Quantum Mechanics is *derivable* from Special Relativity. By using the 4-Vectors of SR, which are 4D (1,0)-Tensors, one can derive the principles of QM. Therefore, the standard QM "axioms" are actually emergent from the rules of SR. Hence [**SR→QM**] The 4D <Time·Space>-splitting into 1D Temporal + 3D Spatial components plays an integral role in understanding these relations.

Key Words: Special Relativity (SR), Quantum Mechanics (QM), Quantum-Classical relation, Quantum Emergence, SRQM, SR→QM

Introduction:

There are currently two main foundational bodies of physics theory used to model reality as we know it:
Relativity {General (GR) + Special (SR)} & Quantum Mechanics (QM). Typically, GR ~ astronomical scale & QM ~ atomic scale.

1) Relativity uses tensor mathematics to calculate the properties of 4D Spacetime. GR is used for physical systems in which the mass of objects is very large. Essentially this means: "Mass tells spacetime how to curve, spacetime curvature tells mass how to move". SR is a "special" limiting-case of GR for low curvature ~ low mass. SR tells us that certain physical properties once thought totally independent are actually dual to one another: (time:space),(energy:momentum), etc. Tensor mathematics has the concept that tensor component-values can relativistically vary such that measurements between events are independent of arbitrarily-imposed coordinate-systems, ex. consider 2D pictures of a 3D object from different angles/distances. The pics look different, yet are all the same object. Integral to physical relativistic tensors is that {4D <Time·Space> = 1D Temporal (t) + 3D Spatial (x,y,z)} entities have specific ways of splitting into the various natural, measurable components, depending on the type of tensor that they are represented by:

4-Scalar S	(1) Invariant Lorentz Scalar, same for all frames {s}	1 {4D (0,0)-Tensor} component
4-Vector V^μ	$(1\ ^{0}+3\ ^{j})$-splitting into $\{v^t,v^x,v^y,v^z\}$	4 {4D (1,0)-Tensor} components
4-Tensor, Symmetric $T^{\mu\nu}$	$(1\ ^{00}+3\ ^{0k}+3\ ^{j=k}+3\ ^{j\neq k})$-splitting into $\{t^{tt},t^{tx},t^{ty},t^{tz},t^{xx},t^{yy},t^{zz},t^{xy},t^{xz},t^{yz}\}$	10 {4D (2,0)-Tensor} components

There are relativistic Symmetries/Operations in nature which leave the interval-measurement between events unchanged (invariant) and lead to fundamental Conservation Laws. These use active or passive transformations, including changes of coordinate basis. The Poincaré Group {Lorentz Group $\Lambda^\mu{}_\nu$ + SpaceTime Translation Group ΔX^μ} is the Full SpaceTime Symmetry Group and provides the 10 Isometries which match the {Symmetric 4D (2,0)-Tensor [1+3+3+3]-splitting (10)} & {4D (1,0)-Tensor (1+3)-splitting (4)}. SR 4-Vectors have a linear mapping ($V^{\mu'} = \Lambda^{\mu'}{}_\nu V^\nu + \Delta X^{\mu'}$) which preserves their magnitude: ($V^{\mu'}V_{\mu'} = V^\nu V_\nu$). 4-Position only Lorentz, not Poincaré.

[↻]

Lorentz Group ($\Lambda^\mu{}_\nu$) Symmetry → Conservation of 4-AngularMomentum $\boldsymbol{M} = M^{\mu\nu} = R^\mu \wedge P^\nu = [[m^{\mu\nu}]] = [[0,\mathbf{n}],[\mathbf{n}^T,\mathbf{l}=\mathbf{r}\wedge\mathbf{p}]]$: Isotropy
The spatial part: 3 Space-Space-Rotation ($\Lambda^\mu{}_\nu \to R^\mu{}_\nu$) Symmetry → Conservation of 3-angular-momentum $\mathbf{l} = l^k$ same all directions θ,φ
The mixed part: 3 Space-Time-Boost ($\Lambda^\mu{}_\nu \to B^\mu{}_\nu$) Symmetry → Conservation of 3-mass-moment $\mathbf{n} = n^k$ ✴

[→]

Spacetime-Translation Group (ΔX^μ) Symmetry → Conservation of 4-LinearMomentum $\mathbf{P} = P^\mu = (p^\mu) = (E/c,\mathbf{p}) = (p^0,p^k)$: Homogeneity
The temporal part: 1 Time-Translation ($\Delta x^0 = c\Delta t$) Symmetry → Conservation of Energy $E = cp^0$ same all extent $\Delta\mathbf{X}$
The spatial part: 3 Space-Translation ($\Delta x^k = \Delta\mathbf{x}$) Symmetry → Conservation of 3-momentum $\mathbf{p} = p^k$ ▦

In addition, the physical properties of ($\mathbf{P}\cdot\mathbf{P}$) → mass ($m_o$) and ($\mathbf{W}\cdot\mathbf{W}$) → spin ($s_o$) are the Casimir Invariants of the Poincaré Group, which are those quantities that commute with all generators of the Poincaré Group. **W** is the Pauli–Lubanski spin pseudovector.

2) Quantum Mechanics typically uses an operator formalism, Hilbert space, and wavefunctions to describe the properties of fundamental particles and their interactions. Many curious and obscure properties arise: non-zero commutation of measurements, wave-particle duality, matter-waves, quantization of energy levels, superposition of states, single-particle interference effects, unitary time-evolution, Born Probability, Bell's Theorem, Heisenberg Uncertainty, CPT Symmetry, entanglement, B-E:F-D statistics, etc.

For both of these main physics systems, Newtonian classical physics can be shown to be a special limiting-case of the given theory. For Relativity, Newtonian physics emerges when {GR limit-case low-curvature $g^{\mu\nu} \to \eta^{\mu\nu}$} and {SR limit-case low-velocity $|\mathbf{v}| << c$}. For QM, Newtonian physics emerges when {QM limit-case low-divergence $\hbar|\nabla\cdot\mathbf{p}| << (\mathbf{p}\cdot\mathbf{p})$ or $|\nabla\cdot\mathbf{k}| << (\mathbf{k}\cdot\mathbf{k})$ or $S_{action} >> \hbar$}.

For many decades, physicists have been trying to unite these two main theories. While QM appears to be totally compatible with SR in the form of Relativistic Quantum Mechanics (RQM), it does not "seem" to be compatible with GR. The main attempts at unification have been to assume QM as fundamental and try to "quantize gravity" in various ways, i.e. to impose the mathematical rules of QM onto that of GR, basically without success.

In this work, a novel pathway, [**SRQM**] or [**SR→QM**], will be shown that unites the two theories in a way that is mathematically elegant and precise, and that explains why the previous attempts have been unsuccessful. Essentially, GR is taken as fundamental instead of QM. Relativity uses the mathematics of 4-Vectors = 4D (1,0)-Tensors to describe the properties and relations of physical objects and concepts, including both relativistic and quantum physics. Quantum "axioms" are instead actually SR derived principles.

Consider the following idea, with each a special limiting-case subset of the former:
GR→{limit-case $g^{\mu\nu} \to \eta^{\mu\nu}$}→SR→RQM→{limit-case $|\mathbf{v}| << c$}→QM→{limit-case $\hbar|\nabla\cdot\mathbf{p}| << (\mathbf{p}\cdot\mathbf{p})$}→(EM & CM)
The main idea of [**SRQM**] is that the rules of RQM and QM can be **derived** from the rules of SR.

Thus, this treatise explains the following empirical observations:
Why GR works so well in its realm of applicability {massive large-scale systems}.
Why QM works so well in its realm of applicability {micro-scale systems and special macroscopic systems, ex. superfluids}.
i.e. The tangent space to GR curvature at any point is locally Minkowskian, and thus QM works for small volumes...
Why RQM explains physical effects that QM-without-SR cannot, and with greater accuracy those that basic QM can explain.
Why attempts to "quantize gravity" fail {essentially, everyone has been trying to put the cart (QM) before the horse (GR)}.
Why all attempts to modify GR keep conflicting with experimental data {because to-date, GR is apparently still fundamental}.
Why QM works with SR as RQM, but not with GR {because QM is derivable from SR, hence a manifestation of SR rules}.
In other words, the "special case" rules of QM are not something that can be imposed on GR, which is the "general" parent theory.
How Minkowski Space, 4-Vectors, and Lorentz Invariants play vital roles in RQM, & give the SRQM Interpretation of QM.

Major clues from experiment, observation, and mathematics that is actually related to reality:
The components of 4-Vectors and 4-Tensors are experimentally measurable elements/properties of physical reality.
Both General Relativity (GR) and Special Relativity (SR) have passed very stringent tests of multiple varieties.
Relativistic Quantum Mechanics (RQM) and standard Quantum Mechanics (QM) have passed all tests within their realms of validity:
RQM describes and explains phenomena that standard QM cannot.
Mass **and** Spin are Casimir Invariants of Poincaré Invariance, which comes from 4D SR Minkowski Space, not QM.
Hence, neither mass nor spin require a quantum axiom for their existence.
To-date, there is no observational/experimental indication that quantum effects "alter" the fundamentals of either SR or GR.
To-date, there have been no repeatable violations of Poincaré-Lorentz Invariance, nor of Local Position Invariance, nor of CPT Symmetry, nor of the Standard Model formulation (neutrino masses notwithstanding). This rules out many alternative gravity theories.
In fact, in all known experiments where both SR/GR and QM are present, QM respects the principles of SR/GR,
whereas SR/GR modify the results of QM, ex. Dirac RQM Wave Eqn. vs Schrödinger QM Wave Eqn.
All tested quantum-level particles, atoms, isotopes, molecules, super-positions, spin-states, excited-states, etc. obey:
GR's {Universality of Free-Fall & Equivalence Principle} and SR's { $E = mc^2$ & lightspeed (c) communication/signaling limit}.
On the other hand, GR gravity *does* induce changes in quantum interference patterns and hence modifies QM.
For instance, quantum-level atomic-clocks are used to measure gravitational "Doppler" red:blue-shift effects.
i.e. GR gravitational frequency-shift (gravity time-dilation) alters atomic=quantum-level timing. Think about that for a moment...
While quantum entanglement is somewhat mysterious, there is no FTL-communication-with nor alteration-of distant particles.
Getting a Stern-Gerlach "up" |here⟩ doesn't cause the distant entangled particle to suddenly start physically moving "down" |there⟩.
The universe appears to be both: Homogeneous = same all places ▪ (SpaceTime-Translation ΔX^μ Symmetry) over displacement $\Delta\mathbf{X}$
and Isotropic = same all directions ∗ (Lorentz $\Lambda^\mu{}_\nu$ Symmetry) over {angle θ, hyperbolic angle φ}.
The main Schrödinger relation is just a special case of complex plane waves on objects (4-Vectors) already connected in standard SR.
All Lorentz Scalar Product connections between SR 4-Vectors are Lorentz Invariants, and typically fundamental constants with,
to-date, no consistent evidence of change over time during the age of the universe nor of varying value throughout spatial extent.
CM typically uses a Phase Space description; QM typically uses a Hilbert Space description. However:
CM can be done with a Hilbert Space description: see Koopman-von Neumann Classical Mechanics.
QM can be done without a Hilbert Space description: see Phase Space Formulation of Quantum Mechanics.
Hence, Hilbert space is mathematical, and with its associated properties, does not require a quantum axiom for its existence.
Particles of both CM and QM can be described by a wave-like theory: see the Hamilton-Jacobi Equations.
Measurement of Planck's constant (h) can be done with experiments that do not need quantum theory for the measurement, just SR.
In other words, the actual measurement process uses empirical, non-QM-theory-dependent components {don't need Schrödinger eqn}.
50+ years of unsuccessful attempts to "quantize gravity".
To-date, no new particles found outside of the Standard Model by LHC or other experiments, just the expected SM Higgs Boson.

