Top Ten Lists have been created for a variety of categories. Here is a **Physics Top Ten** List, although perhaps in an unexpected way. Using physics to describe our universe, "Ten" occurs in a "number" of very important contexts. Thus... The Physics Top Ten! :)

There are in 4D SpaceTime: (10) Symmetries, (10) Invariances, (10) Isometries, (10) Conservation Laws ↔ (10) Noether Symmetries, (10) Independent Relativistic-Particle Parameters (using points), (10) Independent Relativistic-Fluid Parameters (using densities), (10) {(6)=$(4^2-4)/2$ AntiSymmetric 4-Tensor Angular [↻] (components + (4)=(1+3) 4-Vector Linear [→] components}, (10)=$(4^2+4)/2$ Symmetric 4-Tensor components, (10) Independent (PPN) Parameterized Post-Newtonian formalism variable parameters describing the increasingly unlikely deviations from GR.
These <Time·Space> concepts are intimately related to one another and reveal deep, important facts about how our universe operates.

SpaceTime Symmetry is based on Group Theory. 4D SpaceTime uses the Poincaré Symmetry Group, a Lie Group, which has (10) parameters. It is the also known as the Inhomogeneous Lorentz Group, and is the tensor product of the (Homogeneous) Lorentz Symmetry Group with (6) parameters and the SpaceTime-Translation Symmetry Group with (4) parameters. $P(10)=L(6)+ST(4)$.
Lorentz Symmetry provides: [↻] SpaceTime Isotropy: Same all directions : rotation angle θ, boost hyper-angle φ (∗).
SpaceTime-Translation Symmetry provides: [→] SpaceTime Homogeneity: Same all extent : 4-Displacement $\Delta\mathbf{X}$ [▦].

Invariance is based on the Poincaré Group linear mapping ($V^{\mu'} = \Lambda^{\mu'}{}_{\nu} V^{\nu} + \Delta V[\Delta X^{\mu'}]$) which preserves the 4D Interval-Magnitude: ($V^{\mu'}V_{\mu'} = V^{\nu}V_{\nu}$). Note this is similar to algebraic linear mapping x'=A*x+B[Δx]. Observers in different inertial reference frames will obtain the same magnitude results, although they will usually measure different individual component values. Ex.: getting the same invariant mass m_o but measuring different energies and momenta ($E/c=mc$,$\mathbf{p}$). The Lorentz Transform 4-Tensor $\Lambda^{\mu'}{}_{\nu}$ has (6) parameters {3 boost + 3 rotate} and the 4-Displacement SpaceTime Transform [$\Delta X^{\mu'}$] has (4) parameters {1 time + 3 space}, for a total of (10).

Isometry is the concept that various operations can be done to a system and still get the same measurement result $|\Delta\mathbf{X}|$, the SpaceTime Interval or "4D Distance" between <events>. ($\Delta X^{\mu'}\Delta X_{\mu'} = \Delta X^{\nu}\Delta X_{\nu}$) For example, you can rotate a system (3), translate a system in space (3), translate a system in time (1), or boost a system to a uniform velocity (3), for a total of (10). Isometry = "Same Measure". This is related to the concept of active (change the physical system) and passive (change the coordinate system) transformations.

Conservation Laws are the broad principles that govern physical systems. They describe those properties which remain unchanged in proper time for a closed system. Noether's Theorem: "If a system has a continuous symmetry property, then there are corresponding quantities whose values are conserved in time." The most famous is the Conservation of Energy (1), from time translation, followed by the Conservation of Linear Momentum (3) from space translation, and Conservation of Angular Momentum (3) from spatial rotation. Another, less well-known, is the Conservation of Mass Moment (3) from temporal-spatial Lorentz boosts. Total (10) & (10).

Particle Parameters are those independent variables that fully describe the SpaceTime dynamics of a point particle. These include the [↻] 4-AngularMomentum $M^{\alpha\beta}$, which has a total of (6)=$(4^2-4)/2$ independent parameters due to being an anti-symmetric 4D (2,0)-Tensor, and the [→] 4-LinearMomentum P^{μ}, which has a total of (4) parameters due to being a 4D (1,0)-Tensor=4-Vector. Combined, there are a total of (10) independent parameters. One can also think of this as particle Spin-momentum + Linear-momentum.

Fluid Parameters are those independent variables that fully describe the SpaceTime dynamics of a fluid. The components of the Relativistic Fluid Stress-Energy 4-Tensor $T_{\text{relfluid}}{}^{\mu\nu}$ include energy-density (1), heat-flux (3), pressure (1), and viscous-shear (5). Likewise, a symmetric 4D (2,0)-Tensor has a total of (10)=$(4^2+4)/2$ independent parameters. Just as a note, it is technically a Stress-EnergyDensity 4-Tensor, as each component has units of [$kg/(m{\cdot}s^2) = J/m^3 = N/m^2 = Pa$ = {energy-density} = {pressure} = {stress}]

4D Tensor Components are those independent components of a generic 4D Tensor. For a regular, generic 2-index tensor there are $N*N = N^2$ components and for a regular, generic 1-index tensor there are N components. Any 2-index tensor $T^{\mu\nu}$ can be decomposed into symmetric $S^{\mu\nu}$ and anti-symmetric $A^{\mu\nu}$ parts using the following procedure. Define $S^{\mu\nu} = (T^{\mu\nu} + T^{\nu\mu})/2$ and $A^{\mu\nu} = (T^{\mu\nu} - T^{\nu\mu})/2$.
$S^{\mu\nu} + A^{\mu\nu} = (T^{\mu\nu} + T^{\nu\mu})/2 + (T^{\mu\nu} - T^{\nu\mu})/2 = 2T^{\mu\nu}/2 + T^{\nu\mu}/2 - T^{\nu\mu}/2 = T^{\mu\nu}$.
By inspection (just switch indices), the symmetric 4-Tensor $S^{\mu\nu} = +S^{\nu\mu}$ and the anti-symmetric 4-Tensor $A^{\mu\nu} = -A^{\nu\mu}$.
Viewing the tensors as [N×N] matrices:
The symmetric tensor has $(N^2+N)/2$ independent components. The lower triangle = upper triangle. Diagonal has N components.
The anti-symmetric tensor has $(N^2-N)/2$ independent components. The lower triangle = - upper triangle. Diagonal has all zeroes.
So, for 4D tensors we obtain:
The symmetric tensor has $(4^2+4)/2$ = (10) independent components.
The anti-symmetric tensor has $(4^2-4)/2$ = (6) independent components + 4-Vector (4) components = (10) independent components.

Parameterized Post-Newtonian formalism (PPN), is a formulation that explicitly details the parameters in which a general theory of gravity can differ from Newtonian gravity. It is used as a tool to compare Newtonian and Einsteinian gravity in the limit of a weak gravitational field {$g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$ with $|h^{\mu\nu}|$ small} and in which objects move slowly compared to the speed of light {$|v| \ll c$}.
In general, PPN formalism can be applied to all (inc. alternative) metric theories of gravitation in which the massive bodies satisfy the Einstein Equivalence Principle (EEP). There are (10) Parameters. To-date, GR is the best fit to measured data and observations.

This wondrous number (10) occurs because of a few reasons: Our spacetime has a 4D Poincaré Group Symmetry, and 4D Tensor mathematics provides an excellent mathematical framework to describe the <events> of our universe. The Poincaré Group naturally splits into <Time·Space> components, with each tensor splitting into temporal and/or spatial and/or mixed components. The two Casimir Invariants of the Poincaré Group lead naturally to a linear [→] (mass-momentum) & angular [↻] (spin-momentum) splitting. Let's look at some of the physical, geometric, tensorial objects describing these <events>.

4-Vectors = 4D (1,0)-Tensors:

4-Position	$X^\mu = (ct,\mathbf{x}) = \mathbf{X} \in$ <event>	[m]	$(ct,\mathbf{x}) \to (ct,x,y,z)$ only Lorentz, not Poincaré Invariant Alt. $R^\mu = \mathbf{R}$
4-Displacement	$\Delta X^\mu = (c\Delta t,\Delta\mathbf{x}) = \Delta\mathbf{X}$	[m]	Finite $\Delta\mathbf{X} = \mathbf{X_2} - \mathbf{X_1}$ fully Poincaré Invariant
4-Gradient	$\partial^\mu = (\partial_t/c,-\nabla) = \partial = \partial/\partial X_\mu$	[1/m]	$X_\mu = \eta_{\mu\nu} X^\nu : X^\mu = \eta^{\mu\nu} X_\nu$: 4D One-form $\partial_\mu = (\partial_t/c,\nabla)$
4-Velocity	$U^\mu = \gamma(c,\mathbf{u}) = \mathbf{U} = d\mathbf{X}/d\tau$	[m/s]	Lorentz Gamma Factor $\gamma = 1/\sqrt{[1-(u/c)^2]} = dt/d\tau$
4-Acceleration	$A^\mu = \gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a}) = \mathbf{A}$	[m/s²]	$= d\mathbf{U}/d\tau = d^2\mathbf{X}/d\tau^2$
4-"Unit"Temporal	$\bar{T}^\mu = \gamma(1,\boldsymbol{\beta}) = \mathbf{U}/c$	[1]	Dimensionless primitive 4-Vector, w/ 4-"Unit"Spatial
4-LinearMomentum	$P^\mu = (E/c{=}mc,\mathbf{p}) = \mathbf{P} = m_o\mathbf{U}$	[(kg·m)/s = N·s]	m_o = RestMass : $E = mc^2 = \gamma m_o c^2 = \gamma E_o$

4-Tensors,Upper = 4D (2,0)-Tensors:

Minkowski 4D Metric	$\eta^{\mu\nu} = \partial^\mu[X^\nu] = V^{\mu\nu} + H^{\mu\nu}$	→ Diag[+1,-1,-1,-1]	[1]
Kronecker Delta (2,0)	$\delta^{\mu\nu}$	= Diag[+1,+1,+1,+1]	[1] $[\delta^{\mu\nu}]$ = {1 if $^{\mu=\nu}$, 0 otherwise}
Temporal Projection (2,0)	$V^{\mu\nu} = U^\mu U^\nu/c^2 = \bar{T}^\mu\bar{T}^\nu$	→ Diag[+1,0,0,0]	[1] (V)ertical Projection on LightCone Diagram
Spatial Projection (2,0)	$H^{\mu\nu} = \eta^{\mu\nu}$ - $V^{\mu\nu}$	→ Diag[0,-1,-1,-1]	[1] (H)orizontal Projection on LightCone Diagram
4-AngularTensor	$\omega^{\mu\nu}$		[{rad}] encoding 3 rotation angles + 3 boost angles
4-AngularMomentum	$M^{\alpha\beta} = X^\alpha P^\beta - X^\beta P^\alpha = \mathbf{X}\wedge\mathbf{P}$		[(kg·m²)/s = N·m·s = J·s = Action]
Relativistic Fluid Stress-Energy(Density) 4-Tensors $T_{RelFluid}{}^{\mu\nu} : T_{PerfectFluid}{}^{\mu\nu}$			[kg/(m·s²) = J/m³ = N/m² = Pa]

4-Tensors,Mixed = 4D (1,1)-Tensors:

Lorentz Transform	$\Lambda^{\mu'}{}_\nu = \partial_\nu[X^{\mu'}] = \partial X^{\mu'}/\partial X^\nu$		[1] =Rotation $R^{\mu'}{}_\nu$ or Boost $B^{\mu'}{}_\nu$ or Flip $F^{\mu'}{}_\nu$ or Combo
Kronecker Delta (1,1)	$\delta^\mu{}_\nu = \eta^\mu{}_\nu = g^\mu{}_\nu$	= Diag[+1,+1,+1,+1]	[1] $[\delta^\mu{}_\nu]$ = {1 if $^{\mu=\nu}$, 0 otherwise}
Temporal Projection (1,1)	$V^\mu{}_\nu = U^\mu U_\nu/c^2 = \bar{T}^\mu\bar{T}_\nu$	→ Diag[+1,0,0,0]	[1] (V)ertical Projection on LightCone Diagram
Spatial Projection (1,1)	$H^\mu{}_\nu = \eta^\mu{}_\nu$ - $V^\mu{}_\nu = \eta_{\sigma\nu} H^{\mu\sigma}$	→ Diag[0,+1,+1,+1]	[1] (H)orizontal Projection on LightCone Diagram

4-Tensors,Lower = 4D (0,2)-Tensors:

Kronecker Delta (0,2)	$\delta_{\mu\nu}$	= Diag[+1,+1,+1,+1]	[1] $[\delta_{\mu\nu}]$ = {1 if $^{\mu=\nu}$, 0 otherwise}
Temporal Projection (0,2)	$V_{\mu\nu} = U_\mu U_\nu/c^2 = \bar{T}_\mu\bar{T}_\nu$	→ Diag[+1,0,0,0]	[1] (V)ertical Projection on LightCone Diagram
Spatial Projection (0,2)	$H_{\mu\nu} = \eta_{\mu\nu}$ - $V_{\mu\nu}$	→ Diag[0,-1,-1,-1]	[1] (H)orizontal Projection on LightCone Diagram

4-Tensors can be transformed among {Upper,Mixed,Lower} forms by Tensor Index Raising:Lowering using the Minkowski Metric $\boldsymbol{\eta}$. Another tensor object which comes up often is the Levi-Civita Totally AntiSymmetric Tensor: 3D $\varepsilon_{ij}{}^k$: 4D $\varepsilon_{\mu\nu\rho}{}^\sigma$: other combinations.

