

Here are the most-used Derivative Rules, mostly derived from the Natural Log (Ln) & Constant Rules, which are assumed ( $y'=dy/dx$  :  $x'=dx/dx=1$ )

**$y=\ln(x)$**   
 $dy=dx/x$   
 $y'=x'/x$   
**Natural Log (Ln) Rule**

**$y=c$**   
 $y'=0$   
**Constant Rule**

**$y=\log_a(x)$**   
 $y=\log_a(x=a^z)$   
 $y=z*\log_a(a)$   
 $y=z$   
 $dy=dz$   
 $dx=\ln(a)a^z dz$   
 $dx=\ln(a)x dy$   
 $dy=dx/[x*\ln(a)]$   
 $y'=1/[x*\ln(a)]$   
**Log Rule**

**$y=a+b$**   
 $\ln(y)=\ln(a+b)$   
 $dy/y=(da+db)/(a+b)$   
 $dy=y(da+db)/(a+b)$   
 $dy=(a+b)(da+db)/(a+b)$   
 $dy=(da+db)$   
 $y'=a'+b'$   
**Sum Rule**

**$y=a-b$**   
 $\ln(y)=\ln(a-b)$   
 $dy/y=(da-db)/(a-b)$   
 $dy=y(da-db)/(a-b)$   
 $dy=(a-b)(da-db)/(a-b)$   
 $dy=(da-db)$   
 $y'=a'-b'$   
**Difference Rule**

**$y=ab$**   
 $\ln(y)=\ln(ab)=\ln(a)+\ln(b)$   
 $dy/y=da/a+db/b$   
 $dy=y[da/a+db/b]$   
 $dy=(ab)[da/a+db/b]$   
 $dy=(b)da+(a)db$   
 $y'=(b)a'+(a)b'$   
**Product Rule**

**$y=cx$  (Special case:  $c=\text{constant}$ )**  
 $y'=(x)c'+(c)x'$   
 $y'=(x)(0)+(c)x'$   
 $y'=(c)x'$   
**Constant\*Function Rule**

**$y=a/b$**   
 $\ln(y)=\ln(a/b)=\ln(a)-\ln(b)$   
 $dy/y=da/a-db/b$   
 $dy=y[da/a-db/b]$   
 $dy=(a/b)[da/(a)-db/(b)]$   
 $dy=[da/(b)-(a)db/(b^2)]$   
 $dy=[(b)da/(b^2)-(a)db/(b^2)]$   
 $dy=[(b)da-(a)db]/b^2$   
 $y'=[(b)a'-(a)b']/b^2$   
**Quotient Rule**

**$y=a^b$**   
 $\ln(y)=\ln(a^b)=b*\ln(a)$   
 $dy/y=db*\ln(a)+b*(da/a)$   
 $dy=y[db*\ln(a)+b*(da/a)]$   
 $dy=(a^b)[db*\ln(a)+b*(da/a)]$   
 $dy=\ln(a)*(a^b)db+b*a^{(b-1)}da$   
 $y'=\ln(a)*(a^b)b'+b*a^{(b-1)}a'$   
**Exponential & Power Rule**

**$y=x^b$  (Special case:  $b=\text{constant}$ )**  
 $y'=\ln(x)*(x^b)b'+b*x^{(b-1)}x'$   
 $y'=\ln(x)*(x^b)(0)+b*x^{(b-1)}x'$   
 $y'=b*x^{(b-1)}x'$   
**Power Rule**

**$y=a^x$  (Special case:  $a=\text{constant}$ )**  
 $y'=\ln(a)*(a^x)x'+x*a^{(x-1)}a'$   
 $y'=\ln(a)*(a^x)x'+x*a^{(x-1)}(0)$   
 $y'=\ln(a)*(a^x)x'$   
**Exponential Rule**

**$y=x^x$  (Special case:  $a=b=x$ )**  
 $dy=\ln(x)*(x^x)dx+x*x^{(x-1)}dx$   
 $dy=\ln(x)*(x^x)dx+x^x dx$   
 $dy=[\ln(x)+1]*(x^x)dx$   
 $y'=[\ln(x)+1]*(x^x)x'$   
**Hyper-Exponent Rule**