Other SRQM sources:
Main website: SRQM - QM from SR - Simple RoadMap (.html)
PDF slideshow presentation/book: http://scirealm.org/SRQM.pdf
This Summary: http://scirealm.org/SRQM-Summary.pdf **Most current version**
Alternate discussion at: http://scirealm.org/SRQM.html
SRQM Flyer: http://scirealm.org/SRQM_Flyer.pdf
See: 4-Vectors & Lorentz Scalars Reference for more info on Four-Vectors (4-Vectors) in general
See: John's Online RPN Scientific Calculator, using Complex Math
See: SRQM - Online SR 4-Vector & Tensor Calculator

https://www.researchgate.net/project/SR--QM-Special-Relativity--Quantum-Mechanics-Project
https://www.researchgate.net/project/Classical-Foundations-of-Quantum-Mechanics, with William P. Rice

Notation & Conventions:
{Temporal, 0^{th} component, Positive(+), SI} = Metric Signature (+,-,-,-) with [SI Units]
4D "Flat" <Time·Space> SR:Minkowski Metric $\eta_{\mu\nu} = \eta^{\mu\nu} \rightarrow$Diagonal[+1,-1,-1,-1]$_{(Cartesian)}$; $\eta^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu}$ = Diagonal[+1,+1,+1,+1] = $I_{[4]} = g^{\mu}{}_{\nu}$
Component Coloring Mnemonic: Temporal (blue) + Spatial (red) give Mixed SpaceTime (purple)
4-Vector $\mathbf{A} = A^{\mu} = (a^{\mu}) = (a^0,\mathbf{a}) = (a^0,a^k) = (a^0,a^1,a^2,a^3) \rightarrow (a^t,a^x,a^y,a^z)_{[Cartesian]} \rightarrow (a^t,a^r,a^{\theta},a^{\varphi})_{[spherical]}$
3-vector $\mathbf{a} = a^k = (a^k) = (\mathbf{a}) = (a^k) = (a^1,a^2,a^3) \rightarrow (a^x,a^y,a^z)_{[Cartesian]} \rightarrow (a^r,a^{\theta},a^{\varphi})_{[spherical]}$
4-Scalar S (1) Invariant Lorentz Scalar, same for all frames {s} 1 {4D (0,0)-Tensor} component
4-Vector V^{μ} ($1^{\,0}+3^{\,j}$)-splitting into $\{v^t,v^x,v^y,v^z\}$ 4 {4D (1,0)-Tensor} components
4-Tensor, Symmetric $T^{\mu\nu}$ ($1^{\,00}+3^{\,0k}+3^{\,j=k}+3^{\,j\neq k}$)-splitting into $\{t^{tt},t^{tx},t^{ty},t^{tz},t^{xx},t^{yy},t^{zz},t^{xy},t^{xz},t^{yz}\}$ 10 {4D (2,0)-Tensor} components
Technically, these are all 4-Tensors = 4D Tensors; specify precisely using the #D (m,n)-Tensor notation {# dims, m ^{upper}, n $_{lower}$ indices}
<Time·Space> 4-Vector Name matches its spatial 3-vector component name
SR 4-Vector = (**spacetime 4-Vector**) = (temporal 3-scalar, **spatial 3-vector**) = 4D (1+3)-splitting into (v^t,v^x,v^y,v^z)
4-Vectors (4D) in **bold** UPPERCASE: ex. **A**
3-vectors (3D) in **bold** lowercase: ex. **a** : sometimes with vector=over-arrow $\bar{\mathbf{a}}$
Temporal scalars (1D) in non-bold, usually lowercase, 0^{th} component: ex. a^0 , a_0 "Count from 1, but index from 0 :-) "
Individual non-grouped components of 4-Tensors in non-bold: ex. $\mathbf{A} = (a^0,a^1,a^2,a^3) = (a^0,\mathbf{a})$
Rest scalars (invariants) normal in non-bold, denoted with naught ($_o$): ex. a_o "A rest-frame is a valid relativistic concept"
Tensor-index-notation normal in non-bold: ex. $A^{\mu} = (a^{\mu}) = (a^0,a^j) = (a^0,a^1,a^2,a^3)$
4D Tensors use Greek indices: ex. { μ, ν, σ, ρ, ...} : ex. 4-Position $R^{\mu} = (r^{\mu}) = (r^0,r^1,r^2,r^3)$, with 4 possible values in index {0,1,2,3}
3D tensors use Latin indices: ex. { i, j, k, ...} : ex. 3-position $r^k = (r^k) = (r^1,r^2,r^3)$, with 3 possible values in index {1,2,3}
Upper indices 4-Vector $A^{\mu} = (a^{\mu}) = (a^0,a^j)$: Lower indices 4-CoVector $B_{\mu} = (b_{\mu}) = (b_0,b_j)$
Index lowering/raising via Minkowski Metric $\boldsymbol{\eta}$: ex. $R_{\mu} = \eta_{\mu\nu}R^{\nu}$ or $\partial^{\mu} = \eta^{\mu\nu}\partial_{\nu}$
Relativistic Gamma $\gamma = 1/\sqrt{(1 - \boldsymbol{\beta}\cdot\boldsymbol{\beta})}$: Relativistic $\boldsymbol{\beta} = \mathbf{u}/c = \{0..1\}\hat{\mathbf{n}}$: ProperTimeDerivative $(d/d\tau) = \gamma(d/dt) = (\mathbf{U}\cdot\boldsymbol{\partial})$
LightSpeed Factor (c) in temporal component as required to make all dimensional units of a 4-Vector's components match
4-Vector: $\mathbf{A} = \overline{\mathbf{A}} = A^{\mu}$: ex. 4-Momentum $\mathbf{P} = P^{\mu} = (E/c,\mathbf{p}) = (mc,m\mathbf{u}) = m_o\mathbf{U}$
4-CoVector = OneForm: $\underline{\mathbf{A}} = A_{\mu}$: ex. 4D GradientOneForm $\boldsymbol{\partial} = \partial_{\mu} = (\partial_t/c,\boldsymbol{\nabla}) = (\partial/\partial R^{\mu})$
Null 4-Vector $\mathbf{N} \sim (|\mathbf{a}|,\mathbf{a}) = a(1,\hat{\mathbf{n}})$, with Lorentz Scalar Invariant $\mathbf{N}\cdot\mathbf{N} = N^{\mu}N_{\mu} = 0$ = "Null"
SR:Metric Convention: Temporal-0th-Positive (+,-,-,-), Time-Positive, Particle-Physics, West-Coast, Time-Like, Mostly-Minuses
Signature that is used herein: (+,-,-,-)

Alternate ways/styles of writing 4-Vector and 4-Tensor expressions in Physics:
=====
$(\mathbf{A}\cdot\mathbf{B})$ is a 4-Vector style, which uses vector-notation {ex. inner product "dot=·" or exterior product "wedge=^"},
and is typically more compact, always using **bold** UPPERCASE to represent the 4-vector, ex. $(\mathbf{A}\cdot\mathbf{B}) = (A^{\mu}\eta_{\mu\nu}B^{\nu})$,
and **bold** lowercase to represent 3-vectors, ex. $(\mathbf{a}\cdot\mathbf{b}) = (a^j\delta_{jk}b^k)$. There are 4D analogs to the standard 3D vector rules.
ex. The 4D Gauss' Theorem in SR is: $\int_{\Omega} d^4\mathbf{X}\,(\boldsymbol{\partial}\cdot\mathbf{V}) = \oint_{\partial\Omega} dS\,(\mathbf{V}\cdot\mathbf{N})$
with:
Ω as a 4D simply-connected region of Minkowski SpaceTime
$\partial\Omega = S$ as its 3D boundary with its own 3D Volume element dS and outward-pointing 4-UnitHyperSurfaceNormal **N**.
$d^4\mathbf{X}$ as a 4D Infinitesimal Volume Element
V as an arbitrary 4-Vector
=====
$(A^{\mu}\eta_{\mu\nu}B^{\nu})$ is a Ricci-Calculus style, which uses tensor-index-notation and is useful for more complicated expressions, especially to clarify those expressions involving tensors with more than one index, such as the Faraday EM Tensor $F^{\mu\nu} = (\partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}) = (\boldsymbol{\partial}\wedge\mathbf{A})$.
Some tensor rules include concepts such as:
Index lowering/raising with a Metric $\boldsymbol{g}$, ex. $g^{\mu\nu}$ or $g_{\mu\nu}$: typically using SR limit-case $\boldsymbol{g}\rightarrow\boldsymbol{\eta}$ for "flat" Minkowski Metric
Einstein summation convention {paired lower/upper indices are summed over} ex. $A^{\mu}B_{\mu} = A^0B_0+A^1B_1+A^2B_2+A^3B_3$
Symmetric:AntiSymmetric Tensor decomposition $\{T^{\mu\nu} = S^{\mu\nu} + A^{\mu\nu}\}$, with $S^{\mu\nu} = (T^{\mu\nu}+T^{\nu\mu})/2$ and $A^{\mu\nu} = (T^{\mu\nu}-T^{\nu\mu})/2$
Tensor Contraction of Symmetric with AntiSymmetric yields zero $\{S^{\mu\nu}A_{\mu\nu} = 0\}$, from $\{ S^{\mu\nu} = +S^{\nu\mu} \}$ and $\{ A^{\mu\nu} = -A^{\nu\mu} \}$
Proof: $S^{\mu\nu}A_{\mu\nu} = S^{\nu\mu}A_{\nu\mu} = (S^{\nu\mu})(A_{\nu\mu}) = (+S^{\mu\nu})(-A_{\mu\nu}) = -S^{\mu\nu}A_{\mu\nu} = 0$, since { C = -C = 0 } 10 components 6 components
Kronecker Delta $\delta^{\mu}{}_{\nu}$ = Diagonal[+1,+1,+1,+1] = 4D Identity = $I_{[4]} = \eta^{\mu}{}_{\nu} = g^{\mu}{}_{\nu}$
=====
This paper uses a mix of the two styles, as both are useful in various circumstances.