Integral to physical, relativistic tensors is that {4D <Time·Space> = 1D Temporal (t) + 3D Spatial (x,y,z)} entities have specific ways of splitting into their various natural, measurable components, depending on the type of tensor that they are represented by:

4-Scalar	S	= S	(1) Invariant Lorentz Scalar, same for all frames {s} or {s_o}	1 {4D (0,0)-Tensor} component
4-Vector	**V**	= V^μ	($1^{\,0}+3^{\,j}$)-splitting into {v^t,v^x,v^y,v^z}	4 {4D (1,0)-Tensor} components
4-Tensor,$_{AntiSymmetric}$	$\boldsymbol{T_{asym}}$	= $T_{asym}{}^{\mu\nu}$	($3^{\,0j}+3^{\,j\neq k}$)-splitting into {$t^{tx},t^{ty},t^{tz},t^{xy},t^{xz},t^{yz}$} w/ all $^{j=k}$ comps=0	6 {4D (2,0)-Tensor} components
4-Tensor,$_{Symmetric}$	$\boldsymbol{T_{sym}}$	= $T_{sym}{}^{\mu\nu}$	($1^{\,00}+3^{\,0j}+3^{\,j=k}+3^{\,j\neq k}$)-splitting into {$t^{tt},t^{tx},t^{ty},t^{tz},t^{xx},t^{yy},t^{zz},t^{xy},t^{xz},t^{yz}$}	10 {4D (2,0)-Tensor} components
4-Tensor,$_{Generic}$	$\boldsymbol{T}$	= $T^{\mu\nu}$	[[$t^{tt},t^{tx},t^{ty},t^{tz}$],[$t^{xt},t^{xx},t^{xy},t^{xz}$],[$t^{yt},t^{yx},t^{yy},t^{yz}$],[$t^{zt},t^{zx},t^{zy},t^{zz}$]]	16 {4D (2,0)-Tensor} components

There are relativistic Symmetries/Operations in nature which leave the interval-measurement between events unchanged (invariant) and lead to fundamental Conservation Laws. These can use active or passive transformations, including changes of coordinate basis. SR 4-Vectors have a Poincaré Group linear mapping ($V^{\mu'} = \Lambda^{\mu'}{}_\nu V^\nu + \Delta V[\Delta X^{\mu'}]$) which preserves interval-magnitude: ($V^{\mu'}V_{\mu'} = V^\nu V_\nu$).
The Poincaré Group {Lorentz Group $\Lambda^\mu{}_\nu$+ SpaceTime Translation Group ΔX^μ} is the Full SpaceTime Symmetry Group. $\mathbf{R}^{1,3} \ltimes O(1,3)$
(10) Isometries match the {Anti-Symmetric 4D (2,0)-Tensor [3+3]-splitting→(6) + 4D (1,0)-Tensor (1+3)-splitting→(4)}.
(10) Isometries match the {Symmetric 4D (2,0)-Tensor [1+3+3+3]-splitting→(10)}.

Relativistic Particle, Scalar Cartesian form:

#	Type	Symmetry	Conservation Law	# Parameters	Symmetry
1	Mixed	Lorentz Boost $\Lambda^{\mu'}_{\nu} \to B^{\mu'}_{\nu}[t,x]$	mass-moment n^{tx}	1	Isotropy-t,x : boost along x, φ_x
2	Mixed	Lorentz Boost $\Lambda^{\mu'}_{\nu} \to B^{\mu'}_{\nu}[t,y]$	mass-moment n^{ty}	1	Isotropy-t,y : boost along y, φ_y
3	Mixed	Lorentz Boost $\Lambda^{\mu'}_{\nu} \to B^{\mu'}_{\nu}[t,z]$	mass-moment n^{tz}	1	Isotropy-t,z : boost along z, φ_z
4	Spatial	Lorentz Rotate $\Lambda^{\mu'}_{\nu} \to R^{\mu'}_{\nu}[x,y]$	angular-momentum l^{xy}	1	Isotropy-x,y : rotate about z, θ_z
5	Spatial	Lorentz Rotate $\Lambda^{\mu'}_{\nu} \to R^{\mu'}_{\nu}[x,z]$	angular-momentum l^{xz}	1	Isotropy-x,z : rotate about y, θ_y
6	Spatial	Lorentz Rotate $\Lambda^{\mu'}_{\nu} \to R^{\mu'}_{\nu}[y,z]$	angular-momentum l^{yz}	1	Isotropy-y,z : rotate about x, θ_x
7	Temporal	Translate Time $\Delta X^{\mu'} \to c\Delta t$	energy $E = cp^t$	1	Homogeneity-t : translate Δt
8	Spatial	Translate Space $\Delta X^{\mu'} \to \Delta x$	linear-momentum p^x	1	Homogeneity-x : translate Δx
9	Spatial	Translate Space $\Delta X^{\mu'} \to \Delta y$	linear-momentum p^y	1	Homogeneity-y : translate Δy
10	Spatial	Translate Space $\Delta X^{\mu'} \to \Delta z$	linear-momentum p^z	1	Homogeneity-z : translate Δz
				(10)	

Relativistic Particle, Vector Form:

#	Type	Symmetry	Conservation Law	# Parameters	Symmetry
1	Mixed	Lorentz Boost $\Lambda^{\mu'}_{\nu} \to B^{\mu'}_{\nu}$	3-mass-moment $\mathbf{n}=m^{0j}/c$	3	Isotropy-t,$\hat{\mathbf{n}}$: boost along $\hat{\mathbf{n}}$, φ
2	Spatial	Lorentz Rotate $\Lambda^{\mu'}_{\nu} \to R^{\mu'}_{\nu}$	3-angular-momentum $\mathbf{l}=\varepsilon_{ij}{}^{k}m^{ij}/2$	3	Isotropy-$\hat{\mathbf{n}}_1$,$\hat{\mathbf{n}}_2$: rotate about $\hat{\mathbf{n}}_3$, θ
3	Temporal	Translate Time $\Delta X^{\mu'} \to c\Delta t$	energy $E = cp^0$	1	Homogeneity-t : in time Δt
4	Spatial	Translate Space $\Delta X^{\mu'} \to \Delta\mathbf{x}$	3-linear-momentum $\mathbf{p}$	3	Homogeneity-$\hat{\mathbf{n}}$: in space $\Delta\mathbf{x}$
				(10)	

Relativistic Particle, Tensor Form: 4D Anti-Symmetric (2,0)-Tensor + 4D (1,0)-Tensor

#	Type	Symmetry	Conservation Law	# Parameters	Symmetry
1	Mixed	Lorentz Transform $\Lambda^{\mu'}_{\nu}$	4-AngularMomentum $M^{\mu\nu}=X^{\mu}\wedge P^{\nu}$	6 [⥁]	Isotropy-Spacetime θ,φ (∗)
2	Mixed	SpaceTime Translate $\Delta X^{\mu'}$	4-LinearMomentum P^{μ}	4 [→]	Homogeneity-Spacetime $\Delta\mathbf{X}$ [▦]
				(10)	

Relativistic Fluid Stress-Energy Tensor $T^{\mu\nu}$: 4D Symmetric (2,0)-Tensor

#	Type	Tensor Component, index	Tensor Component, type	# Parameters	
1	Temporal	t^{00}	energy-density $\rho_e = \rho_m c^2 = t^{00}$	1	
2	Mixed	t^{0j} and t^{i0} with $t^{0k} = t^{k0}$	3-heat-flux $\mathbf{q} = ct^{i0}$	3	
3	Spatial	$t^{i=j} = p\delta^{ij}$	isotropic pressure $p = -(1/3)H_{\mu\nu}T^{\mu\nu}$	1	spatial diagonal components
4	Spatial	$t^{ij} = \Pi^{ij}$	anisotropic 6-viscous-shear Π^{ij}	5	only 5 independent
				(10)	

[↻]
Lorentz Group ($\Lambda^{\mu}{}_{\nu}$) Symmetry → Conservation of 4-AngularMomentum $\boldsymbol{M} = M^{\mu\nu} = R^{\mu} \wedge P^{\nu} = [[m^{\mu\nu}]] = [[0,-c\mathbf{n}],[c\mathbf{n}^T,\mathbf{l}=\mathbf{r}\wedge\mathbf{p}]]$:Isotropy
The spatial part: 3 Space-Space-Rotation ($\Lambda^{\mu}{}_{\nu}\to R^{\mu}{}_{\nu}$) Symmetry → Conservation of 3-angular-momentum $\mathbf{l} = l^k$ same all directions
The mixed part: 3 Space-Time-Boost ($\Lambda^{\mu}{}_{\nu}\to B^{\mu}{}_{\nu}$) Symmetry → Conservation of 3-mass-moment $\mathbf{n} = n^k$ θ,φ (∗)

[→]
Spacetime-Translation Group (ΔX^{μ}) Symmetry → Conservation of 4-LinearMomentum $\mathbf{P} = P^{\mu} = (p^{\mu}) = (p^0,p^k) = (E/c,\mathbf{p})$: Homogeneity
The temporal part: 1 Time-Translation ($\Delta x^0 = c\Delta t$) Symmetry → Conservation of energy $E = cp^0$ same all extent
The spatial part: 3 Space-Translation ($\Delta x^k = \Delta\mathbf{x}$) Symmetry → Conservation of 3-momentum $\mathbf{p} = p^k$ $\Delta\mathbf{X}$ [▦]

P: the 4-Momentum & **W**: the Pauli–Lubanski 4-Spin pseudovector. give ($\mathbf{P\cdot P}$) → mass (m_o) and ($\mathbf{W\cdot W}$) → spin (s_o), which are the two Casimir Invariants of the Poincaré Group, i.e. the quantities that commute with all generators of the Poincaré Group $\mathbf{R}^{1,3} \ltimes O(1,3)$. Technically, ($\mathbf{P\cdot P}$) gives a 4-Momentum magnitude (moving mass) and ($\mathbf{W\cdot W}$) gives a 4-SpinMomentum magnitude (moving spin).

