Bells's Spaceship Paradox: Resolved https://en.wikipedia.org/wiki/Bell's_spaceship_paradox

Bell's Spaceship Paradox is based on a mistaken assumption that seems reasonable in classical physics, that identical spatial accelerations leave distances unchanged in the altered frame. Careful analysis, however, shows that proper spacetime accelerations, also known as hyperbolic boosts, and described by the 4-Vectors, esp. the 4-Acceleration, must be used. This is a result of Poincaré and Lorentz Invariances. In fact, the same applies to rotations if you examine the math.

KeyWords: Bell's Spaceship Paradox, Special Relativity (SR), Poincaré & Lorentz Invariance, Boosts, Rotations, 4-Vectors, Tensors

Bell's Paradox: Consider two identically constructed ships (same lengths, same engine thrusts, same accelerations) at rest in an inertial start frame. They face the same direction, one behind the other. The ships are connected to one another by a taut string, aft to bow. At a prearranged time in the start frame, both ships simultaneously thrust, and maintain identical accelerations in that frame. Their velocities with respect to the initial frame are always equal throughout the remainder of the experiment. This means, by definition, *that with respect to the initial frame,* the distance between the two ships does not change, even when they speed up to relativistic velocities. However, oddly enough, this setup leads to the ships having a relativistically-changed distance from one another in their own frames. In the start frame, the string seems to stay taut and connected; in the boosted frame, the string must snap. **What is the reality?**

We can examine this using tensors, which are singularly adept at handling frames-of-reference and spacetime invariances.
The physical 4-Vectors used in SR are 4D (1,0)-tensors, which are 1-upper-index tensors, and are geometrically Lorentz invariant.
4-Position $\mathbf{R} = R^{\mu} = (ct,\mathbf{r}) = (ct,x,y,z)$. It has internal components of time (t), & 3-position $\mathbf{r} = r^k = (x,y,z)$, which may vary with frame.
4-Displacement $\Delta\mathbf{R} = \Delta R^{\mu} = (c\Delta t,\Delta\mathbf{r}) = (ct_2\text{-}ct_1,\mathbf{r}_2\text{-}\mathbf{r}_1) = \mathbf{R}_2\text{-}\mathbf{R}_1$, and is fully Poincaré Invariant.

Poincaré Invariance gives linear transformations between inertial frames. This has 4 equations, one for each dimension: $\{t,x,y,z\}$
$\Lambda^{\mu'}{}_{\nu}$ is a Lorentz Transform (a matrix or dyadic), and $\mathbf{S'} = S^{\mu'}$ is a SpaceTime Translation (yet another 4-Vector).
Poincaré Transformation Equation:
$R^{\mu'} = \Lambda^{\mu'}{}_{\nu} R^{\nu} + S^{\mu'}$

Each of these equations is just like a standard linear transform along a single line or dimension:
$x' = A x + B$, with A the multiplicative component and B the additive component.

This linearity implies that there is a 4D invariant inner product for 4-Displacements:
$\Delta R^{\mu'}\Delta R_{\mu'} = \Delta R^{\nu}\Delta R_{\nu} = \Delta R^{\alpha}\eta_{\alpha\beta}\Delta R^{\beta} = (c\Delta t')^2 - (\Delta x')^2 - (\Delta y')^2 - (\Delta z')^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$

The beauty of the tensor formalism is that the Lorentz Transform Matrix $\mathbf{\Lambda} = \Lambda^{\mu}{}_{\nu}$ can apply to space-space transforms (rotations) or to space-time transforms (boosts). So, for example, the $\Lambda^{\mu}{}_{\nu}$ can be set to trade spatial-x with spatial-y (rotations), or it can be set to trade spatial-x with temporal-t (boosts), or to several more complex cases including combinations of rotations and boosts.