The following 4-Vectors { 4D (1,0)-Tensors } are all elements of classical SR and EM:

4-Position	$\mathbf{R} = R^\mu = (ct,\mathbf{r})$	[m]	Alt. $\mathbf{X} = X^\mu = (ct,\mathbf{x})$ only Lorentz, not Poincaré Invariant
4-Displacement	$\Delta\mathbf{R} = \Delta R^\mu = (c\Delta t,\Delta\mathbf{r})$	[m]	Finite $\Delta\mathbf{R} = \mathbf{R}_2 - \mathbf{R}_1$ fully Poincaré Invariant
4-Differential	$d\mathbf{R} = dR^\mu = (cdt,d\mathbf{r})$	[m]	Infinitesimal fully Poincaré Invariant
4-Velocity	$\mathbf{U} = U^\mu = \gamma(c,\mathbf{u})$	[m/s]	
4-Momentum	$\mathbf{P} = P^\mu = (E/c,\mathbf{p})$	[kg·m/s = N·s]	
4-WaveVector	$\mathbf{K} = K^\mu = (\omega/c,\mathbf{k})$	[{rad}/m]	
4-Gradient	$\boldsymbol{\partial} = \partial^\mu = (\partial_t/c,-\nabla)$	[1/m]	There is also a 4-VelocityGradient $\boldsymbol{\partial}_U$ [s/m]
4-(Dust)NumberFlux	$\mathbf{N} = N^\mu = (nc,\mathbf{n})$	[#/(m²·s) = (#/m³)·(m/s)]	
4-Current(Density)=4-ChargeFlux	$\mathbf{J} = J^\mu = (\rho c,\mathbf{j})$	[C/(m²·s) = (C/m³)·(m/s) = A/m²]	
4-(EM)VectorPotential	$\mathbf{A} = A^\mu = (\varphi/c,\mathbf{a})$	[kg·m/(C·s) = T·m]	Alt. $\mathbf{A}_{EM} = A_{EM}{}^\mu = (\varphi_{EM}/c,\mathbf{a}_{EM})$
4-(Minkowski)Force	$\mathbf{F} = F^\mu = \gamma(\dot{E}/c,\mathbf{f})$	[kg·m/s² = N]	

The mathematical Lorentz Scalar Product of two generic 4-Vectors $\mathbf{A} = A^\mu = (a^0,\mathbf{a})$ and $\mathbf{B} = B^\mu = (b^0,\mathbf{b})$ is:

$(\mathbf{A}\cdot\mathbf{B}) = A^\mu\eta_{\mu\nu}B^\nu = A_\nu B^\nu = A^\mu B_\mu = (a^0b^0 - \mathbf{a}\cdot\mathbf{b}) = (a^0{}_o b^0{}_o)$ which is a Lorentz Scalar Invariant { 4D (0,0)-Tensor }

Applying this rule to the individual SR 4-Vectors gives the following SR Lorentz Invariants (rest values indicated with naught $_o$):

$(\mathbf{R}\cdot\mathbf{R}) = (ct)^2 - \mathbf{r}\cdot\mathbf{r} = (ct_o)^2 = (c\tau)^2 = (i\lvert\mathbf{r}_o\rvert)^2$	: Proper Time ($t_o = \tau$), Proper Length ($\lvert\mathbf{r}_o\rvert$)	[s], [m]	{conversion factor c}
$(\mathbf{U}\cdot\mathbf{U}) = \gamma^2[c^2 - \mathbf{u}\cdot\mathbf{u}] = c^2$	: LightSpeed (c)	[m/s]	
$(\mathbf{P}\cdot\mathbf{P}) = (E/c)^2 - \mathbf{p}\cdot\mathbf{p} = (E_o/c)^2 = (m_oc)^2$	: Rest Energy (E_o), Rest Mass (m_o)	[kg·m²/s² = J], [kg]	{conversion factor c²}
$(\mathbf{K}\cdot\mathbf{K}) = (\omega/c)^2 - \mathbf{k}\cdot\mathbf{k} = (\omega_o/c)^2$	: Rest Angular Frequency (ω_o)	[{rad}/s]	
$(\boldsymbol{\partial}\cdot\boldsymbol{\partial}) = (\partial_t/c)^2 - \nabla\cdot\nabla = (\partial_{to}/c)^2 = (\partial_\tau/c)^2$	: d'Alembertian 4D Wave Equation $(\boldsymbol{\partial}\cdot\boldsymbol{\partial})$	[1/m²]	
$(\mathbf{N}\cdot\mathbf{N}) = (nc)^2 - \mathbf{n}\cdot\mathbf{n} = (n_oc)^2$	: Rest Number Density (n_o)	[#/m³]	
$(\mathbf{J}\cdot\mathbf{J}) = (\rho c)^2 - \mathbf{j}\cdot\mathbf{j} = (\rho_oc)^2$	: Rest Charge Density (ρ_o)	[C/m³]	
$(\mathbf{A}\cdot\mathbf{A}) = (\varphi/c)^2 - \mathbf{a}\cdot\mathbf{a} = (\varphi_o/c)^2$	: Rest Electric Potential (φ_o)	[kg·m²/(C·s²) = V = J/C]	
$(\mathbf{F}\cdot\mathbf{F}) = \gamma^2[(\dot{E}/c)^2 - \mathbf{f}\cdot\mathbf{f}] = (\dot{E}_o/c)^2$	: Rest Power ($\dot{E}_o$)	[kg·m²/s³ = W = J/s]	
$(\mathbf{U}\cdot\boldsymbol{\partial}) = \gamma[\partial_t + \mathbf{u}\cdot\nabla] = \gamma d/dt = d/d\tau$	: Proper Time Derivative ($d/d\tau$)	[1/s]	

The SR 4-Vectors also have some fundamental relations between one another (again using rest naught $_o$ notation):

4-Position	$\mathbf{R} = R^\mu = (ct,\mathbf{r}) \in$ <Event> $\in$ <Time·Space>	
4-Velocity	$\mathbf{U} = U^\mu = \gamma(c,\mathbf{u}=\dot{\mathbf{r}}) = (\mathbf{U}\cdot\boldsymbol{\partial})\mathbf{R} = (d/d\tau)\mathbf{R} = d\mathbf{R}/d\tau = \gamma d\mathbf{R}/dt$	$\mathbf{u} = \dot{\mathbf{r}} = d\mathbf{r}/dt$: $(\mathbf{U}\cdot\boldsymbol{\partial})=(d/d\tau)=\gamma(d/dt)$
4-Momentum	$\mathbf{P} = P^\mu = (E/c=mc,\mathbf{p}=m\mathbf{u}) = (E_o/c^2)\mathbf{U} = m_o\mathbf{U}$	$E = mc^2$: $E_o = m_oc^2$
4-WaveVector	$\mathbf{K} = K^\mu = (\omega/c=1/c\mathcal{T},\mathbf{k}=\omega\hat{\mathbf{n}}/v_{phase}=\hat{\mathbf{n}}/\lambda\!\!\!^{-}=\omega\mathbf{u}/c^2) = (\omega_o/c^2)\mathbf{U}$	$\hat{\mathbf{n}}$ = unit-direction 3-vector
4-Gradient	$\boldsymbol{\partial} = \partial^\mu = (\partial_t/c,-\nabla) = (\partial/\partial R_\mu)$	$\nabla = ({}^{\partial}/_{\partial x},{}^{\partial}/_{\partial y},{}^{\partial}/_{\partial z}) = (\partial_x,\partial_y,\partial_z)$ $R_\mu = \eta_{\mu\nu}R^\nu$
4-(Dust)NumberFlux	$\mathbf{N} = N^\mu = (nc,\mathbf{n}=n\mathbf{u}) = n_o\mathbf{U}$	
4-Current(Density)=4-ChargeFlux	$\mathbf{J} = J^\mu = (\rho c,\mathbf{j}=\rho\mathbf{u}) = \rho_o\mathbf{U}$	
4-(EM)VectorPotential	$\mathbf{A} = A^\mu = (\varphi/c,\mathbf{a}=\varphi\mathbf{u}/c^2) = (\varphi_o/c^2)\mathbf{U}$	
4-(Minkowski)Force	$\mathbf{F} = F^\mu = \gamma(\dot{E}/c,\mathbf{f}=\dot{\mathbf{p}}) = (\mathbf{U}\cdot\boldsymbol{\partial})\mathbf{P} = (d/d\tau)\mathbf{P} = d\mathbf{P}/d\tau = \gamma d\mathbf{P}/dt$	$\dot{E} = dE/dt$, $\mathbf{f} = \dot{\mathbf{p}} = d\mathbf{p}/dt$
Faraday EM 4-Tensor	$F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha = (\boldsymbol{\partial}\wedge\mathbf{A})$	$F^{\alpha\beta}$ = AntiSymmetric 4D(2,0)-Tensor
Maxwell Equation with Source	$\boldsymbol{\partial}\cdot F^{\alpha\beta} = (\boldsymbol{\partial}\cdot\boldsymbol{\partial})\mathbf{A} - \boldsymbol{\partial}(\boldsymbol{\partial}\cdot\mathbf{A}) = (\mu_o)\mathbf{J}$	{or $(\boldsymbol{\partial}\cdot\boldsymbol{\partial})\mathbf{A} = (\mu_o)\mathbf{J}$ in Lorenz-Gauge $(\boldsymbol{\partial}\cdot\mathbf{A}) = 0$}
4D Lorentz Force Equation	$\mathbf{U}\cdot F^{\alpha\beta} = (1/q)\mathbf{F} = (1/q)d\mathbf{P}/d\tau$	q = EM charge
Conservation of EM Current(Density)	$\boldsymbol{\partial}\cdot\mathbf{J} = 0$	$\partial_\alpha\partial_\beta F^{\alpha\beta} = 0 = (\mu_o)\boldsymbol{\partial}\cdot\mathbf{J}$

Beautiful proof based on Symmetric:AntiSymmetric tensors

Fundamental Constants (Lorentz Invariants) which elegantly appear from SR 4-Vector formalism:

LightSpeed (Vacuum)	(c)	[m/s]	
RestMass=InvariantMass	(m_o)	[kg]	Varies depending on particle type
EM charge (=e for electron)	(q)	[C]	Varies depending on particle type
Electric Constant/Permittivity (Vacuum)	(ε_o)	[F/m = C²·s²/kg·m³]	$(\varepsilon_o\mu_o) = 1/c^2$
Magnetic Constant/Permeability (Vacuum)	(μ_o)	[H/m = kg·m/C²]	$(\varepsilon_o\mu_o) = 1/c^2$
Planck's Reduced = Dirac's Constant	($\hbar$)	[J·s = (kg·m/s)·(m) = Action]	We shall see this in the next section…
Boltzmann's Constant	(k_B)	[J/°K·m²·s = kg/°K·s³]	See the full presentation SRQM-RoadMap

Constants with (Vacuum) are considered in their non-interacting state, their effective values change with matter-interaction.
ex. Photon speed varies in a medium, such as going through a prism, which gives rise to diffraction effects.
Note: (c) is large, but never → ∞; ($\hbar$) is small, but never → 0; Always use realistic limits, ex. { $\lvert\mathbf{v}\rvert \ll c$ } for SR→CM, RQM→ QM

One can divide two SR 4-Vectors $\mathbf{A} = A^{\mu} = (a^0,\mathbf{a})$ and $\mathbf{B} = B^{\mu} = (b^0,\mathbf{b})$ by using a third intermediary 4-Vector $\mathbf{V} = V^{\mu} = (v^0,\mathbf{v})$:
$|\mathbf{A}|/|\mathbf{B}| = (\mathbf{A}\cdot\mathbf{V})/(\mathbf{B}\cdot\mathbf{V}) = (a^0{}_o v^0{}_o)/(b^0{}_o v^0{}_o) = (a^0{}_o/b^0{}_o)$ which is a Lorentz Scalar Invariant { 4D (0,0)-Tensor }