Notes on Poincaré SpaceTime Group, Tensor Linear Mapping, Lie Group, etc.:
SR 4-Vectors have a Poincaré Group linear mapping ($V^{\mu'} = \Lambda^{\mu'}{}_{\nu} V^{\nu} + \Delta V[\Delta X^{\mu'}]$) which preserves interval-magnitude: ($V^{\mu'}V_{\mu'} = V^{\nu}V_{\nu}$).
Poincaré = ISO(1,3) or $(\mathbb{R}\oplus\mathbb{R}^3)\rtimes O(1,3)$ or $\mathbb{R}^4\rtimes SU_2(\mathbb{C})$

The Poincaré Group is a Lie Group, and can be written as a Unitary Operation: $U(\Lambda^{\mu'}{}_{\nu} : \Delta X^{\mu'}) = e^{\wedge}[({}^{i}/_{2\hbar})\omega_{\mu\nu}M^{\mu\nu}]$ [↻] $e^{\wedge}[({}^{i}/_{\hbar})\Delta X_{\mu}P^{\mu}]$ [→] with:
4-LinearMomentum P^{μ} [kg·m/s] as generator of SpaceTime-Translation-Transforms [→]
with 4-Displacement ΔX^{μ} [m] encoding the 1 + 3 = 4 displacements

4-AngularMomentum M^{μ} [kg·m²/s] as generator of Lorentz-Transforms [↻]
with anti-symmetric Angular 4-Tensor $\omega^{\mu\nu}$ [1 or {rad}] encoding the 3 rotation angles + 3 boost hyper-angles

[$\Delta X_{\mu}P^{\mu}$] and [$\omega_{\mu\nu}M^{\mu\nu}$] and (ħ) all have dimensional-units of [Action = kg·m²/s = J·s] :
Infinitesimal versions: $T^{\mu'} = 1^{\mu'} + \Delta X^{\mu'} + \ldots$: $\Lambda^{\mu'}{}_{\nu} = \delta^{\mu'}{}_{\nu} + \omega^{\mu'}{}_{\nu} + \ldots$

4-AngularMomentum $M^{\mu\nu} = \mathbf{X}\wedge\mathbf{P} = X^{\mu}P^{\nu} - X^{\nu}P^{\mu}$
= Generator of Lorentz Transformations (6)
= { $\Lambda^{\mu'}{}_{\nu}\to R^{\mu'}{}_{\nu}$ Rotations (3) + $\Lambda^{\mu'}{}_{\nu}\to B^{\mu'}{}_{\nu}$ Boosts (3) }

4-LinearMomentum $P^{\mu} = \mathbf{P}$
= Generator of Translation Transformations (4)
= { $\Delta X^{\mu'}\to(c\Delta t,\mathbf{0})$ Time (1) + $\Delta X^{\mu'}\to(0,\Delta\mathbf{x})$ Space (3) }

Symmetric:AntiSymmetric Tensor decomposition $\{T^{\mu\nu} = S^{\mu\nu} + A^{\mu\nu}\}$, with $S^{\mu\nu} = (T^{\mu\nu}+T^{\nu\mu})/2$ and $A^{\mu\nu} = (T^{\mu\nu}-T^{\nu\mu})/2$
Tensor Contraction of Symmetric with AntiSymmetric yields zero $\{S^{\mu\nu}A_{\mu\nu} = 0\}$, from { $S^{\mu\nu} = +S^{\nu\mu}$ } and { $A^{\mu\nu} = -A^{\nu\mu}$ } parts of { $T^{\mu\nu}$ }
Proof: 10 components + 6 components = 16 comps
$S^{\mu\nu}A_{\mu\nu}$ ={swaping dummy indices}→ $S^{\nu\mu}A_{\nu\mu} = (S^{\nu\mu})(A_{\nu\mu}) = (+S^{\mu\nu})(A_{\nu\mu}) = (+S^{\mu\nu})(-A_{\mu\nu}) = -S^{\mu\nu}A_{\mu\nu} = 0$, since { C = -C = 0 }
The Symmetric Tensor is further decomposed into an Isotropic part $S_{iso}{}^{\mu\nu} = (S^{\alpha}{}_{\alpha}/4)\eta^{\mu\nu}$ and zero-trace Anisotropic part $S_{aniso}{}^{\mu\nu} = S^{\mu\nu} - S_{iso}{}^{\mu\nu}$
So, $\{T^{\mu\nu} = S_{iso}{}^{\mu\nu} + S_{aniso}{}^{\mu\nu} + A^{\mu\nu}\}$ This is manifestly invariant: The Poincaré Group Symmetry operations respect these decompositions, meaning that boosts, rotations, etc. don't intermix them, unlike the (temporal+mixed+spatial)-splittings, which can get intermixed.

Lorentz Transform $\Lambda^{\mu'}{}_{\nu}\to B^{\mu'}{}_{\nu}(\boldsymbol{\beta})$ or $B^{\mu'}{}_{\nu}(\varphi_{hyperangle},\hat{\mathbf{n}})$ Boost : Symmetric Mixed 4-Tensor $B^T = B : B^{-1} = B[-\boldsymbol{\beta}]$: 3 parameters
$[\gamma \quad , \quad -\gamma\beta^0{}_j \quad]$ Trace of Boost = {4..Infinity}
$[-\gamma\beta^i{}_0 \, , \; (\gamma-1)\beta^i\beta_j/(\boldsymbol{\beta\cdot\beta})+\delta^i{}_j]$

Lorentz Transform $\Lambda^{\mu'}{}_{\nu}\to R^{\mu'}{}_{\nu}(\theta_{angle},\hat{\mathbf{n}})$ Rotation : Non-symmetric Mixed 4-Tensor : Orthogonal $R^T = R^{-1} : R^{-1} = R[-\theta]$: 3 parameters
$[\ 1 \ , \quad 0^0{}_j \quad]$ Trace of Rotation = {0..4}
$[0^i{}_0 \, , \, (\delta^i{}_j - n^in_j)\cos(\theta) - (\varepsilon^i{}_{jk}n^k)\sin(\theta) + n^in_j]$

Lorentz Transform $\Lambda^{\mu'}{}_{\nu}\to I^{\mu'}{}_{\nu} = \delta^{\mu'}{}_{\nu} = \mathrm{Diag}[1,1,1,1]$ Identity : Symmetric Mixed 4-Tensor : $B[\boldsymbol{\beta}=\mathbf{0}] = R[\theta=0]$: 0 parameters
$[\ 1 \ , \ 0^0{}_j]$ Trace of Identity = {4}
$[0^i{}_0 \, , \ \delta^i{}_j]$
The Boost and Rotation forms "meet" each other at the Identity Transform, at the Trace[Lorentz] = 4.

There are also various combinations of Discrete Flips $\Lambda^{\mu'}{}_{\nu}\to F^{\mu'}{}_{\nu}$ of coordinates: Trace = {-4,-2,0,2,4}
which when paired (ex. t→-t & x→-x , y=y, z=z) make Proper Discrete Transforms Det = +1.

4D-Tensor Theory of Measurements (SR Poincaré Invariance = Lorentz $\Lambda^{\mu'}{}_{\nu}$ Invariance + SpaceTime Translation $\Delta X^{\mu'}$ Invariance):
SR 4-Vectors have a Poincaré Group 4D linear mapping {technically a linear-affine transformation due to the additive constant} ($V^{\mu'} = \Lambda^{\mu'}{}_{\nu} V^{\nu} + \Delta V[\Delta X^{\mu'}]$) which preserves interval-magnitude: ($V^{\mu'} V_{\mu'} = V^{\nu} V_{\nu} = \mathbf{V}\cdot\mathbf{V}$), which is a 4-Scalar calculated from 4-Vectors.

This idea basically says the following:
A measurement made in one reference frame has an affine relationship to the same measurement made in a different reference frame.
In other words, it is a linear relation (a) with a possible additive constant (b) mapping on (**X**): **X'**=a**X**+b {the eqn. of a line}
This means that there are certain transformations:symmetries that one can do and still get the same invariant measurement interval.

Active SR Transformations (the system being measured is changed in a certain way, the reference coordinate frame is not changed):
Rotate the system of objects. 3 Ex. Pick an angle:axis and rotate whole experiment by that amount. $\Lambda^{\mu'}{}_{\nu} \rightarrow R^{\mu'}{}_{\nu}\ (\theta_{angle} : \hat{\mathbf{n}})$
Boost the system of objects. 3 Ex. Have the whole experiment move uniformly on a linear track. $\Lambda^{\mu'}{}_{\nu} \rightarrow B^{\mu'}{}_{\nu}\ (\boldsymbol{\beta})$ or $B^{\mu'}{}_{\nu}\ (\varphi_{hyperangle} : \hat{\mathbf{n}})$
Translate the system of objects in space. 3 Ex. Move everything 2 meters to the left. $\Delta X^{\mu'} \rightarrow \Delta\mathbf{x}$
Translate the system of objects in time. 1 Ex. Wait 3 minutes and then measure. $\Delta X^{\mu'} \rightarrow \Delta ct$

Passive SR Transformations (the system being measured is not changed, the reference coordinate frame is changed in a certain way):
Rotate the reference coordinate frame. Ex. Rotate your measuring rods about some axis, then do measurement.
Boost the reference coordinate frame. Ex. Have the measuring rods uniformly move on a linear track, then do measurement.
Translate the reference coordinate frame in space. Ex. Move the measuring rods 5 feet south, then do measurement.
Translate the reference coordinate frame in time. Ex. Take pic of object. Wait 7 seconds, then do measurement on pic.

Note: There is a symmetry to the active:passive transforms, whether the measured object or the measuring system gets “changed”.

The (10) one-parameter groups can be expressed directly as exponentials of the generators.
Poincaré Algebra is the Lie Algebra of the Poincaré Group.

$U[I, (a^0,0)] = e\hat{}(ia^0 \cdot H) = e\hat{}(ia^0 \cdot p^0)$: (1) Hamiltonian (Energy) H = Temporal Momentum
$U[I, (0,\lambda\hat{\mathbf{a}})] = e\hat{}(-i\lambda\hat{\mathbf{a}}\cdot\mathbf{p})$: (3) Linear Momentum **p**
$U[\Lambda(i\lambda\hat{\boldsymbol{\theta}}/2), 0] = e\hat{}(i\lambda\hat{\boldsymbol{\theta}}\cdot\mathbf{j})$: (3) Angular Momentum **j** = **l**
$U[\Lambda(\lambda\hat{\boldsymbol{\varphi}}/2), 0] = e\hat{}(i\lambda\hat{\boldsymbol{\varphi}}\cdot\mathbf{k})$: (3) Dynamic Mass Moment **k** = **n**
The Poincaré Algebra is the Lie Algebra of the Poincaré Group:
Total of {1+3+3+3 = (1+3)+(3+3) = 4+6 = 10} Invariances from Poincaré Symmetry
$U(\Lambda^{\mu'}{}_{\nu} : \Delta X^{\mu'}) = e\hat{}[({}^{i}/_{2\hbar})\omega_{\mu\nu}M^{\mu\nu}]\ \ e\hat{}[({}^{i}/_{\hbar})\Delta X_{\mu}P^{\mu}]$

Conservation of 4-LinearMomentum $P^{\mu} = \mathbf{P}$: (1 + 3) = (4) Laws {Linear [→]}
  Conservation of scalar 1-energy E (temporal)
  Conservation of 3-vector 3-momentum **p** (spatial)

Conservation of 4-AngularMomentum $M^{\mu\nu} = \mathbf{X}\hat{}\mathbf{P} = X^{\mu}P^{\nu} - X^{\nu}P^{\mu}$: (3 + 3) = (6) Laws {Angular [↻]}
  Conservation of relativistic 3-mass-moment **n** (temporal-spatial)
  Conservation of angular 3-momentum **l** (spatial-spatial)

==

[→] 4-LinearMomentum 4-Vector : 4 Independent parameters
$P^\mu = (E/c = mc, \mathbf{p}) = \mathbf{P}$ [kg·m/s = N·s]