For an x,y-rotation, we get **Λ→R**otation:
[x'] = [cos(α) -sin(α)][x]
[y'] [sin(α) cos(α)][y]

For an x,t-boost, we get **Λ→B**oost:
[ct'] = [cosh(ω) -sinh(ω)][ct] = [γ -γβ][ct]
[x'] [-sinh(ω) cosh(ω)][x] = [-γβ γ][x]

For identity, we get **Λ→I**dentity:
[x'] = [1 0][x]
[y'] [0 1][y]

We get the following invariants, those scalars which observers in all frames must agree on, even though components may differ:

Full Lorentz:	$(ct')^2 - (x')^2 - (y')^2 - (z')^2 = (ct)^2 - (x)^2 - (y)^2 - (z)^2$	because Determinant$[\Lambda^{\mu}{}_{\nu}] = +1$ for continuous, proper
Rotations x,y:	$(x')^2 + (y')^2 = (x)^2 + (y)^2$	because $\cos^2 + \sin^2 = +1$
Boosts x,t:	$(ct')^2 - (x')^2 = (ct)^2 - (x)^2$	because $\cosh^2 - \sinh^2 = +1$

Interestingly, the Trace$[\Lambda^{\mu}{}_{\nu}]$ identifies the transform type. Trace={0..4} are rotations, {4} is identity, {4..Infinity} are boosts.
This also makes sense because a Rotation of 0, or N*2π = Identity, and a Boost of 0 = Identity
The Rotation meets the Boost at the Identity.

The difference in signs comes from: 4D "Flat" <Time·Space> SR:Minkowski Metric $\eta_{\mu\nu} = \eta^{\mu\nu} \to \text{Diagonal}[+1,-1,-1,-1]$(Cartesian coordinates)

Before getting into the spacetime/boost version of the paradox, let's start with a related experiment that is much easier to conceptualize and analyze, and which will ultimately point out the intricate connection of space to time.

Consider the same situation as set-up in the paradox, but with two model ships in a laboratory on a turn-table, or Lazy-Susan. The two ships are again of the same length, lined up one behind another, and connected by a taut string.

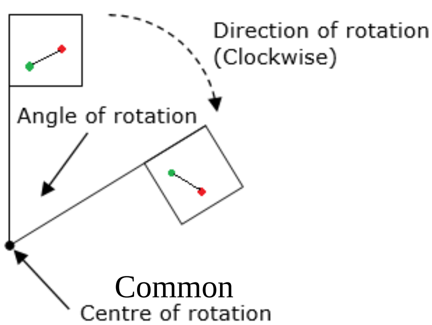
$$\begin{bmatrix}x'\\ y'\end{bmatrix} = \begin{bmatrix}\cos(\alpha) & -\sin(\alpha)\\ \sin(\alpha) & \cos(\alpha)\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$$ = **Rotation Transformation**

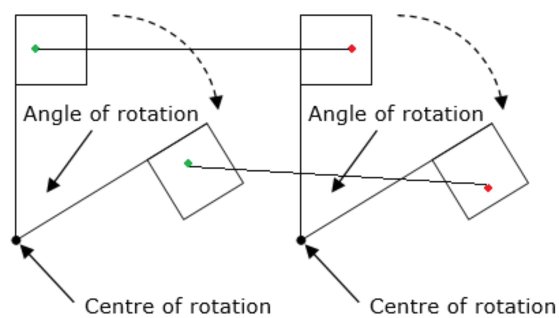
In the 1st scenario, rotate the turn-table by some angle (α). The two ships will both have their headings rotated by (α) degrees, the distances are preserved in the rotated frame, preserving the invariances, and the string remains attached.

In the 2nd scenario, we lock the turn-table so that it can't turn. Instead, rotate each ship individually by (α) degrees about independent Centers of Rotation. In this case, the ships will both still have their headings rotated by (α) degrees, but they are no longer aligned head to tail (instead are somewhat sideways to one another), the distances between the ships are not preserved, and the string connecting them breaks.

What is happening?

In both cases, the ships have indeed rotated by the same angle (α).





However, in the 1st scenario, the two ships have a Common Center of Rotation. They share a common frame of reference in both the initial and final (rotated) frame. The distances between all points of the ships is maintained. Invariance is maintained.

In the 2nd scenario, even though they rotate by the same amount (α), they are using their own independent Centers of Rotation. The final placements have not preserved the distances, and the string breaks. Invariance is not maintained, they are not in the same frames.