Applying the division rule with certain SR 4-Vector combinations leads to the following relations:
$(\mathbf{P}\cdot\mathbf{U})/(\mathbf{K}\cdot\mathbf{U}) = \gamma(E - \mathbf{p}\cdot\mathbf{u})/[\gamma(\omega - \mathbf{k}\cdot\mathbf{u})] = E_o/\omega_o \quad \rightarrow |\mathbf{P}|/|\mathbf{K}| = E_o/\omega_o$
$(\mathbf{P}\cdot\mathbf{K})/(\mathbf{K}\cdot\mathbf{K}) = (E\omega/c^2 - \mathbf{p}\cdot\mathbf{k})/[(\omega/c)^2 - \mathbf{k}\cdot\mathbf{k}] = m_o\omega_o/(\omega_o/c)^2 \quad \rightarrow |\mathbf{P}|/|\mathbf{K}| = E_o/\omega_o$
$(\mathbf{P}\cdot\mathbf{P})/(\mathbf{K}\cdot\mathbf{P}) = (E^2/c^2 - \mathbf{p}\cdot\mathbf{p})/(E\omega/c^2 - \mathbf{p}\cdot\mathbf{k}) = (m_oc)^2/(m_o\omega_o) \quad \rightarrow |\mathbf{P}|/|\mathbf{K}| = E_o/\omega_o$
$(\mathbf{P}\cdot\mathbf{R})/(\mathbf{K}\cdot\mathbf{R}) = (Et - \mathbf{p}\cdot\mathbf{r})/(\omega t - \mathbf{k}\cdot\mathbf{r}) = (-S_{action,free\ particle})/(-\Phi_{phase,planewave}) \quad \rightarrow |\mathbf{P}|/|\mathbf{K}| = E_o/\omega_o$
Also
$\mathbf{K} = (\omega_o/c^2)\mathbf{U}$ or $\mathbf{U} = (c^2/\omega_o)\mathbf{K}$
Thus
$\mathbf{P} = (E_o/c^2)\mathbf{U} = (E_o/c^2)(c^2/\omega_o)\mathbf{K} = (E_o/\omega_o)\mathbf{K}$: $\mathbf{P} = (E_o/\omega_o)\mathbf{K} \quad \rightarrow |\mathbf{P}|/|\mathbf{K}| = E_o/\omega_o$

Analysis of Dirac's Constant (ħ) : Planck's Constant (h), with (ħ = h/2π) in the context of SRQM:
It is an empirical fact that the Lorentz Scalar Invariant $E_o/\omega_o = \gamma E_o/\gamma\omega_o = E/\omega = (\hbar)$ [J·s]
for all known experimental measurements. The SR 4D Tensor rules show that one doesn't need a quantum axiom for this.
(ħ) is actually an empirically-measurable quantity, just like (c), (e), (G), (k_B), (μ_o) or the other fundamental constants, which are also Lorentz Scalar Invariants. (ħ) can be measured classically from the photoelectric effect, from the inverse photoelectric effect, from LED's (injection electroluminescence), from Atomic Line Spectra (Bohr Atom), from the Duane-Hunt Law in Bremsstrahlung, from Electron Diffraction in crystals, from the Watt/Kibble-Balance, from Incandescent Blackbody Intensity-Temperature Relations, from Compton Scattering, from the Stern-Gerlach experiment, etc.

Regarding physics simulations of measurements of Dirac's : Planck's Constant (ħ : h), see:
==
http://scirealm.org/Physics-PlanckConstantViaAtomicLineSpectra.html
http://scirealm.org/Physics-PlanckConstantViaComptonScattering.html
http://scirealm.org/Physics-PlanckConstantViaElectronDiffraction.html
http://scirealm.org/Physics-PlanckConstantViaIncandescence.html
http://scirealm.org/Physics-PlanckConstantViaLEDs.html
http://scirealm.org/Physics-PlanckConstantViaSternGerlach.html
==

This implies the following SR (wave-like ∿) relation:
$\mathbf{P} = \hbar\mathbf{K} = (E/c,\mathbf{p}) = \hbar(\omega/c,\mathbf{k})$
The temporal part $\{E = \hbar\omega\}$ gives Einstein's photoelectric quantum postulate.
The spatial part $\{\mathbf{p} = \hbar\mathbf{k}\}$ gives de Broglie's matter-wave quantum postulate.

This is very similar to Einstein's other SR (particle-like ·) relation:
$\mathbf{P} = m_o\mathbf{U} = (E/c,\mathbf{p}) = m_o\gamma(c,\mathbf{u}) = m(c,\mathbf{u}) = (mc,m\mathbf{u}) = (E/c^2)(c,\mathbf{u}) = (E_o/c^2)\gamma(c,\mathbf{u}) = (E_o/c^2)\mathbf{U}$
The temporal part $\{E = \gamma m_oc^2 = mc^2 = \gamma E_o\}$ gives Einstein's famous energy:mass relation (in both rest & relativistic forms).
The spatial part $\{\mathbf{p} = \gamma m_o\mathbf{u} = m\mathbf{u} = E\mathbf{u}/c^2 = \gamma E_o\mathbf{u}/c^2\}$ gives Einstein's relativistic momentum.

The 4-WaveVector **K** exists in all physics/mathematical contexts as a solution of the 4D Invariant d'Alembertian $(\partial\cdot\partial) = (\partial_t/c)^2 - \nabla\cdot\nabla$
It is an empirical:mathematical fact that all waves (classical/relativistic/EM/quantum/purely-mathematical) can be modeled using complex planewaves which obey a principle of superposition: Wave $\Psi = \Sigma_n [\psi_n]$ = sum of complex planewaves.
An individual wavefunction of form $\psi = (a)e^{\wedge}\{\pm i(\mathbf{K}\cdot\mathbf{X})\} = (a)e^{\wedge}\{\pm i\varphi\} = (a)e^{\wedge}\{\pm iS_{action}/\hbar\}$ has amplitude (a) that can be:
4D (0,0)-Tensor A {ex. Quantum Scalar}
4D (1,0)-Tensor A^{μ} {ex. EM/Photonic}
4D (2,0)-Tensor $A^{\mu\nu}$ {ex. Gravitational Wave}
This gives the mathematical 4-Vector relation:
$\partial = -i\mathbf{K} = (\partial_t/c,-\nabla) = -i(\omega/c,\mathbf{k})$
The temporal part $\{\partial_t = -i\omega\}$ or $\{\omega = i\partial_t\}$ gives temporal:frequency complex planewave change:operator
The spatial part $\{\nabla = i\mathbf{k}\}$ or $\{\mathbf{k} = -i\nabla\}$ gives spatial:wavenumber complex planewave change:operator

$\psi = (a)e^{\wedge}\{\pm i(\mathbf{K}\cdot\mathbf{X})\}$; There exists also $\psi^* = (a^*)e^{\wedge}\{\mp i(\mathbf{K}\cdot\mathbf{X})\}$, giving $\psi^*\psi = (a^*)(a)$, independent of the phase part $(\mathbf{K}\cdot\mathbf{X})$
$\partial[\psi] = \partial[(a)e^{\wedge}\{\pm i(\mathbf{K}\cdot\mathbf{X})\}] = \pm i\mathbf{K}[(a)e^{\wedge}\{\pm i(\mathbf{K}\cdot\mathbf{X})\}] = \pm i\mathbf{K}[\psi]$, with the minus sign $\{\partial = -i\mathbf{K}\}$ typically chosen for historical reasons.
$\partial[\psi^*\psi] = (\partial[\psi^*]\psi + \psi^*\partial[\psi]) = (\mp i\mathbf{K}\psi^*\psi \pm i\mathbf{K}\psi^*\psi) = 0 = \partial[(a^*)(a)]$, giving conservation of probability $[\psi^*\psi]$.

SR Wave Energy-Momentum, Dispersion, & Velocity Relations using 4D Tensors:

4-Vectors = 4D (1,0)-Tensors **Properties**

4-Position	$\mathbf{R} = R^\mu = (ct,\mathbf{r})$	[m]	c = Invariant LightSpeed
4-Velocity	$\mathbf{U} = U^\mu = \gamma(c,\mathbf{u})$	[m/s]	$\gamma = 1/\text{Sqrt}[1-(u/c)^2]$
4-Momentum	$\mathbf{P} = P^\mu = (E/c,\mathbf{p})$	[kg·m/s]	
4-WaveVector	$\mathbf{K} = K^\mu = (\omega/c,\mathbf{k} = \omega\hat{\mathbf{n}}/v_{phase})$	[{rad}/m]	$v_{phase} = \omega/k$

4-Vectors = 4D (1,0)-Tensors **Relations**

4-Velocity	$\mathbf{U} = d\mathbf{R}/d\tau$	τ = Proper Time
4-Momentum	$\mathbf{P} = (E_o/c^2)\mathbf{U} = m_o\mathbf{U} = \hbar\mathbf{K}$	E_o = Rest Energy, m_o = Rest mass, $\hbar$ = Planck's const
4-WaveVector	$\mathbf{K} = (\omega_o/c^2)\mathbf{U}$	ω_o = Rest Angular Frequency

$\mathbf{K} = (\omega/c,\mathbf{k} = \omega\hat{\mathbf{n}}/v_{phase}) = (\omega_o/c^2)\mathbf{U} = (\omega_o/c^2)\gamma(c,\mathbf{u}) = (\gamma\omega_o/c^2)(c,\mathbf{u}) = (\gamma\omega_o/c,\gamma\omega_o\mathbf{u}/c^2)$

The temporal part: $\omega = \gamma\omega_o$ {Relativistic ω : Rest ω_o} ang.-frequency

The spatial part: $\mathbf{k} = \omega\hat{\mathbf{n}}/v_{phase} = \gamma\omega_o\mathbf{u}/c^2 = \omega\mathbf{u}/c^2$ {Relativistic **k**} 3-wavenumber

$\hat{\mathbf{n}}/v_{phase} = \mathbf{u}/c^2$

$\mathbf{u} = c^2\hat{\mathbf{n}}/v_{phase}$

$|v_{phase} * u| = c^2$

$\mathbf{P} = (E/c,\mathbf{p}) = (E_o/c^2)\mathbf{U} = (E_o/c^2)\gamma(c,\mathbf{u}) = (\gamma E_o/c^2)(c,\mathbf{u}) = (\gamma E_o/c,\gamma E_o\mathbf{u}/c^2)$

The temporal part: $E = \gamma E_o$ {Relativistic E : Rest E_o} energy

The spatial part: $\mathbf{p} = \gamma E_o\mathbf{u}/c^2 = E\mathbf{u}/c^2 = Ec^2\hat{\mathbf{n}}/(c^2\, v_{phase}) = E\hat{\mathbf{n}}/v_{phase}$ {Relativistic **p**} 3-momentum

$= \gamma m_o\mathbf{u} \quad = m\mathbf{u}$

$\mathbf{P} = (E/c,\mathbf{p}) = \hbar\mathbf{K} = \hbar(\omega/c,\mathbf{k} = \omega\hat{\mathbf{n}}/v_{phase})$

The temporal part: $E = \hbar\omega$ The Einstein photoelectric eqn.

The spatial part: $\mathbf{p} = \hbar\mathbf{k} = \hbar\omega\hat{\mathbf{n}}/v_{phase} = E\hat{\mathbf{n}}/v_{phase}$ The de Broglie matter-wave eqn.

So, 3-momentum $|\mathbf{p}|$ = Energy E / phase-velocity v_{phase} in general.

