistakenly applied outside of its range-of-validity.

(from Wikipedia)
No-Communication Theorem/No-Signaling:
A no-go theorem from quantum information theory which states that, during measurement of an entangled quantum state, it is not possible for one observer, by making a measurement of a subsystem of the total state, to communicate information to another observer. The theorem shows that quantum correlations do not lead to what could be referred to as "spooky communication at a distance". SRQM: There is no FTL signaling/communication.

No-Teleportation Theorem:
The no-teleportation theorem stems from the Heisenberg uncertainty principle and the EPR paradox: although a qubit $|\psi\rangle$ can be imagined to be a specific direction on the Bloch sphere, that direction cannot be measured precisely, for the general case $|\psi\rangle$. The no-teleportation theorem is implied by the no-cloning theorem.
SRQM: Ket states are informational, not physical.

No-Cloning Theorem:
In physics, the no-cloning theorem states that it is impossible to create an identical copy of an arbitrary unknown quantum state. This no-go theorem of quantum mechanics proves the impossibility of a simple perfect non-disturbing measurement scheme. The no-cloning theorem is normally stated and proven for pure states; the no-broadcast theorem generalizes this result to mixed states. SRQM: Measurements are arrangements of particles that interact with a subject particle.

No-Broadcast Theorem:
Since quantum states cannot be copied in general, they cannot be broadcast. Here, the word "broadcast" is used in the sense of conveying the state to two or more recipients. For multiple recipients to each receive the state, there must be, in some sense, a way of duplicating the state. The no-broadcast theorem generalizes the no-cloning theorem for mixed states. The no-cloning theorem says that it is impossible to create two copies of an unknown state given a single copy of the state. SRQM: Conservation of worldlines.

No-Deleting Theorem:
In physics, the no-deleting theorem of quantum information theory is a no-go theorem which states that, in general, given two copies of some arbitrary quantum state, it is impossible to delete one of the copies. It is a time-reversed dual to the no-cloning theorem, which states that arbitrary states cannot be copied. SRQM: Conservation of worldlines.

No-Hiding Theorem:
The no-hiding theorem is the ultimate proof of the conservation of quantum information. The importance of the no-hiding theorem is that it proves the conservation of wave function in quantum theory.
SRQM: Conservation of worldlines. RQM wavefunctions are Lorentz 4-Scalars (spin=0), 4-Spinors (spin=1/2), 4-Vectors (spin=1), all of which are Lorentz Invariant.
SRQM Interpretation:
Quantum Information

We should not be surprised by the "quantum" probabilities being correct instead of "classical" probabilities in the EPR/Bell-Inequalities experiments. Classical thinking (in both CM and QM) has a number of fallacies when it is mistakenly applied outside of its range-of-validity.

Quantum information (qubits) differs strongly from classical information, epitomized by the bit, in many striking and unfamiliar ways. Among these are the following:

A unit of quantum information is the qubit. Unlike classical digital states (which are discrete), a qubit is continuous-valued, describable by a direction on the Bloch sphere. Despite being continuously valued in this way, a qubit is the smallest possible unit of quantum information, as despite the qubit state being continuously-valued, it is impossible to measure the value precisely.

A qubit cannot be (wholly) converted into classical bits; that is, it cannot be "read". This is the no-teleportation theorem.

Despite the awkwardly-named no-teleportation theorem, qubits can be moved from one physical particle to another, by means of quantum teleportation. That is, qubits can be transported, independently of the underlying physical particle. SRQM: Ket states are informational, not physical.

An arbitrary qubit can neither be copied, nor destroyed. This is the content of the no-cloning theorem and the no-deleting theorem. SRQM: Conservation of worldlines.

Although a single qubit can be transported from place to place (e.g. via quantum teleportation), it cannot be delivered to multiple recipients; this is the no-broadcast theorem, and is essentially implied by the no-cloning theorem. SRQM: Conservation of worldlines.

Qubits can be changed, by applying linear transformations or quantum gates to them, to alter their state. While classical gates correspond to the familiar operations of Boolean logic, quantum gates are physical unitary operators that in the case of qubits correspond to rotations of the Bloch sphere.

Due to the volatility of quantum systems and the impossibility of copying states, the storing of quantum information is much more difficult than storing classical information. Nevertheless, with the use of quantum error correction quantum information can still be reliably stored in principle. The existence of quantum error correcting codes has also led to the possibility of fault tolerant quantum computation.

Classical bits can be encoded into and subsequently retrieved from configurations of qubits, through the use of quantum gates. By itself, a single qubit can convey no more than one bit of accessible classical information about its preparation. This is Holevo's theorem. However, in superdense coding a sender, by acting on one of two entangled qubits, can convey two bits of accessible information about their joint state to a receiver.

Quantum information can be moved about, in a quantum channel, analogous to the concept of a classical communications channel. Quantum messages have a finite size, measured in qubits; quantum channels have a finite channel capacity, measured in qubits per second.
Minkowski still applies in local GR

QM is a local phenomenon

The QM Schrödinger Equation is not fundamental. It is just the low-energy limiting-case of the RQM Klein-Gordon Equation. All of the standard QM Axioms are shown to be empirically measured constants or emergent properties of SR. It is a bad approach to start with NRQM as an axiomatic starting point and try to generalize it to RQM, in the same way that one cannot start with CM and derive SR. Since QM *can* be derived from SR, this partially explains the difficulty of uniting QM with GR: QM is not a “separate formalism” outside of SR that can be used to “quantize” just anything...

Strictly speaking, the use of the Minkowski space to describe physical systems over finite distances applies only in the SR limit of systems without significant gravitation. In the case of significant gravitation, SpaceTime becomes curved and one must abandon SR in favor of the full theory of GR.