An important point here is that in just regular standard rotation, the points that are further from the Center of Rotation must accelerate harder to maintain coordinated distances as the object is physically rotated. The reason I mention this is that it becomes very important in understanding what happens in the boost.

Also, it is important to note that the x-components and y-components as seen in any frame don't physically "do" anything to the ships.

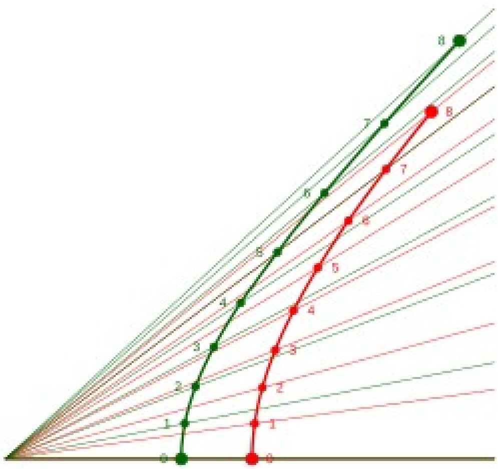
If you look at an object from the from the front, eg. x-axis, then you look at the same object from the side, eg. y-axis, you haven't physically altered the object itself. You are simply changing a frame of reference. The only thing that can "alter" the object is differential accelerations, which cause forces/stresses in the object. Length contractions in x and length contractions in y are perceptual only.

Now, let's examine the original setup, with the real ships in space.

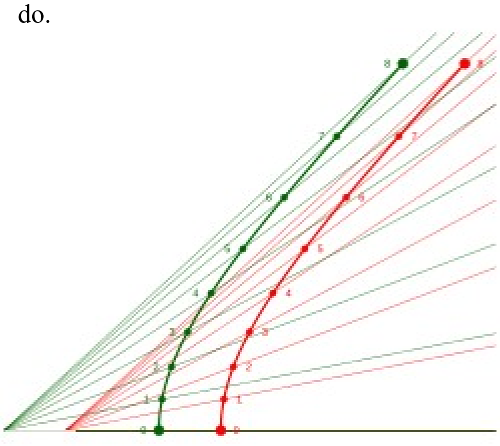
$$\begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} \cosh(\omega) & -\sinh(\omega) \\ -\sinh(\omega) & \cosh(\omega) \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix}$$ = **Boost Transformation**

In both cases, the ships will "hyperbolically rotate", or boost, by the same spacetime "hyperbolic angle" (ω), just like the spatial rotations using the same spatial angle (α).

In the 1st scenario, let the ships boost with a common frame, a proper hyperbolic acceleration, in which they share the same Rindler Point, which is like a hyperbolic Center of Rotation. In this case, the distances in the ship's boosted reference frame are preserved, the invariances are maintained, and the string doesn't break. As it turns out, in this scenario, they do NOT use identical start-frame acceleration profiles. The ship in back, closer to the Rindler Point, must accelerate harder. Note that in the start frame the ships seem to be getting closer, from Lorentz length contraction, but they are actually maintaining distance in the ship frame of reference.



In the 2nd scenario, the ships boost with identical acceleration profiles in the start frame. However, this means that they are each using their own separate, non-identical, Rindler Points. This means that distances between the ships are not preserved in the ship reference frames, and the string snaps, in exactly the same way as that of rotations using their own separate, non-identical Centers of Rotation do.



The Minkowski diagram for this is somewhat strange. In the start frame, it seems that the ships are staying equidistant. However, in the upper part of the graph, the ships are moving, which means the start frame is actually measuring a contracted length according to the Minkowski diagram, which means that the MCRF frame is actually longer than it was in the start frame. The ships are accelerating away from one another in that frame and the string snaps. This is not due to the relativistic length contraction, which is just apparent measurement from the start frame; it is due to how the ships are accelerating in their MCRF's (Momentarily Co-moving Rest Frames), which produces stress in the string, causing it to break.

If you look at an object from the from a resting inertial frame, then you look at the same object from a moving inertial frame, you haven't physically altered the object itself. You are simply changing a frame of reference. The only thing that can "alter" the object is differential accelerations, which cause forces/stresses in the object. Length contractions (eg. x-axis) and time dilations (t-axis) are perceptual only.