$(\mathbf{U}\cdot\mathbf{U}) = \gamma^2[(c)^2 - \mathbf{u}\cdot\mathbf{u}] = (c)^2\gamma^2[1 - \mathbf{u}\cdot\mathbf{u}/c^2] = (c)^2$

$(\mathbf{P}\cdot\mathbf{P}) = [(E/c)^2 - \mathbf{p}\cdot\mathbf{p}] = (E_o/c)^2$

$(E/c)^2 = \mathbf{p}\cdot\mathbf{p} + (E_o/c)^2$

$(E)^2 = \mathbf{p}\cdot\mathbf{p}c^2 + (E_o)^2$

$E^2 = (pc)^2 + (E_o)^2$

$E = \sqrt{[(pc)^2 + (E_o)^2]}$: If photonic, (E_o=0), then $E = |\mathbf{p}|c$, which gives $u_{photon} = v_{phase,photon} = c$, from $|v_{phase} * u| = c^2$

$(\mathbf{K}\cdot\mathbf{K}) = [(\omega/c)^2 - \mathbf{k}\cdot\mathbf{k}] = (\omega_o/c)^2$

$(\omega/c)^2 = k^2 + (\omega_o/c)^2$

$d[(\omega/c)^2] = d[k^2 + (\omega_o/c)^2]$

$2\omega\, d\omega/c^2 = 2k\, dk + 0$

$\omega\, d\omega/c^2 = k\, dk$

$d\omega/dk = c^2 k/\omega = c^2/v_{phase} = u$

Recapping:

$\omega/k = v_{phase}$ From the formal definition of 4-WaveVector $\mathbf{K} = K^\mu = (\omega/c,\mathbf{k} = \omega\hat{\mathbf{n}}/v_{phase})$

$d\omega/dk = u = v_{group} = v_{particle}$ Derived from Lorentz Scalar Product $(\mathbf{K}\cdot\mathbf{K}) = [(\omega/c)^2 - \mathbf{k}\cdot\mathbf{k}] = (\omega_o/c)^2$

$\mathbf{u} = c^2\hat{\mathbf{n}}/v_{phase}$ The relation between particle and wave velocities, from $\mathbf{K} = (\omega_o/c^2)U$

To recap, the SR 4-Vectors have some very simple, fundamental, and Invariant Lorentz Scalar relations between one another:

4-Position $\mathbf{R} = R^\mu = (ct,\mathbf{r}) \in$ <Event> $\in$ <Time·Space>
4-Velocity $\mathbf{U} = U^\mu = \gamma(c,\mathbf{u}) = (\mathbf{U}\cdot\partial)\mathbf{R}=(d/d\tau)\mathbf{R}$
4-Momentum $\mathbf{P} = P^\mu = (E/c,\mathbf{p}) = (m_o)\mathbf{U}$
4-WaveVector $\mathbf{K} = K^\mu = (\omega/c,\mathbf{k}) = (1/\hbar)\mathbf{P}$
4-Gradient $\partial = \partial^\mu = (\partial_t/c,-\nabla) = (-i)\mathbf{K}$

The main point of work above is to show that the last two relations, $\{\mathbf{K} = (1/\hbar)\mathbf{P}\}$ and $\{\partial = -i\mathbf{K}\}$, alternately $\{\mathbf{P} = \hbar\mathbf{K}\}$ and $\{\mathbf{K}= i\partial\}$, are of the same character as the other relations in this group. They are derivable from SR and/or purely-mathematical principles, and do not require quantum axioms for their existence. These will then lead to wave-particle duality and other derived quantum "axioms".

The Lorentz Scalar Products of these SR-related 4-Vectors gives the following chain of Invariants:

$(\mathbf{R}\cdot\mathbf{R}) = (c\tau)^2$
$(\mathbf{U}\cdot\mathbf{U}) = (c)^2$
$(\mathbf{P}\cdot\mathbf{P}) = (m_oc)^2$
$(\mathbf{K}\cdot\mathbf{K}) = (m_oc/\hbar)^2$: $(\hbar/m_oc) = ƛ = (c/\omega_o)$ is the reduced Compton wavelength [length = m]
$(\partial\cdot\partial) = (im_oc/\hbar)^2 = -(m_oc/\hbar)^2 = -(\omega_o/c)^2$: The fundamental quantum Klein-Gordon (KG) RQM wave relation: **[SR→QM]**

Each step is a logical progression, taking into account the simple relation between each of these SR 4-Vectors.
In the same way that the Relativistic 4D Euler-Lagrange Relation $(\mathbf{U}\cdot\partial_\mathbf{R})[\partial_\mathbf{U}] = (d/d\tau)[\partial_\mathbf{U}] = \partial_\mathbf{R}$ {itself a variation of $(d/d\tau)[\mathbf{R}] = \mathbf{U}$} implies that there can exist a Lagrangian function (L) that solves it, the KG relation $(\partial\cdot\partial) = (im_oc/\hbar)^2 = -(m_oc/\hbar)^2 = -(\omega_o/c)^2$ implies that there can exist a "wavefunction" (Ψ) which solves it. One does not need a presupposed quantum axiom. The Klein-Gordon relation gives a relativistic, 2nd order, linear Partial Differential Eqn (PDE). The fact that it is a linear PDE leads to the principle of quantum superposition. The standard Schrödinger quantum wave equation is the non-relativistic ($|\mathbf{v}|<<c$) limit-case of the KG relativistic quantum wave equation, which continues to show superposition.

The Klein-Gordon Eqn. $\{(\partial\cdot\partial) - (im_oc/\hbar)^2 = 0\}$ is itself the Relativistic Quantum (RQM) Equation for spin=0 particles (4-Scalars).
Factoring the KG Eqn. $\{(\partial + im_oc/\hbar)\cdot(\partial - im_oc/\hbar) = 0\}$ leads to the RQM Dirac Eqn. for spin=1/2 particles (4-Spinors).
Applying the KG Eqn. to a 4-Vector field leads to the RQM Proca Eqn. for spin=1 particles (4-Vectors).
Taking the low-velocity-limit ($|\mathbf{v}|<<c$) of the KG leads to the standard QM non-relativistic Schrödinger Eqn., for spin=0 (4-Scalar).
Taking the low-velocity-limit ($|\mathbf{v}|<<c$) of the Dirac leads to the standard QM non-relativistic Pauli Eqn., for spin=1/2 (4-Spinor).
Setting RestMass $\{m_o \to 0\}$ gives the RQM Free Wave (4-Scalar), Weyl (4-Spinor), and Free Maxwell Standard EM (4-Vector) Eqns.
In all of these cases, the equations can be modified to work with various potentials/interactions by using more SR 4-Vectors, and more empirically-found relations between them, ex. the Minimal Coupling Relations $\{\mathbf{P} = \mathbf{P_T} - q\mathbf{A}\}$:
4-TotalMomentum $\{\mathbf{P_T} = (H/c{=}E_T/c,\mathbf{p_T}) = \mathbf{P} + q\mathbf{A}\}$, with 4-Momentum $\mathbf{P}$, EM charge (q), and 4-(EM)VectorPotential $\mathbf{A}$
Also note that generating QM from RQM (via a low-energy limit) is much more natural and mathematically well-defined than attempting to "relativize or generalize" a given NRQM equation. Facts assumed from a non-relativistic equation may or may not be applicable to a relativistic one, whereas the relativistic facts are still true in the low-velocity limiting-cases. This leads again to the idea that QM is an approximation-only of the more general RQM, just as SR is an approximation-only of the more general GR.

The standard Schrödinger QM Relations derived from SR:

Again, we examine these SR 4-Vector relations derived above…
$\mathbf{P} = \hbar\mathbf{K}$ (a relation which is entirely empirical, based on just SR arguments, shown above)
$\mathbf{K} = i\partial$ (which is a relation for complex plane-waves, used in classical EM)

combining...
$\mathbf{P} = i\hbar\partial = (E/c,\mathbf{p}) = i\hbar(\partial_t/c,-\nabla)$
The temporal part $\{E = i\hbar\partial_t = i\hbar\ \partial/\partial t\}$ gives unitary QM time evolution.
The spatial part $\{\mathbf{p} = -i\hbar\nabla\}$ gives the QM momentum operator.

These are the main quantum relations used in standard QM calculations, as well as in RQM.
Just as a note:
The 4-Momentum **P** is used in purely-relativistic particle collision calculations.
The 4-WaveVector **K** is used in purely-relativistic Doppler effect calculations.
Both are used in the relativistic Compton effect photon-electron scattering calculations.
The 4-Gradient ∂ is used in several purely-relativistic settings: charge conservation $(\partial\cdot\mathbf{J}) = 0$, particle # conservation $(\partial\cdot\mathbf{N}) = 0$, Lorenz EM Gauge $(\partial\cdot\mathbf{A}) = 0$, invariant d'Alembertian $(\partial\cdot\partial)$, proper time derivative $(\mathbf{U}\cdot\partial)$, Minkowski Metric $\partial^\mu[R^\nu] = \eta^{\mu\nu}$, etc.
These facts show that the tensorial 4-Vectors are from SR, and not QM axioms.

Non-zero SR→QM Commutation Relation between position $\mathbf{x} = x^j$ and momentum $\mathbf{p} = p^k$:

4-Position $\mathbf{R} = R^\mu = (ct,\mathbf{r}) = (ct,x,y,z) = \mathbf{X} = X^\mu$ — $\mathbf{X}$ is common alt. form of $\mathbf{R}$

4-Gradient $\partial = \partial^\mu = (\partial_t/c,-\nabla) = (\partial/_{c\partial t},-\partial/_{\partial x},-\partial/_{\partial y},-\partial/_{\partial z}) = (\partial_t/c,-\partial_x,-\partial_y,-\partial_z) = \partial/\partial X_\mu$ — with $X_\mu = \eta_{\mu\nu}X^\nu$

Let { f } be an arbitrary SR function.

$\mathbf{X}[f] = \mathbf{X}f$ & $\partial[f] = \partial[f]$ are the primitive relations. The following is basic calculus.

$\mathbf{X}[\partial[f]] = \mathbf{X}\partial[f]$
$\partial[\mathbf{X}[f]] = \partial[\mathbf{X}f] = \partial[\mathbf{X}]f + \mathbf{X}\partial[f]$
$\partial[\mathbf{X}f] - \mathbf{X}\partial[f] = \partial[\mathbf{X}]f$
$\partial[\mathbf{X}[f]] - \mathbf{X}[\partial[f]] = \partial[\mathbf{X}]f$

Recognizing the commutation relation $\mathbf{[A,B]}f = \mathbf{A}[\mathbf{B}[f]] - \mathbf{B}[\mathbf{A}[f]]$ and using commutator notation...

$\mathbf{[\partial,X]}f = \partial[\mathbf{X}]f$ {temporarily using **bold [,] commutator** and non-bold [ ] function-indicator for clarity}

And since { f } was an arbitrary SR function, we can remove it (or set it to unity), which leaves the functional form:

$\mathbf{[\partial,X]} = \partial[\mathbf{X}] = (\partial_t/c,-\nabla)[(ct,\mathbf{r})] = (\partial_t/c,-\partial_x,-\partial_y,-\partial_z)[(ct,x,y,z)] = \text{Diag}[+1,-1,-1,-1]_{(\text{Cartesian})} = \eta^{\mu\nu}$ = Minkowski Metric

Thus,

$\mathbf{[\partial,X]} = \mathbf{[}\partial^\mu\mathbf{,}X^\nu\mathbf{]} = \partial^\mu[X^\nu] = \eta^{\mu\nu}$ = Minkowski Metric

At this point, we have established purely mathematically, that there exists in SR a non-zero commutation relation between the SR 4-Gradient ∂ and SR 4-Position $\mathbf{X}$.
Note also that { $\mathbf{X}[f] = \mathbf{X}f$ } doesn't actually say that $\mathbf{X}$ is an operator.
It just says that {an X next to an f} = {X times f}.
$\mathbf{X}$ could be an operator or just a numerical vector.
The 4-Gradient ∂ is definitely an operator, because it is already an operator:function in pure SR, and uses basic calculus rules.