==

[⥁] 4-AngularMomentum 4-Tensor,$_{\text{Antisymmetric}}$: $M^{\alpha\beta} = -M^{\beta\alpha}$: 6 Independent parameters
$M^{\alpha\beta} = X^\alpha P^\beta - X^\beta P^\alpha = \mathbf{X \wedge P}$ [kg·m²/s = N·m·s = J·s = Action]
→
$[M^{tt}\ M^{tx}\ M^{ty}\ M^{tz}]$
$[M^{xt}\ M^{xx}\ M^{xy}\ M^{xz}]$
$[M^{yt}\ M^{yx}\ M^{yy}\ M^{yz}]$
$[M^{zt}\ M^{zx}\ M^{zy}\ M^{zz}]$
=
$[\ 0\ \ x^0p^1-x^1p^0\ \ x^0p^2-x^2p^0\ \ x^0p^3-x^3p^0]$
$[x^1p^0-x^0p^1\ \ 0\ \ x^1p^2-x^2p^1\ \ x^1p^3-x^3p^1\ ]$
$[x^2p^0-x^0p^2\ \ x^2p^1-x^1p^2\ \ 0\ \ x^2p^3-x^3p^2\ ]$
$[x^3p^0-x^0p^3\ \ x^3p^1-x^1p^3\ \ x^3p^2-x^2p^3\ \ 0\ ]$
=
$[\ 0\ \ ctp^x-xE/c\ \ ctp^y-yE/c\ \ ctp^z-zE/c]$
$[xE/c-ctp^x\ \ 0\ \ xp^y-yp^x\ \ xp^z-zp^x\ ]$
$[yE/c-ctp^y\ \ yp^x-xp^y\ \ 0\ \ yp^z-zp^y\ ]$
$[zE/c-ctp^z\ \ zp^x-xp^z\ \ zp^y-yp^z\ \ 0\ ]$
=
$[\ 0\ \ c(tp^x-xm)\ \ c(tp^y-ym)\ \ c(tp^z-zm)]$
$[c(xm-tp^x)\ \ 0\ \ xp^y-yp^x\ \ xp^z-zp^x\ ]$
$[c(ym-tp^y)\ \ yp^x-xp^y\ \ 0\ \ yp^z-zp^y\ ]$
$[c(zm-tp^z)\ \ zp^x-xp^z\ \ zp^y-yp^z\ \ 0\ ]$
=
$[0\ \ -cn^x\ \ -cn^y\ \ -cn^z]$
$[+cn^x\ \ 0\ \ +l^z\ \ -l^y\ ]$
$[+cn^y\ \ -l^z\ \ 0\ \ +l^x\ ]$
$[+cn^z\ \ +l^y\ \ -l^x\ \ 0\ ]$
=
$[\ 0\ ,\ -cn^{0j}\ ]$
$[\ +cn^{i0},\ \varepsilon^{ij}{}_k l^k\ ]$
=
$[\ 0\ ,\ -c\mathbf{n}\ ]$
$[+c\mathbf{n}^T,\ \mathbf{x \wedge p}\ ]$

==

A particle can be part of a system, with 4-AngularMomentum (6 parameters).
Then, the whole system can be moving uniformly with a 4-Momentum (4 parameters).
Total is (10) Independent parameters.

==

Full Relativistic Fluid Stress-Energy(Density) 4-Tensor,$_{\text{Symmetric}}$: $T^{\alpha\beta} = +T^{\beta\alpha}$: (10) Independent parameters

$T_{RelFluid}{}^{\mu\nu} = (\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu} + (\bar{T}^{\mu}H^{\nu}{}_{\sigma}Q^{\sigma} + Q^{\sigma}H^{\mu}{}_{\sigma}\bar{T}^{\nu})/c + \Pi^{\mu\nu}$ [kg/m·s² = J/m³ = N/m² = Pa]

→

$$\begin{bmatrix} T^{tt} & T^{tx} & T^{ty} & T^{tz} \\ T^{xt} & T^{xx} & T^{xy} & T^{xz} \\ T^{yt} & T^{yx} & T^{yy} & T^{yz} \\ T^{zt} & T^{zx} & T^{zy} & T^{zz} \end{bmatrix}$$

=

$$\begin{bmatrix} \rho_e=\rho_m c^2 & q^{01}/c & q^{02}/c & q^{03}/c \\ q^{10}/c & p + \Pi^{11} & \Pi^{12} & \Pi^{13} \\ q^{20}/c & \Pi^{21} & p + \Pi^{22} & \Pi^{23} \\ q^{30}/c & \Pi^{31} & \Pi^{32} & p + \Pi^{33} \end{bmatrix}$$

=

$$\begin{bmatrix} \rho_e=\rho_m c^2 & q^{x}/c & q^{y}/c & q^{z}/c \\ q^{x}/c & p + \Pi^{xx} & \Pi^{xy} & \Pi^{xz} \\ q^{y}/c & \Pi^{yx} & p + \Pi^{yy} & \Pi^{yz} \\ q^{z}/c & \Pi^{zx} & \Pi^{zy} & p + \Pi^{zz} \end{bmatrix}$$

=

$$\begin{bmatrix} \rho_e=\rho_m c^2, & \mathbf{q}/c \\ \mathbf{q}^T/c, & p\delta^{ij} + \Pi^{ij} \end{bmatrix}$$

==

(temporal:mixed:spatial) splitting

1 Temporal:Temporal EnergyDensity $(\rho_{eo})=V_{\mu\nu}T^{\mu\nu}$

3 Temporal:Spatial HeatEnergy Flux $(Q^{\mu})=c\bar{T}_{\nu}T^{\mu\nu}$

1 Spatial:Spatial Isotropic Pressure $(p_o)=(-1/3)H_{\mu\nu}T^{\mu\nu}$

5 Spatial:Spatial Anisotropic Stress $(\Pi^{\mu\nu})=H^{\mu}{}_{\alpha}H^{\nu}{}_{\beta}T^{\alpha\beta} + (p_o)H^{\mu\nu}$

(10) Total Independent components

Full Relativistic Fluid Stress-Energy(Density) Q^{σ} = 4-HeatFluxVector, $\Pi^{\mu\nu}$ = ViscousShear

$T^{\mu\nu} = (\rho_{eo})V^{\mu\nu} + (-p_o)H^{\mu\nu} + (\bar{T}^{\mu}H^{\nu}{}_{\sigma}Q^{\sigma} + Q^{\sigma}H^{\mu}{}_{\sigma}\bar{T}^{\nu})/c + \Pi^{\mu\nu} \rightarrow [[\rho_e,q^{0j}/c],[q^{i0}/c,p\delta^{ij} + \Pi^{ij}]]_{\{MCRF\}}$

(ρ_{eo}) = (Temporal) EnergyDensity 4-Scalar : 1 independent component

(p_o) = (Spatial) Isotropic Pressure 4-Scalar : 1 independent component

$(\bar{T}^{\mu})$ = UnitTemporal 4-Vector

(Q^{μ}) = HeatEnergyFlux 4-Vector w/ $Q^{\mu}\bar{T}_{\mu} = 0$: 3 independent components aka. MomentumDensity

$(\Pi^{\mu\nu})$ = ViscousShear 4-Tensor w/ $\Pi^{\mu\nu}\bar{T}_{\mu} = 0^{\nu}$: 5 indep. components aka. AnisotropicStress (traceless $Tr[\Pi^{\mu\nu}]=0$ and $\Pi^{\mu\nu}=H^{\mu}{}_{\rho}H^{\nu}{}_{\sigma}\Pi^{\sigma\rho}$)

$(V^{\mu\nu})$ = (Temporal) (V)ertical Projection 4-Tensor

$(H^{\mu\nu})$ = (Spatial) (H)orizontal Projection 4-Tensor

The Full ViscousShear Tensor has 6 total components.

We have done a decomposition into an Isotropic (with trace) and Anisotropic (without trace = traceless = zero trace) part.

The Isotropic part is the Trace of the Tensor, and the Anisotropic part is the (Full – Isotropic) part.

We can do this because the Trace operation is itself a single component (scalar) tensor invariant.

Define Isotropic $I^{\mu\nu} = (\eta^{\mu\nu}/4)\mathrm{Trace}[T^{\mu\nu}]$, with the $\mathrm{Trace}[T^{\mu\nu}] = \eta_{\mu\nu} T^{\mu\nu}$. $\mathrm{Trace}[I^{\mu\nu}] = \eta_{\mu\nu} I^{\mu\nu} = \eta_{\mu\nu} (\eta^{\mu\nu}/4)\mathrm{Trace}[T^{\mu\nu}] = \mathrm{Trace}[T^{\mu\nu}]$

Define Anisotropic $A^{\mu\nu} = T^{\mu\nu} - I^{\mu\nu}$. Since $\mathrm{Trace}[T^{\mu\nu}] = \mathrm{Trace}[I^{\mu\nu}]$, $\mathrm{Trace}[A^{\mu\nu}] = 0$

$T^{\mu\nu} = I^{\mu\nu} + A^{\mu\nu} = I^{\mu\nu} + (T^{\mu\nu} - I^{\mu\nu}) = T^{\mu\nu}$

In this case, the Anisotropic part has 5 components, and the Isotropic part has 1 component

There are also the (10) parameters of the Parameterized Post-Newtonian (PPN) formalism, [from Wikipedia] which is used as a tool to compare Newtonian and Einsteinian gravity in the limit in which the gravitational field is weak and generated by objects moving slowly compared to the speed of light. In general, PPN formalism can be applied to all metric theories of gravitation in which all bodies satisfy the Einstein equivalence principle (EEP). The speed of light remains constant in PPN formalism and it assumes that the metric tensor is always symmetric.

1	γ	How much space curvature g_{ij} is produced by unit rest mass?
2	β	How much non-linearity is there in the superposition law for gravity g_{00}?
3	β_1	How much gravity is produced by unit kinetic energy $(1/2)\rho_0 v^2$?
4	β_2	How much gravity is produced by unit gravitational potential energy ρ_0/U?
5	β_3	How much gravity is produced by unit internal energy $\rho_0\Pi$?
6	β_4	How much gravity is produced by unit pressure p?
7	ζ	Difference between radial and transverse kinetic energy on gravity
8	η	Difference between radial and transverse stress on gravity
9	Δ_1	How much dragging of inertial frames g_{0j} is produced by unit momentum $\rho_0 v$?
10	Δ_2	Difference between radial and transverse momentum on dragging of inertial frames

$g_{\mu\nu}$ is the 4×4 symmetric metric tensor with Greek indices μ & ν going from {0..3}.
An index of 0 will indicate the temporal direction and Latin indices i & j going from {1..3} will indicate spatial directions.

In Einstein's theory, the values of these parameters are chosen (1) to fit Newton's Law of gravity in the limit of velocities and mass approaching zero, (2) to ensure conservation of energy, mass, momentum, and angular momentum, and (3) to make the equations independent of the reference frame.
In this notation, general relativity has PPN parameters $\gamma = \beta = \beta_1 = \beta_2 = \beta_3 = \beta_4 = \Delta_1 = \Delta_2 = 1$ and $\zeta = \eta = 0$.

These requirements are really just restatements of the other **Physics Top Ten** categories.

==============================

Haiku of the Mystic Numbers

Zero, the center,
Symbol of nothing, yet more,
Granter of balance.

One, the first to count,
Unit by which all measure,
It will describe truth.

Two, the yin and yang,
Members of duets and duels,
Computer logic.

Three, supports aligned,
Significant for structure,
Rigid triangle.

Four, the dimension,
Of space and time united,
Relativity.

Five, the sign of Man,
Digits or tally marks grouped,
The rays of a star.

Six, magic value,
Carbon chemistry of life,
First perfect number.

Seven, humans play,
Musical notes, A through G,
Luck in pair o' dice.

Eight, serious math,
Octonions are the last,
to follow nice rules.

Nine, the last digit,
Days and nights on Yggdrasil,
The count of all realms.

Ten, most mystical,
Poincaré, conservation,
Fearful symmetry.

...

Infinity, end,
The universal wholeness,
Uncountable all.

...

i, imaginary,
Complex number discovered,
Elegant technique.