So, the paradox is resolved. A paradox is a situation is which one or more of a set of assumptions leads to a contradiction. The contradiction here is thinking that identical acceleration profiles and using the same "angle/hyperbolic-angle" lead to preserved displacements in the altered frame.

Even in the case of identical acceleration profiles for the rotations, the string also breaks. To preserve rotation displacements, the accelerations on points further from a common Center of Rotation must be greater. Likewise, to preserve boost displacements, the accelerations on points closer to the Rindler Point must be greater. So, the ship in back must accelerate harder. But the main thing that breaks the string in both cases is not having a shared, common Center of Rotation/Rindler Point. This gives different acceleration profiles in the ship's reference frames, which breaks the string for BOTH rotations and boosts.

We note the similarities in the two types of Transform.
Rotations have $(x)^2 + (y)^2 = R^2$, with R being a constant distance to a Center of Rotation
Boosts have $(ct)^2 - (x)^2 = D^2$, with D being a constant distance to a Rindler Point, which in this case is also an event horizon.

Rotations have circular paths in space, eg. x and y.
Boosts have hyperbolic paths in spacetime, eg. x and t.

A very important aspect of these transformations is the idea of active and passive transforms. An active transform is one in which the objects being measured are physically acted upon. They are accelerated and moved, requiring work. A passive transform is one in which the observer moves (or simply rearranges their measuring system, as all linearly-connected measuring systems are equally valid) and there is no acceleration or motion or work done on the object being measured.

The interesting point here is that active and passive transforms are inverses of one another. They are mathematically identically inverse. Since a passive transform applies to a frame as a whole (the observer moves without affecting the system under observation), it is natural to consider its inverse as more physically natural as well. That would be the situation of a rotation of all objects using a common Center of Rotation, or a boost of all objects using a common Rindler Point. These are the transforms that preserve event distances in the altered frames of reference. In this work, we are actively transforming certain objects in such a way that they maintain their Lorentz Invariance, specifically the Invariant Interval.

General Motion using 4-Vectors: All Lorentz Scalar Products are Invariants

4-Position	$\mathbf{R} = R^\mu = (ct,\mathbf{r})$	$= \mathbf{R} = d^0\mathbf{R}/d\tau^0$	$(\mathbf{R}\cdot\mathbf{R}) = (ct)^2 - \mathbf{r}\cdot\mathbf{r} = (ct_o)^2 = (c\tau)^2 = -(\mathbf{r_o}\cdot\mathbf{r_o})$:either (±),variable
4-Velocity	$\mathbf{U} = U^\mu = \gamma(c,\mathbf{u})$	$= d\mathbf{R}/d\tau = d^1\mathbf{R}/d\tau^1$	$(\mathbf{U}\cdot\mathbf{U}) = (c)^2$:temporal(+),fundamental constant
4-Acceleration	$\mathbf{A} = A^\mu = \gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})$	$= d\mathbf{U}/d\tau = d^2\mathbf{R}/d\tau^2$	$(\mathbf{A}\cdot\mathbf{A}) = -(a_o)^2 = -(\alpha)^2 = (i\alpha)^2$:spatial(-),variable

An alternate form of the 4-Acceleration handy for circular motion:
4-Acceleration $\mathbf{A} = A^\mu = (\gamma^4(\mathbf{a}\cdot\mathbf{u})/c , \gamma^4(\mathbf{a}\cdot\mathbf{u})\mathbf{u}/c^2+\gamma^2\mathbf{a}) = \gamma^2(0,\mathbf{a})_\perp$ if $(\mathbf{a}\cdot\mathbf{u})=0$

Circular Motion: constants {R,Ω,γ}, which constrain the motion to spatial circular arcs.
4-Position $\mathbf{R} = R^\mu = (ct,\mathbf{r} = R\,\hat{\mathbf{r}})$
4-Velocity $\mathbf{U} = U^\mu = \gamma^1(c,\mathbf{u} = R\Omega\,\hat{\boldsymbol{\theta}})$
4-Acceleration $\mathbf{A} = A^\mu = \gamma^2(0,\mathbf{a} = -R\Omega^2\,\hat{\mathbf{r}})$