Now, using these 4-Vectors and the relations between them derived from SR above:

4-Position $\mathbf{R} = R^\mu = (ct,\mathbf{r}) = (x^0,x^i) = \mathbf{X} = X^\mu$
4-Momentum $\mathbf{P} = P^\mu = (E/c,\mathbf{p}) = (p^0,p^i) = \hbar\mathbf{K}$ (a relation which is entirely empirical, based on just SR arguments, shown above)
4-WaveVector $\mathbf{K} = K^\mu = (\omega/c,\mathbf{k}) = (k^0,k^i) = i\partial$ (which is a relation for complex plane-waves, used in classical EM)
4-Gradient $\partial = \partial^\mu = (\partial_t/c,-\nabla) = (\partial^0,\partial^i) = \partial/\partial X_\mu$ with $X_\mu = \eta_{\mu\nu}X^\nu$

$[\partial,\mathbf{X}] = [\partial^\mu,X^\nu] = \eta^{\mu\nu}$
$[i\partial,\mathbf{X}] = [i\partial^\mu,X^\nu] = i\eta^{\mu\nu}$
$[\mathbf{K},\mathbf{X}] = [K^\mu,X^\nu] = i\eta^{\mu\nu}$
$[\hbar\mathbf{K},\mathbf{X}] = [\hbar K^\mu,X^\nu] = i\hbar\eta^{\mu\nu}$
$[\mathbf{P},\mathbf{X}] = [P^\mu,X^\nu] = i\hbar\eta^{\mu\nu}$
$[\mathbf{X},\mathbf{P}] = [X^\mu,P^\nu] = -i\hbar\eta^{\mu\nu}$ This is a major result of **[SR→QM]**.
The temporal part $[x^0,p^0] = [ct,E/c] = [t,E] = -i\hbar\eta^{00} = -i\hbar$ is the "oft-misunderstood" time-energy commutation.
The spatial part $[x^j,p^k] = i\hbar\delta^{jk}$ is the standard QM Canonical Commutation Relation.
The mixed parts $[x^0,p^k] = [x^j,p^0] = \eta^{0k} = \eta^{j0} = 0$, meaning these parts commute normally.

Similar 4-vector arguments lead to the standard angular-momentum quantum commutation relations via 4-AngularMomentum $M^{\mu\nu} = X^\mu \wedge P^\nu$. In fact, the entire Poincaré Algebra (Lie Algebra of the Poincaré Group) can be generated in this fashion.
P^μ is generator of SpaceTime-Translations (ΔX^μ). $M^{\mu\nu}$ is the generator of Lorentz-Transformations ($\Lambda^\mu{}_\nu$). $\eta^{\mu\nu}$ is the Minkowski Metric.

Canonical (Momentum,Position):	$[P^\mu,X^\nu] = i\hbar\eta^{\mu\nu}$	from $[\partial^\mu,X^\nu] = \eta^{\mu\nu}$
[Linear, Linear] Momentum:	$[P^\mu,P^\nu] = [X^\mu,X^\nu] = 0^{\mu\nu}$	from partials commuting $[\partial^\mu,\partial^\nu] = 0^{\mu\nu}$
[Angular, Linear] Momentum:	$[M^{\mu\nu},P^\sigma] = i\hbar(\eta^{\sigma\nu}P^\mu - \eta^{\sigma\mu}P^\nu)$	from $O^{\mu\nu} = X^\mu \wedge \partial^\nu$
[Angular, Angular] Momentum:	$[M^{\mu\nu},M^{\sigma\rho}] = i\hbar(\eta^{\nu\sigma}M^{\mu\rho} + \eta^{\sigma\mu}M^{\rho\nu} + \eta^{\mu\rho}M^{\nu\sigma} + \eta^{\rho\nu}M^{\sigma\mu})$	

The (i) and (ħ) again come from SR. The algebra is all real and overall dimensionless when using only {$\mathbf{X}$ and ∂} in the definitions.
Likewise, the general mathematical uncertainty relations, { $\sigma_A{}^2\sigma_B{}^2 \geq (1/2)|\langle[\mathbf{A},\mathbf{B}]\rangle|$ }, based on commutation relations, lead to the standard physical quantum Heisenberg uncertainty relations. Also note that the commutator order of operations is in accord with SR causality conditions. While spacelike-separated events |here⟩ and |there⟩ may occur in any temporal order, all observers will see the same temporal order of timelike-separated events. Thus, for time-like separations, if measurement-event **A** occurs temporally before measurement-event **B**, then this would be written in operator notation as: $|\Psi'\rangle = \mathbf{A}|\Psi\rangle$ then $|\Psi''\rangle = \mathbf{B}|\Psi'\rangle = \mathbf{BA}|\Psi\rangle$.
The operator order shows the timelike-separated order of measurement-events.
Due to non-zero commutation relations, $\mathbf{AB}|\Psi\rangle$ would likely give a different result.

Using Green's Vector Identity to establish a Conserved Current (could be # {dust or probability}or charged):
Consider the following purely mathematical argument:
$\partial\cdot(f\,\partial[g] - \partial[f]\, g) = f\,\partial\cdot\partial[g] - \partial\cdot\partial[f]\, g$ with (f) and (g) as SR Lorentz Scalar functions

Proof of the 4-Divergence relation:
==========================
$\partial\cdot(f\,\partial[g] - \partial[f]\, g)$
$= \partial\cdot(f\,\partial[g]) - \partial\cdot(\partial[f]\, g)$
$= (f\,\partial\cdot\partial[g] + \partial[f]\cdot\partial[g]) - (\partial[f]\cdot\partial[g] + \partial\cdot\partial[f]\, g)$
$= f\,\partial\cdot\partial[g] - \partial\cdot\partial[f]\, g$
We can also multiply this by a constant Lorentz Invariant Scalar Constant (s), for dimensional units.
$s\,(f\,\partial\cdot\partial[g] - \partial\cdot\partial[f]\, g) = s\,\partial\cdot(f\,\partial[g] - \partial[f]\, g) = \partial\cdot[s(f\,\partial[g] - \partial[f]\, g)] = \partial\cdot\mathbf{J}$
==========================
Thus there mathematically exists a 4-Current **J** derivable from the SR d'Alembertian $(\partial\cdot\partial)$

Now, applied to SR physics... Start with the Klein-Gordon relation derived above from the Lorentz Scalar Product:
$\partial\cdot\partial = (-im_oc/\hbar)^2 = -(m_oc/\hbar)^2$
$\partial\cdot\partial + (m_oc/\hbar)^2 = 0$

Let it act on SR Lorentz Invariant function g
$\partial\cdot\partial[g] + (m_oc/\hbar)^2[g] = 0\,[g]$
Then pre-multiply by f
$[f]\partial\cdot\partial[g] + [f]\,(m_oc/\hbar)^2[g] = [f]\,0\,[g]$
$[f]\partial\cdot\partial[g] + (m_oc/\hbar)^2[f][g] = 0$

Let it act on SR Lorentz Invariant function f
$\partial\cdot\partial[f] + (m_oc/\hbar)^2[f] = 0\,[f]$
Then post-multiply by g
$\partial\cdot\partial[f][g] + (m_oc/\hbar)^2[f][g] = 0\,[f][g]$
$\partial\cdot\partial[f][g] + (m_oc/\hbar)^2[f][g] = 0$

Now, subtract the two equations
$\{[f]\,\partial\cdot\partial[g] + (m_oc/\hbar)^2[f][g] = 0\} - \{\,\partial\cdot\partial[f][g] + (m_oc/\hbar)^2[f][g] = 0\}$
$[f]\,\partial\cdot\partial[g] + (m_oc/\hbar)^2[f][g] - \partial\cdot\partial[f][g] - (m_oc/\hbar)^2[f][g] = 0$
$f\,\partial\cdot\partial[g] - \partial\cdot\partial[f]\, g = 0$

As noted from the mathematical Green's Vector Identity, this can be written as a 4-Divergence
with the additional constraint that it now also equates to 0, meaning that it is a conserved 4-Current **J**.
$s\,\partial\cdot(f\,\partial[g] - \partial[f]\, g) = \partial\cdot s(f\,\partial[g] - \partial[f]\, g) = \partial\cdot[s(f\,\partial[g] - \partial[f]\, g)] = \partial\cdot\mathbf{J} = 0$

Thus, there exists a conserved current 4-Vector, $\mathbf{J}_{\mathbf{prob}} = (\rho_{prob}c,\mathbf{j}_{\mathbf{prob}}) = s(f\,\partial[g] - \partial[f]\, g)$, for which $\partial\cdot\mathbf{J}_{\mathbf{prob}} = 0$,
and which also solves the Klein-Gordon relation, and gives unitary evolution and conservation of probability.

For generality, choose as before $(\partial = -i\mathbf{K})$ with a complex planewave function $g = ae\text{^}{-i(\mathbf{K}\cdot\mathbf{X})} = \psi$,
and choose $f = g^* = ae\text{^}{i(\mathbf{K}\cdot\mathbf{X})} = \psi^*$ as its complex conjugate.
At this point, we choose $s = (i\hbar/2m_o) = (ic^2/2\omega_o)$, which is Lorentz Scalar Invariant, in order to make
the probability have dimensionless units and be normalized to unity in the rest case.
$\mathbf{J}_{\mathbf{prob}} = (\rho_{prob}c,\mathbf{j}_{\mathbf{prob}}) = (i\hbar/2m_o)(\psi^*\partial[\psi] - \partial[\psi^*]\psi) = (ic^2/2\omega_o)(\psi^*\partial[\psi] - \partial[\psi^*]\psi)$

Examine the temporal component, the Relativistic Probability Density
$\rho_{prob}c = (i\hbar/2m_o)(\psi^*\,(\partial_t/c)[\psi] - (\partial_t/c)[\psi^*]\,\psi) = (ic^2/2\omega_o)(\psi^*\,(\partial_t/c)[\psi] - (\partial_t/c)[\psi^*]\,\psi)$
$\rho_{prob} = (i\hbar/2m_oc^2)(\psi^*\,\partial_t[\psi] - \partial_t[\psi^*]\,\psi) = (i/2\omega_o)(\psi^*\,\partial_t[\psi] - \partial_t[\psi^*]\,\psi)$
Assume wave solution in following general form:
$\{\,\psi = A\,f\,[k]\,e^{(-i\omega t)}\,\}$ & $\{\,\psi^* = A^*\,f\,[k]^*\,e^{(+i\omega t)}\,\}$
then
$\{\,\partial_t[\psi] = (-i\omega)A\,f\,[k]\,e^{(-i\omega t)} = (-i\omega)\psi\,\}$ & $\{\,\partial_t[\psi^*] = (+i\omega)A^*\,f\,[k]^*\,e^{(+i\omega t)} = (+i\omega)\psi^*\,\}$
then
$\rho_{prob} = (i/2\omega_o)(\psi^*\,\partial_t[\psi] - \partial_t[\psi^*]\,\psi)$
$\rho_{prob} = (i/2\omega_o)((-i\omega)\psi^*\psi - (+i\omega)\psi^*\psi)$
$\rho_{prob} = (i/2\omega_o)((-2i\omega)\psi^*\psi)$
$\rho_{prob} = (\omega/\omega_o)(\psi^*\psi)$
$\rho_{prob} = (\gamma\omega_o/\omega_o)(\psi^*\psi)$
$\rho_{prob} = (\gamma)(\psi^*\psi) = (\gamma)(\rho_{prob\ o})$
Finally, multiply by charge (q) to get standard SR EM 4-CurrentDensity = 4-ChargeFlux = $\mathbf{J} = (\rho c,\mathbf{j}) = q\mathbf{J}_{\mathbf{prob}} = q(\rho_{prob}c,\mathbf{j}_{\mathbf{prob}})$
One can generalize to include the effects of an EM VectorPotential $\mathbf{A} = (\varphi/c,\mathbf{a})$
4-ProbabilityCurrentDensity $\mathbf{J}_{\mathbf{prob}} = (\rho_{prob}c,\ \mathbf{j}_{\mathbf{prob}}) = (i\hbar/2m_o)(\psi^*\partial[\psi]-\partial[\psi^*]\psi) + (q/m_o)(\psi^*\psi)\mathbf{A}$