π, the circle's sign,
Perfection equidistant,
Endless decimal.

e, natural growth,
$e^{2 \pi i} = 1$,
Math: Truth is Beauty.

JBW 2003-Apr,2022-Dec

Notation / Conventions / Fundamentals:

Tensor Convention {Temporal, 0^{th} Component, Positive(+), SI} = Metric Signature (+,-,-,-) with [SI Dimensional-Units].
aka. {"Time-Positive", "Particle-Physics", "West-Coast", "Mostly-Minuses"} Metric Sign Convention → The "Metric System" :-)

SR <Time·Space>-splitting Component Coloring Mnemonic: Temporal (blue) + Spatial (red) give Mixed SpaceTime (purple)

4D "Flat" <Time·Space> SR:Minkowski Metric $\eta_{\mu\nu} = \eta^{\mu\nu} \to \text{Diagonal}[+1,-1,-1,-1]_{(\text{Cartesian})}$: Generally, $\{g_{\mu\nu}\}=1/\{g^{\mu\nu}\}$ for non-zero
Mixed 4D (1,1)-Tensor form Minkowski Metric $\eta^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} = \text{Diagonal}[+1,+1,+1,+1]_{(\text{Always})} = I_{[4]} = g^{\mu}{}_{\nu}$ = Kronecker Delta = Identity

4-Position $\mathbf{R} = R^{\mu} = (ct,\mathbf{r}) = \eta^{\mu\nu}R_{\nu}$: 4D Position-OneForm $R_{\mu} = (ct,-\mathbf{r}) = \eta_{\mu\nu}R^{\nu}$ [m]
4-Gradient $\boldsymbol{\partial} = \partial^{\mu} = (\partial_t/c,-\nabla) = (\partial/\partial R_{\mu}) = \eta^{\mu\nu}\partial_{\nu}$: 4D Gradient-OneForm $\partial_{\mu} = (\partial_t/c,\nabla) = (\partial/\partial R^{\mu}) = \eta_{\mu\nu}\partial^{\nu}$ [1/m]

$\partial^{\mu}[R^{\nu}] = \boldsymbol{\partial}[\mathbf{R}] = (\partial_t/c,-\nabla)[(ct,\mathbf{r})] \to (\partial_t/c,-\partial_x,-\partial_y,-\partial_z)[(ct,x,y,z)]$ = Jacobian = $\text{Diagonal}[+1,-1,-1,-1]_{(\text{Cartesian})} = \eta^{\mu\nu}$ = SR:Minkowski Metric
$\partial^{\mu}\,\eta_{\mu\nu}\,R^{\nu} = (\boldsymbol{\partial}\cdot\mathbf{R}) = (\partial_t/c,-\nabla)\cdot(ct,\mathbf{r}) \to (\partial_t/c,-\partial_x,-\partial_y,-\partial_z)\cdot(ct,x,y,z) = (\partial_t ct/c+\partial_x x+\partial_y y+\partial_z z) = 4$ = SR 4D SpaceTime Dimension
$\partial_{\nu}[R^{\mu'}] = \partial R^{\mu'}/\partial R^{\nu} = \Lambda^{\mu'}{}_{\nu}$ = SR Lorentz Transformation: $A^{\mu'} = \Lambda^{\mu'}{}_{\nu}A^{\nu}$ = New.Ref.Frame' = LorentzTransf. contracted w/ Old.Ref.Frame

4D Tensors use Greek indices: ex. { μ, ν, σ, ρ, ...} : ex. 4-Position $R^{\mu} = (r^{\mu}) = (r^0,r^1,r^2,r^3)$, with 4 possible index-values {0,1,2,3}
3D tensors use Latin indices: ex. { i, j, k, ...} : ex. 3-position $r^k = (r^k) = (r^1,r^2,r^3)$, with 3 possible index-values {1,2,3}

4-Vector (4D) $\mathbf{A} = A^{\mu} = (a^{\mu}) = (a^0,\mathbf{a}) = (a^0,a^k) = (a^0,a^1,a^2,a^3) \to (a^t,a^x,a^y,a^z)_{[\text{Cartesian:rectangular}]} \to (a^t,a^r,a^{\theta},a^{\varphi})_{[\text{spherical}]}$ → other coordinate basis
3-vector (3D) $\mathbf{a} = a^k = (a^k) = (\mathbf{a}) = (a^k) = (a^1,a^2,a^3) \to (a^x,a^y,a^z)_{[\text{Cartesian:rectangular}]} \to (a^r,a^{\theta},a^{\varphi})_{[\text{spherical}]}$ → other coordinate basis

4-Scalar	$S = S$	(1) Invariant Lorentz Scalar, same for all frames, $\{s\}$ or $\{s_o\}$	1 {4D (0,0)-Tensor} component
4-Vector	$\mathbf{V} = V^{\mu}$	$(1^{\,0}+3^{\,j})$-splitting into $\{v^t,v^x,v^y,v^z\}$	4 {4D (1,0)-Tensor} components
4-Tensor,$_{\text{AntiSymmetric}}$	$\boldsymbol{T_{asym}} = T_{asym}{}^{\mu\nu}$	$(3^{\,0j}+3^{\,j\neq k})$-splitting into $\{t^{tx},t^{ty},t^{tz},t^{xy},t^{xz},t^{yz}\}$ w/ all $^{j=k}$ comps=0	6 {4D (2,0)-Tensor} components
4-Tensor,$_{\text{Symmetric}}$	$\boldsymbol{T_{sym}} = T_{sym}{}^{\mu\nu}$	$(1^{\,00}+3^{\,0j}+3^{\,j=k}+3^{\,j\neq k})$-splitting into $\{t^{tt},t^{tx},t^{ty},t^{tz},t^{xx},t^{yy},t^{zz},t^{xy},t^{xz},t^{yz}\}$	10 {4D (2,0)-Tensor} components

$S = \{s\}$ = number $(1=4^0)$: $\mathbf{V} = V^{\mu} = (v^0,\mathbf{v}=v^i)$ = vector $(4=4^1)$: $\boldsymbol{T} = \boldsymbol{T_{asym}} + \boldsymbol{T_{sym}} = T^{\mu\nu} = [[t^{00},t^{0k}],[t^{j0},t^{jk}]]$ = matrix $(16=4^2)$: etc.
Technically, these are all 4-Tensors = 4D Tensors; specify precisely using the #D (m,n)-Tensor notation {# dims, m $^{\text{upper}}$, n $_{\text{lower}}$ indices}
All SR 4-Tensors obey $T^{\mu 1'\ldots\mu m'} = \Lambda^{\mu 1'}{}_{\nu 1}\,\Lambda^{\mu 2'}{}_{\nu 2}\ldots\Lambda^{\mu m'}{}_{\nu m}\,T^{\nu 1\ldots\nu m}$: m is the # of indices and a separate Lorentz Transform Λ for each index

<Time·Space> 4-Vector Name matches its spatial 3-vector component name: ex. 4-Position $\mathbf{R}$ = (c*time t,3-position $\mathbf{r}$) [length]→[m]
LightSpeed Factor (c) will be in temporal component as required to make all [dimensional-units] of a 4-Vector's components match

SR 4-Vector V = (**4D SpaceTime 4-Vector**) = (1D temporal 3-scalar, **3D spatial 3-vector**) → 4D (1+3)-splitting into (v^t,v^x,v^y,v^z)

Tensor-index-notation in non-bold: ex. $A^{\mu} = (a^{\mu}) = (a^0,a^j) = (a^0,a^1,a^2,a^3)$: ex. $A^{\mu\nu} = [[a^{\mu\nu}]] = [[a^{00},a^{0k}],[a^{j0},a^{jk}]]$
4-Vectors (4D) in **bold** UPPERCASE: ex. $\mathbf{A} = \overline{\mathbf{A}} = (\mathbf{A}) = (a^0,\mathbf{a}) = (a^0,a^1,a^2,a^3)$
3-vectors (3D) in **bold** lowercase: ex. $\mathbf{a} = \overline{\mathbf{a}} = \vec{\mathbf{a}} = (\mathbf{a}) = (\mathbf{a}) = (a^1,a^2,a^3)$
Temporal scalars (1D) in non-bold, usually lowercase, 0^{th} component: ex. a^0 , a_0 "Count from 1, but index from 0 :-) "
Individual non-grouped components of 4-Tensors in non-bold: ex. $\mathbf{A} = (a^0,a^1,a^2,a^3) = (a^0,\mathbf{a})$ Vectors are grouped structures, thus **bold**
Rest scalars (invariants) in non-bold, denoted with naught ($_o$): ex. m_o : from $\mathbf{P} = m_o\mathbf{U}$ "A rest-frame is a valid relativistic concept"

Upper index 4-Vector $\mathbf{A} = \overline{\mathbf{A}} = A^{\mu} = (a^{\mu}) = (a^0,a^j)$: Lower index 4-CoVector $\underline{\mathbf{B}} = B_{\mu} = (b_{\mu}) = (b_0,b_j)$ a.k.a 4-DualVector=4D-OneForm
Index lowering/raising via Minkowski Metric $\boldsymbol{\eta}$: ex. $R_{\mu} = \eta_{\mu\nu}R^{\nu}$ or $\partial^{\mu} = \eta^{\mu\nu}\partial_{\nu}$ or $U^{\mu} = \eta^{\mu\nu}U_{\nu}$ with 4-Velocity $\mathbf{U} = U^{\mu} = \gamma(c,\mathbf{u}) = \gamma c(1,\boldsymbol{\beta})$

SR Relativistic Gamma $\gamma = 1/\sqrt{[1-\boldsymbol{\beta}\cdot\boldsymbol{\beta}]} = dt/d\tau$: Relativistic $\boldsymbol{\beta} = \mathbf{u}/c = \{0..1\}\hat{\mathbf{n}}$: ProperTimeDerivative $(d/d\tau) = \gamma(d/dt) = (\mathbf{U}\cdot\boldsymbol{\partial})$

4D (1,0)-Tensor = 4-Vector: $\mathbf{A} = \overline{\mathbf{A}} = A^{\mu}$: ex. 4-Momentum $\mathbf{P} = P^{\mu} = (E/c,\mathbf{p}) = (mc,m\mathbf{u}) = m_o\mathbf{U} = m_o\gamma(c,\mathbf{u}) = m(c,\mathbf{u})$
4D (0,1)-Tensor = 4-CoVector = 4D-OneForm: $\underline{\mathbf{A}} = A_{\mu}$: ex. 4D GradientOneForm $\partial_{\mu} = (\partial_t/c,\nabla) = (\partial/\partial R^{\mu})$

"Unit"Temporal 4-Vector $\overline{\mathbf{T}} = \gamma(1,\boldsymbol{\beta})$, with Lorentz Scalar Invariant $\overline{\mathbf{T}}\cdot\overline{\mathbf{T}} = T^{\mu}T_{\mu} = \gamma^2[1^2-\boldsymbol{\beta}\cdot\boldsymbol{\beta}] = +1$ $\overline{\mathbf{T}} = \mathbf{U}/c$
Null 4-Vector $\overline{\mathbf{N}} \sim (\pm|\mathbf{a}|,\mathbf{a}) = a(\pm 1,\hat{\mathbf{n}})$, with Lorentz Scalar Invariant $\overline{\mathbf{N}}\cdot\overline{\mathbf{N}} = N^{\mu}N_{\mu} = a^2[1^2-\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}] = 0$
"Unit"Spatial 4-Vector $\overline{\mathbf{S}} = \gamma_{\beta\hat{n}}(\boldsymbol{\beta}\cdot\hat{\mathbf{n}},\hat{\mathbf{n}})$, with Lorentz Scalar Invariant $\overline{\mathbf{S}}\cdot\overline{\mathbf{S}} = S^{\mu}S_{\mu} = \gamma_{\beta\hat{n}}{}^2[(\boldsymbol{\beta}\cdot\hat{\mathbf{n}})^2-\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}] = -1$ $\overline{\mathbf{T}}\cdot\overline{\mathbf{S}} = (\gamma{*}\gamma_{\beta\hat{n}})[\boldsymbol{\beta}\cdot\hat{\mathbf{n}}-\boldsymbol{\beta}\cdot\hat{\mathbf{n}}] = 0$
$\overline{\mathbf{T}}\cdot\overline{\mathbf{S}} = 0 \leftrightarrow (\overline{\mathbf{T}} \perp_{4D} \overline{\mathbf{S}})$

Time-like separated <Events>	Null-like separated <Events>	Space-like separated <Events>
Invariant Temporal Causality=Time-ordering	Invariant Null LightCone	Invariant Spatial Topology=Space-ordering
Moving Clock = ←|Time Dilation|→	| |Invariant LightSpeed (c)| |	Moving Ruler = →|Length Contraction|←
Relativity of Stationarity = non-Topological	Causal & Topological	*Relativity* of Simultaneity = non-Causal

I had wanted to stop above, at page (10) :) However, I had an epiphany: Let's look at the point particle again.