Hyperbolic Motion: constants {Rindler "Dist" $D=c^2/\alpha$}, which constrain the motion to spacetime hyperbolic arcs.
4-Position $\mathbf{R} = R^\mu = (c^2/\alpha)(\sinh[\alpha\tau/c],\cosh[\alpha\tau/c]\,\hat{\mathbf{n}})$
4-Velocity $\mathbf{U} = U^\mu = (c)(\cosh[\alpha\tau/c],\sinh[\alpha\tau/c]\,\hat{\mathbf{n}})$
4-Acceleration $\mathbf{A} = A^\mu = (\alpha)(\sinh[\alpha\tau/c],\cosh[\alpha\tau/c]\,\hat{\mathbf{n}})$

http://scirealm.org/Physics-MinkowskiDiagram.html

Rotations have circular paths in space, eg. x and y.	Boosts have hyperbolic paths in spacetime, eg. x and t.
Lorentz Transformation **Λ→R**ot (**R**)otation = All Spatial: 3-vectors { $\lvert\mathbf{r}\rvert$,$\lvert\mathbf{u}\rvert$,$\lvert\mathbf{a}\rvert$,$\lvert\mathbf{j}\rvert$ } & { R,Ω,γ } all constants $\mathbf{a} = (-\Omega^2)\mathbf{r}$, $\mathbf{j} = (-\Omega^2)\mathbf{u}$ $(\mathbf{a}\cdot\mathbf{u}) = 0 = \gamma'$: 3-acceleration ⊥ 3-velocity $\mathbf{n_1}$ = cos : $\mathbf{n_2}$ = sin	Lorentz Transformation **Λ→B**oost (**B**)oost = Time·Space: 4-Vectors { $\lvert\mathbf{R}\rvert$,$\lvert\mathbf{U}\rvert$,$\lvert\mathbf{A}\rvert$,$\lvert\mathbf{J}\rvert$ } & { D,c,α } all constants $\mathbf{A} = (\alpha^2/c^2)\mathbf{R}$, $\mathbf{J} = (\alpha^2/c^2)\mathbf{U}$ $(\mathbf{A}\cdot\mathbf{U}) = 0$: 4-Acceleration ⊥ 4-Velocity γ = cosh : v = sinh
Circular Case: $-(\mathbf{a}\cdot\mathbf{r}) = (R\Omega)^2 = (\mathbf{u}\cdot\mathbf{u})$ $\lvert\mathbf{a}\rvert = \lvert\mathbf{u}\rvert^2/\lvert\mathbf{r}\rvert = R\Omega^2$ R is fixed distance to Center of Rotation	Hyperbolic Case: $-(\mathbf{A}\cdot\mathbf{R}) = (c)^2 = (\mathbf{U}\cdot\mathbf{U})$ $\lvert\mathbf{A}\rvert = \lvert\mathbf{U}\rvert^2/\lvert\mathbf{R}\rvert = \alpha = c^2/D$ D is fixed distance to Rindler Point (Hyp. Center of Rotation)
Circular Motion: constants {R,Ω,γ} 4-Position $\mathbf{R} = R^\mu = (ct, \mathbf{r} = R\,\hat{\mathbf{r}})$ 4-Velocity $\mathbf{U} = U^\mu = \gamma^1(c, \mathbf{u} = R\Omega\,\hat{\boldsymbol{\theta}})$ 4-Acceleration $\mathbf{A} = A^\mu = \gamma^2(0, \mathbf{a} = -R\Omega^2\,\hat{\mathbf{r}})$	Hyperbolic Motion: constants {Rindler "Dist" $D=c^2/\alpha$} 4-Position $\mathbf{R} = R^\mu = (c^2/\alpha)(\sinh[\alpha\tau/c], \cosh[\alpha\tau/c]\,\hat{\mathbf{n}})$ 4-Velocity $\mathbf{U} = U^\mu = (c)(\cosh[\alpha\tau/c], \sinh[\alpha\tau/c]\,\hat{\mathbf{n}})$ 4-Acceleration $\mathbf{A} = A^\mu = (\alpha)(\sinh[\alpha\tau/c], \cosh[\alpha\tau/c]\,\hat{\mathbf{n}})$
For an x,y-rotation, we get: [x'] = [cos(α) -sin(α)][x] [y'] [sin(α) cos(α)][y]	For an x,t-boost, we get: [ct'] = [cosh(ω) -sinh(ω)][ct] = [γ -γβ][ct] [x'] [-sinh(ω) cosh(ω)][x] = [-γβ γ][x]
Rotation x,y Invariance: $(x')^2 + (y')^2 = (x)^2 + (y)^2$ because $\cos^2 + \sin^2 = +1$	Boost x,t Invariance: $(ct')^2 - (x')^2 = (ct)^2 - (x)^2$ because $\cosh^2 - \sinh^2 = +1$
Generic Rotation Matrix **Λ→R**ot $[1 \quad 0_j \quad]$ $[0^i \quad (\delta^i_j - n^i n_j)\cos(\theta) - (\varepsilon^i_{jk} n^k)\sin(\theta) + n^i n_j]$	Generic Boost Matrix **Λ→B**oost $[\ \gamma \quad -\gamma\beta_j \quad]$ $[-\gamma\beta^i \quad (\gamma-1)\beta^i\beta_j/(\boldsymbol{\beta}\cdot\boldsymbol{\beta}) + \delta^i_j]$