Examine the temporal component:
$\rho_{prob} = (i\hbar/2m_o c^2)(\psi^* \partial_t[\psi] - \partial_t[\psi^*] \psi) + (q/m_o)(\psi^*\psi)(\varphi/c^2)$
$\rho_{prob} \rightarrow (\gamma)(\psi^*\psi) + (\gamma)(q\varphi_o/m_o c^2)(\psi^*\psi) = (\gamma)[1 + q\varphi_o/E_o](\psi^*\psi)$
Typically, particle EM potential energy ($q\varphi_o$) is much less than particle rest energy (E_o), else it could generate new particles.
So, take ($q\varphi_o << E_o$), which gives the EM factor ($q\varphi_o/E_o$) ~ 0
Now, taking the low-velocity limit ($\gamma \rightarrow 1$), $\rho_{prob} = \gamma[1 + \sim 0](\psi^*\psi)$, $\rho_{prob} \rightarrow (\psi^*\psi) = (\rho_{probo})$ for $|\mathbf{v}|<<c$
The Standard Born Probability Interpretation, $(\psi^*\psi) = (\rho_{prob})$, only applies in the low-potential-energy & low-velocity limit
This is why the {non-positive-definite} probabilities and {|probabilities| > 1} in the RQM Klein-Gordon equation gave physicists fits, and is the reason why one must regard the probabilities as charge conservation instead.
The original definition from SR is Continuity of Worldlines, $(\partial\cdot\mathbf{J}_{prob}) = 0$, for which all is good and well in the RQM version.
The definition says there are no external sources or sinks of probability = conservation of probability.
The Born idea that Total $(\rho_{prob}) \rightarrow \text{Sum}[(\psi^*\psi)] = 1$ is just the low-velocity ($|\mathbf{v}|<<c$) QM limit-case.
Only the non-EM rest version has Total $(\rho_{prob\ o}) = \text{Sum}[(\psi^*\psi)] = 1$.
It is not a fundamental axiom, it is an emergent property which is valid only in the NRQM limit.
We now multiply by EM charge (q) to get:
4-"Charge"CurrentDensity $\mathbf{J} = (\rho c, \mathbf{j}) = q\mathbf{J}_{prob} = q(\rho_{prob}c, \mathbf{j}_{prob})$, which is the standard SR EM 4-CurrentDensity

Comparison of SR 4-(Dust)NumberFlux **N** to QM 4-ProbabilityCurrent $\mathbf{J}_{prob}$, the same 4-Vector:
Consider

SR 4-Vector (properties):	SR 4-Vector (relations):
4-Velocity $\mathbf{U} = \gamma(c,\mathbf{u})$	
4-Gradient $\partial = \partial^\mu = (\partial_t/c,-\nabla)$	
4-(Dust)NumberFlux $\mathbf{N} = (nc,\mathbf{n}) = (nc,n\mathbf{u}) = n(c,\mathbf{u}) = n_o\gamma(c,\mathbf{u})$	$\mathbf{N} = n_o\mathbf{U}$
4-Current(Density) $\mathbf{J} = (\rho c,\mathbf{j}) = (\rho c,\rho\mathbf{u}) = \rho(c,\mathbf{u}) = \rho_o\gamma(c,\mathbf{u}) = qn_o\gamma(c,\mathbf{u})$	$\mathbf{J} = \rho_o\mathbf{U} = qn_o\mathbf{U} = q\mathbf{N}$
4-ProbabilityCurrentDensity $\mathbf{J}_{prob} = (\rho_{prob}c, \mathbf{j}_{prob}) = (i\hbar/2m_o)(\psi^*\partial[\psi]-\partial[\psi^*]\psi)$	

4-Vector **N** has dimensional units of [#-flux] and the 4-Scalar rest-number-density (n_o) has dimensional units of [#/volume].
4-Vector $\mathbf{J}_{prob}$ has dimensional units of [#-flux] and the 4-Scalar rest-proability-density (ρ_o) has dimensional units of [#/volume].
This leads to the idea that the QM 4-ProbabilityCurrent $\mathbf{J}_{prob}$ is equivalent to the SR 4-(Dust)NumberFlux **N**.
The concepts are actually quite similar if one considers the fluid approximation of individual particles.
The fluid allows densities that are less than unity, much as probabilities of expected positions of particles are less than unity and only sum to unity over the entire volume.
This argument is further strengthened by noting that in QM one also has $\mathbf{J} = q\mathbf{J}_{prob}$ (see again SR's $\mathbf{J} = q\mathbf{N}$)

Analysis of the Koopman-von Neuman (KvN) formalism, a framework which can give QM or CM:
The idea that Hilbert Space requires a quantum axiom is disproved by the Koopman–von Neumann formulation of classical mechanics, in which Hilbert Space mathematical formulation is successfully applied and results in the classical Liouville equation.
This shows that the Hilbert Space framework is purely mathematical and can be applied to both classical and quantum systems.

The main difference between which system emerges is the commutation relation between position and momentum. In the classical case, one assumes a zero-valued commutation relation. In the quantum case, there is a non-zero commutation relation.
SR, as shown above, gives a non-zero commutation relation, thus leading naturally to the QM case.

SR → RQM → QM → CM Classical Correspondence Principle:
In SR, one finds the Newtonian classical limiting-case approximation by using {$|\mathbf{v}| << c$}. In QM, there have been a variety of approaches to the Newtonian classical limiting-case approximation, including the idea of {number of particles >> 1}, the physics action {$S >> \hbar$}, divergence small compared to system magnitude {$\hbar|\nabla\cdot\mathbf{p}| << (\mathbf{p}\cdot\mathbf{p})$}, etc. In the standard view of the theories of relativity and quantum mechanics, it is interesting to speculate on how the two "different" theories "conspire" to end up at the same classical mechanics physics approximation. However, in the SRQM view, this difficulty disappears. SR leads to RQM via the approach that has been shown. RQM then goes to QM as a limiting-case approximation by using {$|\mathbf{v}| << c$}. QM then goes to CM as a limiting-case in its own manner. There is a single chain of relationships, rather than two different theories "amazingly" approaching the same classical limit-case.

GR→{limit-case $g^{\mu\nu}\rightarrow\eta^{\mu\nu}$}→SR→RQM→{limit-case $|\mathbf{v}|<<c$}→QM→{limit-case $\hbar|\nabla\cdot\mathbf{p}| << (\mathbf{p}\cdot\mathbf{p})$}→(EM & CM)

SR Derivation of CPT Symmetry:

The Lorentz Transformations $\Lambda^\mu{}_\nu$ play a fundamental role in SR. They describe the inherent symmetries of spacetime. The main ones usually mentioned are the continuous transforms, which depend on a parameter and are Proper {Det[continuous $\Lambda^\mu{}_\nu$] = +1}:
The temporal-spatial velocity (B)oost $\Lambda^\mu{}_\nu \to B^\mu{}_\nu$ [$\boldsymbol{\beta}$] or [$\varphi,\hat{\mathbf{n}}$], 3 parameters, uses hyperbolic angles φ {cosh, sinh} or relativistic $\boldsymbol{\beta}$ & γ.
The spatial-spatial (R)otation $\Lambda^\mu{}_\nu \to R^\mu{}_\nu$ [$\theta,\hat{\mathbf{n}}$], 3 parameters, uses the regular angles θ {cos, sin} about some axis $\hat{\mathbf{n}}$.

However, there also exist discrete Lorentz Transforms. One is the 4D Identity $\Lambda^\mu{}_\nu \to \delta^\mu{}_\nu = +I_{[4]}$, which leaves a system completely unchanged. It is a special case of both boost and rotation transforms when their parameters are zero. $B^\mu{}_\nu [|\boldsymbol{\beta}|=\mathbf{0}] = R^\mu{}_\nu [\theta=0,\hat{\mathbf{n}}] = \delta^\mu{}_\nu$.

Most well known of the discrete Lorentz Transforms are the space-reversal (P)arity $\Lambda^\mu{}_\nu \to P^\mu{}_\nu$, which reverses the three spatial coordinates {$\mathbf{x} \to -\mathbf{x}$}, and the (T)imeReversal $\Lambda^\mu{}_\nu \to T^\mu{}_\nu$, which reverses the single temporal coordinate {$t \to -t$}. Less well known are the other Lorentz Transforms which include rotations of a fixed amount and spatial flips. It turns out that one can individually reverse any combination of the standard coordinates and still have a Lorentz Transform {essentially anything that has Det[$\Lambda^\mu{}_\nu$] = ±1}. Reversal of all the coordinates, {$t \to -t$} & {$\mathbf{x} \to -\mathbf{x}$} = {$\mathbf{X} \to -\mathbf{X}$}, gives the transform Combo(PT) $\Lambda^\mu{}_\nu \to (PT)^\mu{}_\nu = C^\mu{}_\nu$
Examination of all possible combinations of Discrete Lorentz Transformations leads to CPT Symmetry.

In other words, one can go from the Identity Transform $+I_{[4]}$ (all +1) to the Negative Identity Transform $-I_{[4]}$ (all -1) by doing a Combo(PT) = (C)hargeReversal Lorentz Transform. This Negative Identity has the interpretation of AntiMatter, without any need of Dirac's formulation using RQM. The Feynman-Stueckelberg AntiMatter Interpretation ~ CPT Interpretation (AntiMatter moving spacetime-backward = NormalMatter moving spacetime-forward) aligns with this.

Two interesting properties of (1,1)-Tensors, of which the Lorentz Transform is an example:
Trace = Sum (Σ) of EigenValues :
Determinant = Product (Π) of EigenValues
As 4D Tensors, each Lorentz Transform $\Lambda^\mu{}_\nu$ has 4 EigenValues (EV's).
Create an Anti-Transform which has all EigenValue Tensor Invariants negated.
Σ[-(EV's)] = -Σ[EV's]: The Anti-Transform has negative Trace of the Transform.
Π[-(EV's)] = $(-1)^4$ Π[EV's] = Π[EV's]: The Anti-Transform has equal Determinant.
The Trace Invariant identifies a "Dual" Negative-Side for all Lorentz Transforms, with Proper Transform=Proper Anti-Transform.

This {NM= NormalMatter , AM= AntiMatter } interpretation can be analyzed using tensor determinant and trace operations.