Relativistic Particle, Tensor Form: 3 SR 4D (1,0)-Tensors = {4-Position, 4-Velocity, 4-Acceleration}

4-Vectors = 4D (1,0)-Tensors:

4-Position $R^\mu = (ct,\mathbf{r}) = \mathbf{R} \in$ <event> [m] $(ct,\mathbf{r}) \to (ct,x,y,z)$ only Lorentz, not Poincaré Invariant

4-Velocity $U^\mu = \gamma(c,\mathbf{u}) = \mathbf{U} = d\mathbf{R}/d\tau$ [m/s] Lorentz Gamma Factor $\gamma = 1/\sqrt{[1-(u/c)^2]} = dt/d\tau$

4-Acceleration $A^\mu = \gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a}) = \mathbf{A} = d\mathbf{U}/d\tau = d^2\mathbf{R}/d\tau^2$ [m/s^2] $A^\mu = (\gamma^4(\mathbf{a}\cdot\mathbf{u})/c,\gamma^4(\mathbf{a}\cdot\mathbf{u})\mathbf{u}/c^2+\gamma^2\mathbf{a}) = \gamma^2(\gamma^2(\mathbf{a}\cdot\mathbf{u})/c,\gamma^2(\mathbf{a}\cdot\mathbf{u})\mathbf{u}/c^2+\mathbf{a})$

Examine the total # of independent components/parameters:
Some are Degrees of Freedom (DoF's): spacetime variables which change along a worldline
Some are Constraints: those parameters which force a particle upon a particular path or worldline

4-Position gives (4) with t:(1) and $\mathbf{r}$:(3)
4-Velocity gives (3) with $\mathbf{u}$:(3), since γ is a function of $\mathbf{u}$
4-Acceleration gives (3) in this context, the only new one is $\mathbf{a}$:(3), γ is a function of $\mathbf{u}$, and $\gamma' = \gamma^3(\mathbf{a}\cdot\mathbf{u})/c^2$ is a function of $\mathbf{a}$ and $\mathbf{u}$
Thus, there are (10) independent parameters $\{t,\mathbf{r},\mathbf{u},\mathbf{a}\}=\{1+3+3+3\}$ which allow the generalized particle dynamics.

Now consider: The symmetric 2-index tensor is responsible for giving covariant (frame-invariant) physics.
There are $(N^2+N)/2$ independent components of a symmetric 2-index tensor for spacetime of dimension N.

If we assume as an axiom that there must be {**position r**, **velocity v**, **acceleration a**} **vectors** + a { time t }scalar to describe the dynamics of a system, for whatever dimension N we might be in, then there are 3(N–1)+1 independent dynamical components.

Solve:
$(N^2+N)/2 = 3(N-1)+1$
$N^2/2+N/2-3N+2=0$
$N^2-5N+4=0$
$(N-4)(N-1)=0 \therefore N = 1 \text{ or } 4$
N = 1 is the case in which there is only time, no space: the void of all mythologies...
N = 4 is our 4D spacetime, with 1D time + 3D space

All other spacetime dimensions N are either (over-determined N=2,3) or (under-determined N>4): See **Ostrogradsky instability**.
ex. N=2:there are 1-position (r^x), 1-velocity (v^x), 1-acceleration (a^x), 1-time t = 4, for $(2^2+2)/2 = 3$ tensor slots
ex. N=3:there are 2-position (r^x,r^y), 2-velocity (v^x,v^y), 2-acceleration (a^x,a^y), 1-time t = 7, for $(3^2+3)/2 = 6$ tensor slots
ex. N=4:there are 3-position (r^x,r^y,r^z), 3-velocity (v^x,v^y,v^z), 3-acceleration (a^x,a^y,a^z), 1-time t = (10), for $(4^2+4)/2 =$ (10) tensor slots
ex. N=5:there are 4-position (r^w,r^x,r^y,r^z), 4-velocity (v^w,v^x,v^y,v^z), 4-acceleration (a^w,a^x,a^y,a^z), 1-time t = 13, for $(5^2+5)/2 = 15$ tensor slots
Observation shows that we inhabit the 4D <Time·Space>. We empirically observe 1 time and 3 space dimensions.

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Likewise there are 10 constants of the motion:
Choose an inertial free-particle frame under which the 3-acceleration $\mathbf{a} = \mathbf{a}_{init} = \mathbf{0}$. This is 3 constraints and 3 constants.
This gives $\gamma' = \gamma^3(\mathbf{a}\cdot\mathbf{u})/c^2 \to 0$, which in turn gives $\mathbf{A} = A^\mu = \gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a}) \to \gamma(c*0,0*\mathbf{u}+\gamma*\mathbf{0}) = \gamma(0,\mathbf{0}) = (0,\mathbf{0}) = \mathbf{A}_{init}$
The 4-Zero $(0,\mathbf{0})$ is still a 4-Zero in any and all inertial reference frames. A Lorentz transform on 4-Zero remains 4-Zero.

$\mathbf{A} = d\mathbf{U}/d\tau : d\mathbf{U} = \mathbf{A}\, d\tau$
$\mathbf{U} = \int \mathbf{A}\, d\tau \to \int (0,\mathbf{0})\, d\tau = (0,\mathbf{0})\tau + \mathbf{U}_{init} = \mathbf{U}_{init}$

The 4-Velocity is thus just a constant for inertial motion, the initial 4-Velocity $\mathbf{U}_{init}$.
We can choose the regular representation of 4-Velocity where the initial 3-velocity $\mathbf{u}_{init}$ is a constant to be determined
4-Velocity $\mathbf{U} = U^\mu = \gamma(c,\mathbf{u}) \to \gamma(c,\mathbf{u}_{init}) = \mathbf{U}_{init}$

$\mathbf{U} = d\mathbf{R}/d\tau : d\mathbf{R} = \mathbf{U}\, d\tau$
$\mathbf{R} = \int \mathbf{U}\, d\tau \to \int \gamma(c,\mathbf{u}_{init})\, d\tau = \int(c,\mathbf{u}_{init})\, \gamma d\tau = \int (c,\mathbf{u}_{init})\, dt = (ct,\mathbf{r} = \mathbf{u}_{init}\, t) + \mathbf{R}_{init} = (ct,\mathbf{r} = \mathbf{u}_{init}\, t) + (ct_{init},\mathbf{r}_{init})$

So, while the physical dynamic equations are general: $\mathbf{A} = d\mathbf{U}/d\tau$ and $\mathbf{U} = d\mathbf{R}/d\tau$,
a free particle still has 7 unknown constants to be found $\mathbf{U}_{init}$:(3) and $\mathbf{R}_{init}$:(1+3=4)
The final eqns of motion for 4D Linear Motion are:
$\mathbf{R} = (ct + ct_{init},\mathbf{r} = \mathbf{u}_{init}t + \mathbf{r}_{init})$: 4 constants t_{init} and $\mathbf{r}_{init}$
$\mathbf{U} = \gamma_{init}(c,\mathbf{u} = \mathbf{u}_{init})$: 3 constants $\mathbf{u}_{init}$
$\mathbf{A} = (0,\mathbf{a} = \mathbf{a}_{init} = \mathbf{0})$: 3 constants $\mathbf{a}_{init}$ [7 DoF's $t,\mathbf{r},\mathbf{u}$] + [3 constraints $\mathbf{a}=\mathbf{0}$] = (10) = [10 constants $t_{init},\mathbf{r}_{init},\mathbf{u}_{init},\mathbf{a}_{init}$]

4D Tensor Considerations:

It is interesting to note that the (10) independent parameters can take several different tensorial forms:

{1} Symmetric 4D (2,0)-Tensor:(10), with symmetry giving the property $[T^{\nu\mu}] = +[T^{\mu\nu}]$
4D Stress-Energy Tensor $T^{\mu\nu}$: has 10 independent parameters $\{\rho_e 1 + p1 + \mathbf{q}3 + \mathbf{\Pi}5\}$ energy-density, pressure, heat, stress
$[\rho_e = \rho_m c^2, \mathbf{q}/c\ ]$
$[\mathbf{q}^T/c,\ p\delta^{ij} + \Pi^{ij}]$ It is useful for describing relativistic fluids

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{1} Anti-Symmetric 4D (2,0)-Tensor:(6) , with anti-symmetry giving the property $[T^{\nu\mu}] = -[T^{\mu\nu}]$
{1} 4-Vector = 4D (1,0)-Tensor:(4)
4-AngularMomentum $M^{\mu\nu}$ [↻]: has 6 independent parameters 3-angular-momentum **l** & 3-mass-moment **n**
4-LinearMomentum P^μ [→]: has 4 independent parameters energy E & 3-momentum **p**
for a total of (10) independent parameters {**l**,**n**,E,**p**}.
There are well known conservation laws associated with these.
They are useful for describing relativistic particles.

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{3} 4-Vectors= 4D (1,0)-Tensors:(4) each, which would normally give (12) independent parameters.
However, there is a universal constant (c) which reduces this to (10).
4-Position R^μ has 4 independent parameters, t & **r**
4-Velocity U^μ has 3 independent parameters **u**, due to the constraint $(\mathbf{U}\cdot\mathbf{U}) = c^2$
4-Acceleration A^μ has 3 or 6 independent parameters, depending on how you examine it.
It is based on 3-velocity **u** and 3-acceleration **a**. Again, there is a constraint $(\mathbf{U}\cdot\mathbf{A}) = 0$
The three 4-Vectors taken as a group, however, have only 10 independent parameters: {t,**r**,**u**,**a**}.
There are 3*4=12 parameters for the 3 4-Vectors, but 2 constraint equations $\{ (\mathbf{U}\cdot\mathbf{U}) = c^2 : (\mathbf{U}\cdot\mathbf{A}) = 0 \}$, leaving just 10 parameters.

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Consider also the following:
What are the minimum number of vectors (in whatever dimension N) required to support life?

If there were only relative positions, then the universe would be totally static and unchanging.
If there were positions and velocities, then there is motion, but no interaction. A gas of non-interacting particles.
However, if there are positions, velocities, and accelerations, then things can speed up, slow down, turn, attract, repel, interact!

Add in a scalar variable called time and you can watch the dance unfold…
===
Is it possible to do something similar with the Anti-Symmetric 2-index tensor?

Consider: The anti-symmetric 2-index tensor is responsible for giving covariant (frame-invariant) spatial rotational physics.
The AngularMomentum 2-index tensor is anti-symmetric.
There are $(N^2–N)/2$ independent components of an anti-symmetric 2-index tensor for spacetime of dimension N.
However, (N–1) of these components are mixed Temporal (the top row of the tensor).
There are only $(N^2–N)/2 – (N–1)$ Spatial-only tensor components (think of these as connections between spatial nodes).
We want these to match the (N–1) possible Spatial dimensions (think of these as spatial nodes).