Poincaré Transformation Equation:
$R^{\mu'} = \Lambda^{\mu'}{}_\nu R^\nu + S^{\mu'}$

Determinant[$\Lambda^{\mu'}{}_\nu$] = +1 for Proper, continuous transforms, like rotations and boosts, can be -1 for Improper, discrete.
Trace[$\Lambda^{\mu'}{}_\nu$] identifies the transform type. Trace={0..4} are rotations, {4} is identity, {4..∞} are boosts.

The combination of the factors gives : Poincaré Invariance
$S^{\mu'}$: Conservation of 4-LinearMomentum **P** : SpaceTime Translation Invariance : Homogeneity in SpaceTime
$\Lambda^{\mu'}{}_\nu$: Conservation of 4-AngularMomentum ***M*** : Lorentz Invariance : Isotropy in SpaceTime

which can be separated into temporal and spatial parts.
$S^{0'}$: Conservation of Energy E : time translation invariance : homogeneity in time measurements
$S^{j'}$: Conservation of linear-momentum **p** : space translation invariance : homogeneity in space measurements
$\Lambda^{\mu'}{}_\nu$ →**R**ot : Conservation of angular-momentum **l** : space-space rotation invariance : isotropy in space-space measurements
$\Lambda^{\mu'}{}_\nu$ →**B**oost : Conservation of Mass Moment **n** : space-time boost invariance : isotropy in space-time measurements

Calculations for Rotations:
Pick a totally arbitrary initial starting point (x_o , y_o) and place the models. We can write a generalization of the positions.

Now, calculate for points with a Common Center of Rotation (COR):
The coordinates of the points are:
$(x_1 , y_1) = (x_o+x_{COR}+r_1 \cos[\theta] , y_o+y_{COR}+r_1 \sin[\theta])$
$(x_2 , y_2) = (x_o+x_{COR}+r_2 \cos[\theta] , y_o+y_{COR}+r_2 \sin[\theta])$

$(\Delta x , \Delta y)$
$= (x_2-x_1 , y_2-y_1)$
$= (x_o+x_{COR}+r_2 \cos[\theta]-x_o-x_{COR}-r_1 \cos[\theta] , y_o+y_{COR}+r_2 \sin[\theta]-y_o-y_{COR}-r_1 \sin[\theta])$
$= (r_2 \cos[\theta]-r_1 \cos[\theta] , r_2 \sin[\theta]-r_1 \sin[\theta])$
$= ((r_2-r_1)\cos[\theta] , (r_2-r_1)\sin[\theta])$

$Dist^2$ = {3D Invariant}
$= (\Delta x^2+\Delta y^2) = (\Delta x'^2+\Delta y'^2)$
$= (r_2-r_1)^2\cos[\theta]^2+(r_2-r_1)^2\sin[\theta]^2$
$= (r_2-r_1)^2\{\cos[\theta]^2+\sin[\theta]^2\}$
$= (r_2-r_1)^2$