Tr[NM-Rotate] = {0...+4}	Tr[NM-Identity] = +4	Tr[NM-Boost] = {+4...+∞}
Tr[AM-Rotate] = {0...−4}	Tr[AM-Identity] = −4	Tr[AM-Boost] = {−4...−∞}

t	x	y	z	Discrete **Normal Matter** (NM) Lorentz Transform Type	Trace	Determinant
+1	+1	+1	+1	NM-Minkowksi Identity : AM-Flip-txyz=AM-Combo(PT)	Tr = +4	Det = +1 Proper
+1	+1	+1	–1	NM-Flip-z	Tr = +2	Det = –1 Improper
+1	+1	–1	+1	NM-Flip-y	Tr = +2	Det = –1 Improper
+1	+1	–1	–1	NM-Flip-yz=NM-Rotate-yz(π)	Tr = 0	Det = +1 Proper
+1	–1	+1	+1	NM-Flip-x	Tr = +2	Det = –1 Improper
+1	–1	+1	–1	NM-Flip-xz=NM-Rotate-xz(π)	Tr = 0	Det = +1 Proper
+1	–1	–1	+1	NM-Flip-xy=NM-Rotate-xy(π)	Tr = 0	Det = +1 Proper
+1	–1	–1	–1	NM-Flip-xyz=NM-ParityInverse:AM-Flip-t=AM-TimeReversal	Tr = –2	Det = –1 Improper
–1	+1	+1	+1	AM-Flip-xyz=AM-ParityInverse:NM-Flip-t=NM-TimeReversal	Tr = +2	Det = –1 Improper
–1	+1	+1	–1	AM-Flip-xy=AM-Rotate-xy(π)	Tr = 0	Det = +1 Proper
–1	+1	–1	+1	AM-Flip-xz=AM-Rotate-xz(π)	Tr = 0	Det = +1 Proper
–1	+1	–1	–1	AM-Flip-x	Tr = –2	Det = –1 Improper
–1	–1	+1	+1	AM-Flip-yz=AM-Rotate-yz(π)	Tr = 0	Det = +1 Proper
–1	–1	+1	–1	AM-Flip-y	Tr = –2	Det = –1 Improper
–1	–1	–1	+1	AM-Flip-z	Tr = –2	Det = –1 Improper
–1	–1	–1	–1	AM-Minkowksi Identity : NM-Flip-txyz=NM-Combo(PT)	Tr = –4	Det = +1 Proper
$\bar{t}$	$\bar{x}$	$\bar{y}$	$\bar{z}$	Discrete **AntiMatter** (AM) Lorentz Transform Type	Trace	Determinant

There is complete (+/-) symmetry, which agrees with all known experiments with NormalMatter ←☯→ AntiMatter to-date.

Grouped and ordered by the trace values, one gets:

Discrete **Normal Matter** (NM) Lorentz Transform Type	Trace	Determinant
NM-Minkowksi Identity $+I_{[4]}$ AM-Flip-txyz=AM-Combo(PT)=AM-NegateIdentity=AM-NegateCharge	Tr = +4	Det = +1 Proper
NM-Flip-t ,NM-Flip-x, NM-Flip-y, NM-Flip-z AM-Flip-xyz=AM-ParityInverse	Tr = +2	Det = –1 Improper
NM-Flip-xy=NM-Rotate-xy(π), NM-Flip-xz=NM-Rotate-xz(π), NM-Flip-yz=NM-Rotate-yz(π) AM-Flip-xy=AM-Rotate-xy(π), AM-Flip-xz=AM-Rotate-xz(π), AM-Flip-yz=AM-Rotate-yz(π)	Tr = 0	Det = +1 Proper
NM-Flip-xyz=NM-ParityInverse AM-Flip-t ,AM-Flip-x, AM-Flip-y, AM-Flip-z	Tr = –2	Det = –1 Improper
NM-Flip-txyz=NM-Combo(PT)=NM-NegateIdentity=NM-NegateCharge AM-Minkowksi Identity $–I_{[4]}$	Tr = –4	Det = +1 Proper
Discrete **AntiMatter** (AM) Lorentz Transform Type	Trace	Determinant

This clearly shows that Combo(PT) Transform is equivalent to a (C)harge Transform, which flips NormalMatter ←☯→ AntiMatter. Also, this (C)harge Transform is Proper, with a determinant of +1, the same as the Boost, Rotation, Flip-TwoCoords Transforms, which means that it occurs in reality. Overall, this is the source of (CPT) Symmetry.

Conclusion:

Using the Tensor calculus (esp. 4-Vector calculus} of Einstein-Minkowski Spacetime it is shown that foundational features of spacetime common to both special relativity and quantum mechanics exist.

(ħ) is shown to be an empirically measurable constant and a Lorentz scalar, just like (c).

The 4-vector relations **P** = ħ**K**, the wave view, and **P** = m_o**U**, the particle view, are shown to be isomorphic in the sense that both are derivable from SR.

The mathematical relation **K** = i∂ and existence of complex wavefunction ψ is shown applicable to all types of waves: classical, quantum, relativistic, EM, purely-mathematical. **K** is a solution of the 4D invariant d'Alembertian wave eqn. (∂·∂).

The waves are all described by 4D Tensor amplitudes (a) and the Lorentz scalar product function $e^{\pm i(\mathbf{K}\cdot\mathbf{X})}$ propagator.

The combination of these relations lead to a KG relativistic quantum wave relation $(\partial\cdot\partial) = (im_oc/\hbar)^2 = -(m_oc/\hbar)^2$ and to the 4-Vector form of the standard Schrödinger relations **P** = iħ∂.

There exists a non-zero commutation relation in SR: $[X^\mu,P^\nu] = -i\hbar\eta^{\mu\nu}$. which gives the standard canonical QM $[x^j,p^k] = i\hbar\delta^{jk}$

There exists a conserved current $\mathbf{J}_{prob}$ in SR, based on a simple vector identity applied to the KG relation.

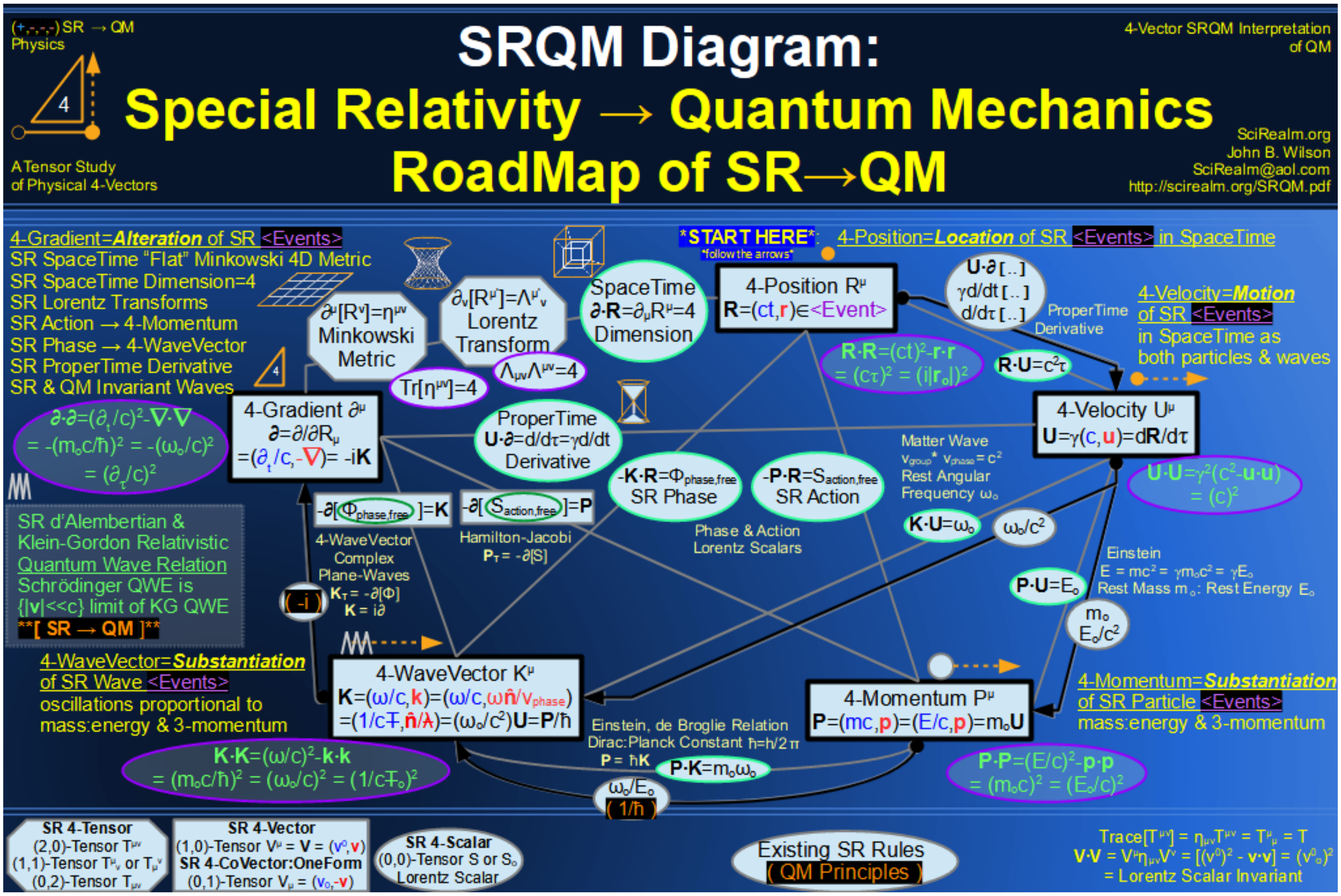
The 4-(Dust)NumberFlux **N** appears to be equivalent to the 4-ProbabilityCurrentDensity $\mathbf{J}_{\mathbf{prob}}$.

The standard Born probability interpretation, $\psi^*\psi = \rho_{prob\ o}$, emerges in the low-potential-energy and low-velocity limit.

CPT Symmetry emerges from an analysis of the mathematical properties of the Lorentz Transformations.

The correspondence principle of both SR and QM to Newtonian classical physics CM is discussed.

One can derive the "axioms" of QM from the principles of SR.



Start with a few SR Physical 4-Vectors:
4-Position $\mathbf{R} = (ct,\mathbf{r})$
4-Velocity $\mathbf{U} = \gamma(c,\mathbf{u})$
4-Momentum $\mathbf{P} = (E/c,\mathbf{p}) = (mc,\mathbf{p})$
4-WaveVector $\mathbf{K} = (\omega/c,\mathbf{k})$
4-Gradient $\partial = (\partial_t/c,-\nabla)$

Note the following relations between SR 4-Vectors:
$\mathbf{U} = d\mathbf{R}/d\tau$
$\mathbf{P} = m_o\mathbf{U} = (E_o/c^2)\mathbf{U}$
$\mathbf{K} = (1/\hbar)\mathbf{P} = (\omega_o/c^2)\mathbf{U}$
$\partial = -i\mathbf{K}$

Form a chain of SR Lorentz Invariant Scalar Equations, based on those relations:
$\mathbf{R}\cdot\mathbf{R} = (c\tau)^2$
$\mathbf{U}\cdot\mathbf{U} = (c)^2$
$\mathbf{P}\cdot\mathbf{P} = (m_oc)^2 = (E_o/c)^2$
$\mathbf{K}\cdot\mathbf{K} = (m_oc/\hbar)^2 = (\omega_o/c)^2$
$\partial\cdot\partial = (-im_oc/\hbar)^2 = -(m_oc/\hbar)^2 = -(\omega_o/c)^2$

This is (RQM) = Relativistic Quantum Mechanics, derived from only:
----
5 of the Standard SR 4-Vectors
4 really simple empirical relations between them
1 SR rule for forming Lorentz Scalar Invariants, i.e. the Minkowski Metric ($\eta_{\mu\nu}$) which gives the Lorentz Scalar Product (·)

4-Creativity ☼ = (Music,**Artwork**) : 4-Universality ∞ = (Eternity,**Infinity**) : 4-Origin **O** = (Now,**Here**)