Solve:
$N(N–1)/2 – (N–1) = (N–1)$
$N(N–1)/2 = 2(N–1)$
$N(N–1) = 4(N–1)$
$(N–4)(N–1) = 0$
Again, we get the cases of: N=1, just time and N=4, the 4D Spacetime that we see!
N = 4 is our 4D spacetime, with 1D time + 3D space

The AngularMomentum [↻] 2-index Tensor has N(N-1)/2 independent components.
The LinearMomentum [→] 1-index Tensor=Vector has N components.
N(N–1)/2 + N = N(N+1)/2 components, which is back to the number in the symmetric 2-index tensor. 6+4=(10) for N=4 in 4D!

Relativistic Motion: {4D General, 3D Circular=4D TimeHelix, 4D Hyperbolic, 4D Linear}

General SR Equations:

$\mathbf{R}^{n\prime} = d\mathbf{R}^{(n-1)\prime}/d\tau = d^{(n-1)}\mathbf{R}/d\tau^{(n-1)}$ $\mathbf{J} = \mathbf{A}' = \mathbf{U}'' = \mathbf{R}'''$ $' = d/d\tau = \gamma d/dt$

$\mathbf{r}^{n\prime} = d\mathbf{r}^{(n-1)\prime}/dt = d^{(n-1)}\mathbf{r}/dt^{(n-1)}$ $\mathbf{j} = \mathbf{a}' = \mathbf{u}'' = \mathbf{r}'''$ $' = d/dt$

$(\mathbf{U}\cdot\mathbf{U}) = c^2$ is temporal, invariant, fundamental constant

$d/d\tau[\mathbf{U}\cdot\mathbf{U}] = d/d\tau[c^2] = 0 = d/d\tau[\mathbf{U}]\cdot\mathbf{U} + \mathbf{U}\cdot d/d\tau[\mathbf{U}] = 2(\mathbf{A}\cdot\mathbf{U}) = 0$

$(\mathbf{A}\cdot\mathbf{U} = 0) \leftrightarrow (\mathbf{A}\perp\mathbf{U})$: 4-Acceleration (normal to worldline) is orthogonal($\perp$) to 4-Velocity (tangent to worldline)

$d/d\tau[\mathbf{A}\cdot\mathbf{U}] = 0 = d/d\tau[\mathbf{A}]\cdot\mathbf{U} + \mathbf{A}\cdot d/d\tau[\mathbf{U}] = \mathbf{J}\cdot\mathbf{U} + \mathbf{A}\cdot\mathbf{A} = \mathbf{J}\cdot\mathbf{U} + -(\alpha)^2$

$(\mathbf{J}\cdot\mathbf{U}) = (\alpha)^2 = -(\mathbf{A}\cdot\mathbf{A})$

For a particle, one can always take $\mathbf{R} \to \mathbf{R} + \mathbf{R}_{init} = (ct,\mathbf{r}) + (ct_{init},\mathbf{r}_{init})$ with $\mathbf{R}_{init}$ = a constant 4-Vector due to Poincaré Invariance

[SR Degrees of Freedom (DoF)] + [# of Constraints] = (10) due to Poincaré Group

General Motion: 10 independent variables = 10 DoF's

4-Position	$\mathbf{R} = R^\mu = (ct,\mathbf{r})$	$= \mathbf{R}$
4-Velocity	$\mathbf{U} = U^\mu = \gamma(c,\mathbf{u})$	$= d\mathbf{R}/d\tau$
4-Acceleration	$\mathbf{A} = A^\mu = \gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a})$	$= d\mathbf{U}/d\tau$
4-Jerk	$\mathbf{J} = J^\mu = \gamma(c(\gamma\gamma')',(\gamma\gamma'\mathbf{u}+\gamma^2\mathbf{a})')$	$= d\mathbf{A}/d\tau$
	$\mathbf{J} = J^\mu = \gamma(c(\gamma'^2+\gamma\gamma''),(\gamma'^2+\gamma\gamma'')\mathbf{u}+\gamma(3\gamma'\mathbf{a}+\gamma\mathbf{j}))$	

All Lorentz Scalar Products are Invariants

$(\mathbf{R}\cdot\mathbf{R}) = (ct)^2 - \mathbf{r}\cdot\mathbf{r} = (ct_o)^2 = (c\tau)^2 = -(\mathbf{r_o}\cdot\mathbf{r_o})$:either(±),variable

$(\mathbf{U}\cdot\mathbf{U}) = (c)^2$:temporal(+), fundamental constant

$(\mathbf{A}\cdot\mathbf{A}) = -(a_o)^2 = -(\alpha)^2 = (i\alpha)^2$:spatial(-), variable

$(\mathbf{J}\cdot\mathbf{J}) = (c\gamma_o'')^2 - (j_o)^2$:either(±),variable

There are 10 DoF's: {3 [**acceleration**] & 3 [**velocity**] & 4 [**4-Position**]} matches 10 Poincaré symmetries:conservation laws
$(V^{\mu'} = \Lambda^{\mu'}{}_\nu V^\nu + \Delta V[\Delta X^{\mu'}])$ 10 = 6 Lorentz + 4 Translations

General Motion: (alt form **A**,**J**) using $\gamma' = \gamma^3\boldsymbol{\beta}'\cdot\boldsymbol{\beta} = \gamma^3(\mathbf{a}\cdot\mathbf{u})/c^2$

4-Acceleration $\mathbf{A} = A^\mu = (\gamma^4(\mathbf{a}\cdot\mathbf{u})/c,\gamma^4(\mathbf{a}\cdot\mathbf{u})\mathbf{u}/c^2+\gamma^2\mathbf{a})$

4-Jerk $\mathbf{J} = J^\mu = \gamma(c(\gamma^6(\mathbf{a}\cdot\mathbf{u})^2/c^4 + \gamma^4[3\gamma^2(\mathbf{a}\cdot\mathbf{u})^2+(\mathbf{a}\cdot\mathbf{u})']/c^2),(\gamma^6(\mathbf{a}\cdot\mathbf{u})^2/c^4 + \gamma^4[3\gamma^2(\mathbf{a}\cdot\mathbf{u})^2+(\mathbf{a}\cdot\mathbf{u})']/c^2)\mathbf{u}+\gamma(3\gamma^3(\mathbf{a}\cdot\mathbf{u})\mathbf{a}/c^2+\gamma\mathbf{j}))$

Imposed condition: Motion w/ Spatial Orthogonality $(\mathbf{a}\cdot\mathbf{u} = 0) \leftrightarrow (\mathbf{a}\perp\mathbf{u})$

4-Acceleration $\mathbf{A} = A^\mu = \gamma^2(0,\mathbf{a})_\perp$ if $(\mathbf{a}\cdot\mathbf{u})=0$ This also gives $\gamma' = \gamma^3(\mathbf{a}\cdot\mathbf{u})/c^2 \to 0$ which gives γ = constant

4-Jerk $\mathbf{J} = J^\mu = \gamma^3(0,\mathbf{j})_\perp$ if $(\mathbf{a}\cdot\mathbf{u})=0$

Circular Motion ⦚ : constants{ $|\mathbf{r}|$, $|\mathbf{u}|$, $|\mathbf{a}|$, $|\mathbf{j}|$ } ↔ { R , Ω , γ } w/ R=Radius, Ω=AngularFrequency, γ=Relativistic Gamma Factor = Constant 3-vector-magnitudes Motion, but known as constant 3-acceleration-magnitude $|\mathbf{a}|$ = 3D Circular = 4D SR TimeHelix

4-Position $\mathbf{R} = R^\mu = (ct=c\gamma\tau,\mathbf{r} = R\hat{\mathbf{r}} = R(\cos[\Omega t+\theta_o]\hat{\mathbf{n}}_1+\sin[\Omega t+\theta_o]\hat{\mathbf{n}}_2)) = \mathbf{R} = d^0\mathbf{R}/d\tau^0$ $\mathbf{R}\cdot\mathbf{R} = (ct^2) - \mathbf{r}\cdot\mathbf{r} = (ct^2)-R^2$

4-Velocity $\mathbf{U} = U^\mu = \gamma^1(c, \mathbf{u} = R\Omega\hat{\boldsymbol{\theta}} = R\Omega(-\sin[\Omega t+\theta_o]\hat{\mathbf{n}}_1+\cos[\Omega t+\theta_o]\hat{\mathbf{n}}_2)) = d\mathbf{R}/d\tau = d^1\mathbf{R}/d\tau^1$ $\mathbf{U}\cdot\mathbf{U} = \gamma^2(c^2 - \mathbf{u}\cdot\mathbf{u}) = (c)^2$

4-Acceleration $\mathbf{A} = A^\mu = \gamma^2(0, \mathbf{a} = -R\Omega^2\hat{\mathbf{r}} = R\Omega^2(-\cos[\Omega t+\theta_o]\hat{\mathbf{n}}_1-\sin[\Omega t+\theta_o]\hat{\mathbf{n}}_2)) = d\mathbf{U}/d\tau = d^2\mathbf{R}/d\tau^2$ $\mathbf{A}\cdot\mathbf{A} = \gamma^4(0^2 - \mathbf{a}\cdot\mathbf{a}) = -\gamma^4 a^2 = -(\alpha)^2$

4-Jerk $\mathbf{J} = J^\mu = \gamma^3(0, \mathbf{j} = -R\Omega^3\hat{\boldsymbol{\theta}} = R\Omega^3(\sin[\Omega t+\theta_o]\hat{\mathbf{n}}_1-\cos[\Omega t+\theta_o]\hat{\mathbf{n}}_2)) = d\mathbf{A}/d\tau = d^3\mathbf{R}/d\tau^3$ $\mathbf{J}\cdot\mathbf{J} = \gamma^6(0^2 - \mathbf{j}\cdot\mathbf{j}) = (c\gamma_o'')^2-(j_o)^2$

○ ↑ ⦚

Circular Motion follows path of ongoing Lorentz Transform $\boldsymbol{\Lambda}\to\mathbf{R}$: (R)otation = Spatial Path+Const Temporal Motion = SR Helix

{ $|\mathbf{r}|$=R , $|\mathbf{u}|$=RΩ , $|\mathbf{a}|$=RΩ^2 , $|\mathbf{j}|$=RΩ^3 } are constants, $(\mathbf{a}\cdot\mathbf{u}) = 0$, $\gamma' = 0$, $\mathbf{a} = (-\Omega^2)\mathbf{r}$, $\mathbf{j} = (-\Omega^2)\mathbf{u}$, $d/d\tau = \gamma d/dt$, $\hat{\mathbf{n}}_1\cdot\hat{\mathbf{n}}_2 = 0$

There are 9 of 10 DoF's: ={1 [initial angle] & 1 [radius] & 3 [**acceleration**] & 4 [**4-Position=location in SpaceTime**]}
={ 1 θ_o , 1 R , 1 Ω , 2 $\hat{\mathbf{n}}_3 = \hat{\mathbf{n}}_1\times\hat{\mathbf{n}}_2$, 4 $(ct_{init},\mathbf{r}_{init})$}

The 1 constraint is: $\mathbf{a} = -(k^2)\mathbf{r} = \mathbf{r}'' = \ddot{\mathbf{r}}$, which gives { $\mathbf{r}$, $\mathbf{u}$, $\mathbf{a}$, $\mathbf{j}$ } ~ ($C^i * \cos[kt] + S^i * \sin[kt]$) for each 3-vector

Boundary Conditions give k=Ω , $\mathbf{a} = -(\Omega^2)\mathbf{r}$, $\mathbf{j} = -(\Omega^2)\mathbf{u}$, $\mathbf{r} = R\hat{\mathbf{r}}$, $\mathbf{u} = R\Omega\hat{\boldsymbol{\theta}}$, $\mathbf{a} = -R\Omega^2\hat{\mathbf{r}}$, $(\mathbf{a}\cdot\mathbf{u}) = 0$, the (cos : sin) mathematics