**$y=x^1=x$  (Special case:  $b='1'$ )**  
 $y'=\ln(x)*(x^b)b'+b*x^{(b-1)}x'$   
 $y'=\ln(x)*(x^1)(0)+1*x^{(1-1)}x'$   
 $y'=1x'$   
**Linear Rule**

**$y=e^x$  (Special case:  $a='e'$ )**  
 $y'=\ln(e)*(e^x)x'$   
 $y'=(e^x)x'$   
**Natural Exponent (e) Rule**

**$y=\sin(x)=(e^{ix}-e^{-ix})/(2i)$**   
 $y'=[(i)e^{ix}-(-i)e^{-ix}]x'/(2i)$   
 $y'=[ie^{ix}+ie^{-ix}]x'/(2i)$   
 $y'=(i)[e^{ix}+e^{-ix}]x'/(2i)$   
 $y'=(e^{ix}+e^{-ix})x'/(2)=\cos(x)x'$   
**Sine Rule**

**$y=\sinh(x)=(e^x-e^{-x})/2$**   
 $y'=[(1)e^x-(-1)e^{-x}]x'/(2)$   
 $y'=(e^x+e^{-x})x'/(2)=\cosh(x)x'$   
**Hyperbolic Sine Rule**

**$y=f(g[x])$**   
 $dy=df$   
 $(dy/dx)=(df/dx)$   
 $(dy/dx)=(df/dx)(dg/dg)$   
 $(dy/dx)=(df/dg)(dg/dx)$   
 $y'=(df/dg)g'$   
**Chain Rule (2 links)**

**$y=f(g[h\{x\}])$**   
 $dy=df$   
 $(dy/dx)=(df/dx)$   
 $(dy/dx)=(df/dx)(dg/dg)(dh/dh)$   
 $(dy/dx)=(df/dg)(dg/dh)(dh/dx)$   
 $y'=(df/dg)(dg/dh)h'$   
**Chain Rule (3 links)**

**$y=\cos(x)=(e^{ix}+e^{-ix})/2$**   
 $y'=[(i)e^{ix}+(-i)e^{-ix}]x'/(2)$   
 $y'=(i)[e^{ix}-e^{-ix}]x'/(2)$   
 $y'=(i^2)[e^{ix}+e^{-ix}]x'/(2i)$   
 $y'=-e^{ix}+e^{-ix}x'/(2)=-\sin(x)x'$   
**Cosine Rule**

**$y=\cosh(x)=(e^x+e^{-x})/2$**   
 $y'=[(1)e^x+(-1)e^{-x}]x'/(2)$   
 $y'=(e^x-e^{-x})x'/(2)=\sinh(x)x'$   
**Hyperbolic Cosine Rule**

**$y=f(g[h\{...\{z(x)...\}\}])$**   
 $dy=df$   
 $(dy/dx)=(df/dx)$   
 $(dy/dx)=(df/dx)(dg/dg)(dh/dh)...(dz/dz)$   
 $(dy/dx)=(df/dg)(dg/dh)...z'$   
**Chain Rule (n links)**

**$y=\tan(x)=\sin(x)/\cos(x)$**   
 $y'=\text{UseQuotientRule}$   
 $y'=[\cos(x)\cos(x)-\sin(x)(-\sin(x))]/\cos^2(x)$   
 $y'=[\cos(x)\cos(x)+\sin(x)(\sin(x))]/\cos^2(x)$   
 $y'=1/\cos^2(x)x' = \sec^2(x)x'$   
**Tangent Rule**

**$y=\tanh(x)=\sinh(x)/\cosh(x)$**   
 $y'=\text{UseQuotientRule}$   
 $y'=[\cosh(x)\cosh(x)-\sinh(x)(\sinh(x))]/\cosh^2(x)$   
 $y'=1/\cosh^2(x)x' = \text{sech}^2(x)x'$   
**Hyperbolic Tangent Rule**