Nevertheless, even in such cases, based on the GR Equivalence Principle, Minkowski space is still a good description in a local region surrounding any point (barring gravitational singularities). More abstractly, we say that in the presence of gravity, SpaceTime is described by a curved 4-dimensional manifold for which the tangent space to any point is a 4-dimensional Minkowski Space. Thus, the structure of Minkowski Space is still essential in the description of GR.

So, even in GR, at the local level things are considered to be Minkowskian: i.e. SR → QM “lives inside the surface” of this local SpaceTime, GR curves the surface.
SRQM Interpretation: Main Result

QM is derivable from SR!

- Hopefully, this interpretation will shed light on why Quantum Gravity has been so elusive. Basically, QM rules of “quantization” don’t apply to GR. They are a manifestation-derivation-from SR. Relativity *is* the “Theory of Measurement” that QM has been looking for.

This would explain why no one has been able to produce a successful theory of “Quantum Gravity”, and why there have been no violations of Lorentz Invariance, CPT, or the Equivalence Principle.

If quantum effects “live” in Minkowski SpaceTime with SR, then GR curvature effects are at a level above the RQM description, and two levels above standard QM. SR+QM are “in” SpaceTime, GR is the “shape” of SpaceTime...

Thus, this SRQM Treatise explains the following:

- Why GR works so well in it's realm of applicability {large scale systems}.
- Why QM works so well in it's realm of applicability {micro scale systems and certain macroscopic systems}. i.e. The tangent space to any point in GR curvature is locally Minkowskian, and thus QM is typically found in small local volumes...
- Why RQM explains more stuff than QM without SR {because QM is just an approximation: the low-velocity limiting-case of RQM}.
- Why all attempts to "quantize gravity" have failed {essentially, everyone has been trying to put the cart (QM) before the horse (GR)}.
- Why all attempts to modify GR keep conflicting with experimental data {because GR is apparently fundamental – passed all tests to-date}.
- Why QM works perfectly well with SR as RQM but not with GR {because QM is derivable from SR, hence a manifestation of SR rules}.
- How Minkowski Space, 4-Vectors, and Lorentz Invariants play vital roles in RQM, and give the SRQM Interpretation of Quantum Mechanics.

SRQM: Special Relativistic Quantum Measurement, Special Relativistic Quantum Mechanics
SRQM: The [ SR → QM ] Interpretation of Quantum Mechanics

Special Relativity (SR) Axioms: Invariant Interval + LightSpeed (c) as Universal Physical Constant lead to SR, although technically SR is itself the Minkowski-Space-Time low-curvature:"flat" limiting-case of GR.

\{ c, \tau, m_0, \hbar, i \} = \{ \text{SpeedOfLight}, \text{ProperTime}, \text{RestMass}, \text{Dirac/PlanckReducedConstant}(h=\hbar/2\pi), \text{ImaginaryNumber} \}:

are all Empirically-Measured SR Lorentz-Invariant Physical and/or Mathematical Constants

\( i = +\sqrt{-1} = (0,1) \) complex#

Standard SR 4-Vectors:

- 4-Position: \( R = (ct, r) \)
- 4-Velocity: \( U = \gamma(c, \mu) \)
- 4-Momentum: \( P = (E/c, p) \)
- 4-WaveVector: \( K = (\omega/c, k) \)
- 4-Gradient: \( \partial = (\partial_t/c, -\nabla) \)

Related by these SR Lorentz Invariants:

\( R \cdot R = (ct)^2 = (|r_0|)^2 \)
\( U \cdot U = (c)^2 \)
\( P \cdot P = (m_0 c)^2 \)
\( K \cdot K = (m_0 c/h)^2 \)
\( \partial \cdot \partial = (-im_0 c/h)^2 = -(m_0 c/h)^2 = \text{QM Relation} \rightarrow \text{RQM} \rightarrow \text{QM} \)

SR + Empirically Measured Physical Constants lead to RQM via the (KG) Klein-Gordon Quantum Eqn, and thence to QM via the low-velocity limit \( |v| \ll c \), giving the Schrödinger Eqn. This fundamental KG Relation also leads to the other Quantum Wave Equations:

\( |v| = c \): massless, no rest-frame, Lorentzian

\( 0 \leq |v| < c \): \( m_0 > 0 \)

spin=0 boson field = 4-Scalar:

\( \{ |v| = c : m_0 = 0 \} \)

Free Scalar Wave (Higgs)

Weyl

Maxwell (EM photonic)

SR 4-Tensor

(2,0)-Tensor \( T^{\mu\nu} \)

(1,1)-Tensor \( T^\mu \), or \( T_\mu \)

(0,2)-Tensor \( T_{\mu\nu} \)

SR 4-Vector

\( V^\mu = V = (v^0, v) \)

SR 4-Vector

S or \( s \)

Lorentz Scalar

SR 4-Scalar

(0,0)-Tensor \( S \)

SRQM: A treatise of SR → QM by John B. Wilson (SciRealm@aol.com)
See also:
http://scirealm.org/SRQM.html (alt discussion)
http://scirealm.org/SRQM-RoadMap.html (main SRQM website)
http://scirealm.org/4Vectors.html (4-Vector study)
http://scirealm.org/SRQM-Tensors.html (Tensor & 4-Vector Calculator)
http://scirealm.org/SciCalculator.html (Complex-capable RPN Calculator)

or Google “SRQM”

http://scirealm.org/SRQM.pdf (this document: most current ver. at SciRealm.org)
The 4-Vector SRQM Interpretation
QM is derivable from SR!

The SRQM or [SR→QM] Interpretation of Quantum Mechanics
A Tensor Study of Physical 4-Vectors

quantum relativity

SRQM = SciRealm QM?
A happy coincidence… :)