Dist = $Sqrt[(r_2-r_1)^2]$, which is independent of the rotation angle (θ)
This means that the two points move in such a way as to maintain distance regardless of the rotation angle.
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Now, calculate for points using Independent Centers of Rotation (COR1 & COR2).
The coordinates of the points are:
$(x_1 , y_1) = (x_o+x_{COR1}+r_1 \cos[\theta] , y_o+y_{COR1}+r_1 \sin[\theta])$
$(x_2 , y_2) = (x_o+x_{COR2}+r_2 \cos[\theta] , y_o+y_{COR2}+r_2 \sin[\theta])$

$(\Delta x , \Delta y)$
$= (x_2-x_1 , y_2-y_1)$
$= (x_o+x_{COR2}+r_2 \cos[\theta]-x_o-x_{COR1}-r_1 \cos[\theta] , y_o+y_{COR2}+r_2 \sin[\theta]-y_o-y_{COR1}-r_1 \sin[\theta])$
$= (x_{COR2}-x_{COR1}+r_2 \cos[\theta]-r_1 \cos[\theta] , y_{COR2}-y_{COR1}+r_2 \sin[\theta]-r_1 \sin[\theta])$
$= (x_{COR2}-x_{COR1}+(r_2-r_1)\cos[\theta] , y_{COR2}-y_{COR1}+(r_2-r_1)\sin[\theta])$
$= (\Delta x_{COR}+(r_2-r_1)\cos[\theta] , \Delta y_{COR}+(r_2-r_1)\sin[\theta])$

$Dist^2$ = {3D Invariant}
$= (\Delta x^2+\Delta y^2) = (\Delta x'^2+\Delta y'^2)$
$= \{\Delta x_{COR}^2+2\Delta x_{COR}(r_2-r_1)\cos[\theta]+(r_2-r_1)^2\cos[\theta]^2\}+\{\Delta y_{COR}^2+2\Delta y_{COR}(r_2-r_1)\sin[\theta]+(r_2-r_1)^2\sin[\theta]^2\}$
$= \Delta x_{COR}^2+\Delta y_{COR}^2+2\Delta x_{COR}(r_2-r_1)\cos[\theta]+2\Delta y_{COR}(r_2-r_1)\sin[\theta]+(r_2-r_1)^2\{\cos[\theta]^2+\sin[\theta]^2\}$
$= \{\Delta x_{COR}^2+\Delta y_{COR}^2\}+2(r_2-r_1)\{\Delta x_{COR} \cos[\theta]+\Delta y_{COR} \sin[\theta]\}+(r_2-r_1)^2$

Dist = $Sqrt[\{\Delta x_{COR}^2+\Delta y_{COR}^2\}+2(r_2-r_1)\{\Delta x_{COR} \cos[\theta]+\Delta y_{COR} \sin[\theta]\}+(r_2-r_1)^2]$
which is not generally independent of the rotation angle (θ).

Can it be made independent?
Yes, set $r_2=r_1$
Then the middle term with angle (θ) dependence drops out, as well as the final term.
The distance between the points will just be the same as the distance between the two COR's.
This is like tying a string between the minute hands of two identical synchronized clocks, which tick in unison.
This is a special degenerate case for which the string doesn't break.

Calculations for Boosts:
Pick a totally arbitrary initial starting point (x_o , y_o) and place the models. We can write a generalization of the positions.

Now, calculate for points with a Common Rindler Point (RP):
The coordinates of the points are:
$(ct_1, x_1) = (ct_o+ct_{RP}+d_1 \cosh[\omega], x_o+x_{RP}+d_1 \sinh[\omega])$
$(ct_2, x_2) = (ct_o+ct_{RP}+d_2 \cosh[\omega], x_o+x_{RP}+d_2 \sinh[\omega])$

$(c\Delta t, \Delta x)$
$= (ct_2-ct_1, x_2-x_1)$
$= (ct_o+ct_{RP}+d_2 \cosh[\omega]-ct_o-ct_{RP}-d_1 \cosh[\omega], x_o+x_{RP}+d_2 \sinh[\omega]-x_o-x_{RP}-d_1 \sinh[\omega])$
$= (d_2 \cosh[\omega]-d_1 \cosh[\omega], d_2 \sinh[\omega]-d_1 \sinh[\omega])$
$= ((d_2-d_1)\cosh[\omega], (d_2-d_1) \sinh[\omega])$

IntervalDist^2 {4D Invariant}
$= (c\Delta t^2-\Delta x^2) = (c\Delta t'^2-\Delta x'^2)$
$= (d_2-d_1)^2\cosh[\omega]^2-(d_2-d_1)^2 \sinh[\omega]^2$
$= (d_2-d_1)^2\{\cosh[\omega]^2-\sinh[\omega]^2\}$
$= (d_2-d_1)^2$
which is independent of the hyperbolic angle (ω).
This means that the two points move in such a way as to maintain invariant interval regardless of the hyperbolic angle.
Now, when measuring spatial distance between points, one sets the $\Delta t=0$ in the MCRF frame,
which means the $-\Delta x^2 = (d_2-d_1)^2$
which means the Δx is constant.
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Now, calculate for points using Independent Rindler Points (RP1 & RP2):
The coordinates of the points are:
$(ct_1, x_1) = (ct_o+ct_{RP1}+d_1 \cosh[\omega], x_o+x_{RP1}+d_1 \sinh[\omega])$
$(ct_2, x_2) = (ct_o+ct_{RP2}+d_2 \cosh[\omega], x_o+x_{RP2}+d_2 \sinh[\omega])$

$(c\Delta t, \Delta x)$
$= (ct_2-ct_1, x_2-x_1)$
$= (ct_o+ct_{RP2}+d_2 \cosh[\omega]-ct_o-ct_{RP1}-d_1 \cosh[\omega], x_o+x_{RP2}+d_2 \sinh[\omega]-x_o-x_{RP1}-d_1 \sinh[\omega])$
$= (ct_{RP2}-ct_{RP1}+d_2 \cosh[\omega]-d_1 \cosh[\omega], x_{RP2}-x_{RP1}+d_2 \sinh[\omega]-d_1 \sinh[\omega])$
$= (c\Delta t_{RP}+(d_2-d_1) \cosh[\omega], \Delta x_{RP}+(d_2-d_1) \sinh[\omega])$

IntervalDist^2 {4D Invariant}
$= (c\Delta t^2-\Delta x^2) = (c\Delta t'^2-\Delta x'^2)$
$= (c\Delta t_{RP})^2+2(c\Delta t_{RP})(d_2-d_1) \cosh[\omega]+(d_2-d_1)^2 \cosh[\omega]^2-(\Delta x_{RP})^2-2(\Delta x_{RP})(d_2-d_1) \sinh[\omega]-(d_2-d_1)^2 \sinh[\omega]^2$
$= (c\Delta t_{RP})^2-(\Delta x_{RP})^2+2(c\Delta t_{RP})(d_2-d_1) \cosh[\omega]-2(\Delta x_{RP})(d_2-d_1) \sinh[\omega]-(d_2-d_1)^2\{\cosh[\omega]^2-\sinh[\omega]^2\}$
$= \{(c\Delta t_{RP})^2-(\Delta x_{RP})^2\}+2(d_2-d_1)\{(c\Delta t_{RP}) \cosh[\omega]-(\Delta x_{RP}) \sinh[\omega]\}+(d_2-d_1)^2$
which is NOT independent of the hyperbolic angle (ω).
Needless to say, the IntervalDist is not preserved in this case, it depends on the MCRF hyperangle ω.
Now, when measuring spatial distance between points, one sets the $\Delta t=0$ in the MCRF frame,
The Δx will vary.

Can it be made independent?
Yes, set $d_2=d_1$, the distance of each ship from its own Rindler Point are equal.
Then the middle term with hyperangle dependence drops out, as well as the final term.
Since the ships start at the same time in the start frame, $\Delta t_{RP}=0$. So, the $\Delta x = \Delta x_{RP}$ for the whole acceleration.
This is the distance between two ships as measured from at any point from the start frame.