$|\mathbf{a}| = |\mathbf{u}|^2/|\mathbf{r}| = (R\Omega^2) = (R\Omega)^2/(R)$ or $-\mathbf{a}\cdot\mathbf{r} = (R\Omega)^2 = \mathbf{u}\cdot\mathbf{u}$ [9 DoF's] + [1 constraint] = (10)

Hyperbolic Motion)×(: constants { $|\mathbf{R}|$, $|\mathbf{U}|$, $|\mathbf{A}|$, $|\mathbf{J}|$ } ↔ { D , c , α } w/ D=c^2/α=Rindler "Distance", c=LightSpeed, α=ProperAccel = Constant 4-Vector-Magnitudes Motion, but known as constant 4-Acceleration-magnitude $|\mathbf{A}|$ = 4D Hyperbolic

4-Position $\mathbf{R} = R^\mu = (c^2/\alpha)(\sinh[\alpha\tau/c+\xi_o],\cosh[\alpha\tau/c+\xi_o]\ \hat{\mathbf{n}}) = \mathbf{R} = d^0\mathbf{R}/d\tau^0$ $\mathbf{R}\cdot\mathbf{R} = (c^2/\alpha)^2(\sinh^2 - \cosh^2\ \hat{\mathbf{n}}\cdot\hat{\mathbf{n}}) = -(c^2/\alpha)^2 = -D^2$

4-Velocity $\mathbf{U} = U^\mu = (c)(\cosh[\alpha\tau/c+\xi_o],\sinh[\alpha\tau/c+\xi_o]\ \hat{\mathbf{n}}) = d\mathbf{R}/d\tau = d^1\mathbf{R}/d\tau^1$ $\mathbf{U}\cdot\mathbf{U} = (c)^2(\cosh^2 - \sinh^2\ \hat{\mathbf{n}}\cdot\hat{\mathbf{n}}) = (c)^2$

4-Acceleration $\mathbf{A} = A^\mu = (\alpha)(\sinh[\alpha\tau/c+\xi_o],\cosh[\alpha\tau/c+\xi_o]\ \hat{\mathbf{n}}) = d\mathbf{U}/d\tau = d^2\mathbf{R}/d\tau^2$ $\mathbf{A}\cdot\mathbf{A} = (\alpha)^2(\sinh^2 - \cosh^2\ \hat{\mathbf{n}}\cdot\hat{\mathbf{n}}) = -(\alpha)^2$

4-Jerk $\mathbf{J} = J^\mu = (\alpha^2/c)(\cosh[\alpha\tau/c+\xi_o],\sinh[\alpha\tau/c+\xi_o]\ \hat{\mathbf{n}}) = d\mathbf{A}/d\tau = d^3\mathbf{R}/d\tau^3$ $\mathbf{J}\cdot\mathbf{J} = (\alpha^2/c)^2(\cosh^2 - \sinh^2\ \hat{\mathbf{n}}\cdot\hat{\mathbf{n}}) = (\alpha^2/c)^2$

↑ ↔)×(

Hyperbolic Motion follows path of ongoing Lorentz Transform $\boldsymbol{\Lambda}\to\mathbf{B}$: (B)oost = Time·Space Path = SR Hyperbolic

{ $|\mathbf{R}|$=D=c^2/α , $|\mathbf{U}|$=c , $|\mathbf{A}|$=α , $|\mathbf{J}|=(\alpha^2/c)^2$ } are constants, $(\mathbf{A}\cdot\mathbf{U})=0$, $\mathbf{A}=(\alpha^2/c^2)\mathbf{R}$, $\mathbf{J}=(\alpha^2/c^2)\mathbf{U}$, γ=cosh : γβ=sinh, τ'=dτ/dt=1/γ : τ=t/γ

There are 8 of 10 DoF's: ={1 [initial hyperangle] & 3 [**proper acceleration**] & 4 [**4-Position=location in SpaceTime**]}
={ 1 ξ_o , 1 α , 2 $\hat{\mathbf{n}}$, 4 $(ct_{init},\mathbf{r}_{init})$}

The constraint eqn. is: $\mathbf{A}=(k^2)\mathbf{R} = \mathbf{R}'' = d^2\mathbf{R}/d\tau^2$, which gives { $\mathbf{R}$, $\mathbf{U}$, $\mathbf{A}$, $\mathbf{J}$ } ~ ($C^\mu * \cosh[k\tau] + S^\mu * \sinh[kt]$) for each 4-Vector

Boundary Conditions give k=(α/c) , $\mathbf{A}=(\alpha^2/c^2)\mathbf{R}$, $\mathbf{J}=(\alpha^2/c^2)\mathbf{U}$, $\mathbf{U}=(c)(\cosh[\alpha\tau/c],\sinh[\alpha\tau/c]\ \hat{\mathbf{n}})$, the (cosh : sinh) mathematics

The 2 constraints: $\mathbf{A}=(\alpha^2/c^2)\mathbf{R}$ splits into (temporal $a^0=(\alpha^2/c^2)r^0$: spatial $\mathbf{a}=(\alpha^2/c^2)\mathbf{r}$)

$|\mathbf{A}| = |\mathbf{U}|^2/|\mathbf{R}| = (\alpha) = (c)^2/(c^2/\alpha) = (c)^2/(D)$ or $-\mathbf{A}\cdot\mathbf{R} = (c)^2 = \mathbf{U}\cdot\mathbf{U}$ [8 DoF's] + [2 constraints] = (10)

Relativistic Motion: {4D General, 3D Circular=4D TimeHelix, 4D Hyperbolic, 4D Linear}

SR 4D Linear Motion ×↗ : constants { $\mathbf{a} = \mathbf{0}$ } with $\mathbf{a}$=3-acceleration
= No Forces Minkowski Metric Motion = 4D Linear

4-Position	$\mathbf{R} = R^\mu = (ct + ct_{init}, \mathbf{r} = \mathbf{u}_{init} t + \mathbf{r}_{init})$	$= \mathbf{R} = d^0\mathbf{R}/d\tau^0$	$\mathbf{R}\cdot\mathbf{R} = (ct^2) - \mathbf{r}\cdot\mathbf{r} = (c\tau)^2$
4-Velocity	$\mathbf{U} = U^\mu = \gamma(c, \mathbf{u} = \mathbf{u}_{init})$	$= d\mathbf{R}/d\tau = d^1\mathbf{R}/d\tau^1$	$\mathbf{U}\cdot\mathbf{U} = \gamma^2(c^2 - \mathbf{u}\cdot\mathbf{u}) = (c)^2$
4-Acceleration	$\mathbf{A} = A^\mu = (0, \mathbf{a} = \mathbf{a}_{init} = \mathbf{0})$	$= d\mathbf{U}/d\tau = d^2\mathbf{R}/d\tau^2$	$\mathbf{A}\cdot\mathbf{A} = (0^2 - \mathbf{a}\cdot\mathbf{a}) = -(\alpha)^2 = 0$
4-Jerk	$\mathbf{J} = J^\mu = (0, \mathbf{j} = \mathbf{0})$	$= d\mathbf{A}/d\tau = d^3\mathbf{R}/d\tau^3$	$\mathbf{J}\cdot\mathbf{J} = (0^2 - \mathbf{j}\cdot\mathbf{j}) = (c\gamma_o'')^2 - (j_o)^2 = 0$
			↑ ↔ ×↗

Linear Motion follows path of ongoing Lorentz Transform $\mathbf{\Lambda} \to \mathbf{I}_{[4]}$: (I)dentity = Time·Space Path = SR Linear
$(\mathbf{a}\cdot\mathbf{u}) = 0$, $\gamma' = 0$, $\gamma'' = 0$, γ = constant, $\mathbf{j} = \mathbf{0}$, $\mathbf{R} = \int \mathbf{U} d\tau \to \mathbf{U}\tau$ for no acceleration = inertial motion
There are 7 of 10 DoF's: ={3 **[initial velocity]** & 4 **[initial 4-Position=location in SpaceTime]**}
={ 3 $\mathbf{u}_{init}$, 4 $(ct_{init}, \mathbf{r}_{init})$}
The 3 constraints are: $\mathbf{a} = \mathbf{0}$, which splits into 3 separate components: $a^x = 0$, $a^y = 0$, $a^z = 0$
[7 DoF's] + [3 constraints] = (10)

To Reiterate, for 4D SR Motion [# DoF's] + [# Constraints] = 10 due to Poincaré Group = 10 due to Symmetric 4D (2,0)-Tensor:

General:	[10 DoF's] + [0 Constraints] = (10)	$\mathbf{A}$ = unconstrained
Circular=4DHelix: ⦚	[9 DoF's] + [1 Constraint] = (10)	$\mathbf{A} = \gamma^2(0, \mathbf{a} = -R\Omega^2\hat{\mathbf{r}}) \leftrightarrow \mathbf{a} = -(\Omega^2)\mathbf{r}$
Hyperbolic:)×(	[8 DoF's] + [2 Constraints] = (10)	$\mathbf{A} = (\alpha^2/c^2)\mathbf{R}$ splits into (temporal $a^0 = (\alpha^2/c^2)r^0$: spatial $\mathbf{a} = (\alpha^2/c^2)\mathbf{r}$
Linear: ×↗	[7 DoF's] + [3 Constraints] = (10)	$\mathbf{A} = (0, \mathbf{0}) \leftrightarrow \mathbf{a} = \mathbf{0}$, which splits into $a^x = 0$, $a^y = 0$, $a^z = 0$

The N*(N-1)/2 situation is the Anti-Symmetric 2-index tensor + an N dimensional vector. The AngularMomentum 2-index Tensor gives rotational motion, and the associated 1-index Tensor = vector gives linear motion. So, the combo of the two gives N*(N-1)/2 + N = N*(N+1)/2, again matching the number of independent components of the Symmetric 2-index tensor.

Examine the various dimension arguments.:

If N=1, (time on a 0d-point), there is only time again:
[0]

If N=2, (time on a 1d-line), there is no possibility of spatial-only rotation, only a mass moment:
[0 n^{tx}]
[-n^{tx} 0]

If N=3, (time on a 2d-plane), the rotation is about a direction that doesn't have a dimension:
[0 n^{tx} n^{ty}]
[-n^{tx} 0 l^{xy}]
[-n^{ty} -l^{xy} 0]
The 1 spatial-only tensor doesn't fit a 2d-spatial vector.

If N=4, (time on a 3d-volume), the rotation is possible in all the possible spatial dimensions:
[0 n^{tx} n^{ty} n^{tz}]
[-n^{tx} 0 l^{xy} l^{xz}]
[-n^{ty} -l^{xy} 0 l^{yz}]
[-n^{tz} -l^{xz} -l^{yz} 0]
with l^{xy} = rotate_about_z, l^{xz} = rotate_about_y, l^{yz} = rotate_about_z. It has # of nodes = # of connections.

If N=5, (time on a 4d-hypervolume), then you have more tensor slots than rotational dimensions:
[0 n^{tx} n^{ty} n^{tz} n^{tw}]
[-n^{tx} 0 l^{xy} l^{xz} l^{xw}]
[-n^{ty} -l^{xy} 0 l^{yz} l^{yw}]
[-n^{tz} -l^{xz} -l^{yz} 0 l^{zw}]
[-n^{tw} -l^{xw} -l^{yw} -l^{zw} 0]
In other words, does l^{xy} = rotate_about_z, or rotate_about_w, or something else. Also, there is no way to alot the spatial-spatial tensor components isotropically to 4d-spatial vectors. In any case, it gives something that we don't observe.

If N=6, then you have more tensor slots than rotational dimensions:
[0 n^{tx} n^{ty} n^{tz} n^{tw} n^{tv}]
[-n^{tx} 0 l^{xy} l^{xz} l^{xw} l^{xv}]
[-n^{ty} -l^{xy} 0 l^{yz} l^{yw} l^{yv}]
[-n^{tz} -l^{xz} -l^{yz} 0 l^{zw} l^{zv}]
[-n^{tw} -l^{xw} -l^{yw} -l^{zw} 0 l^{wv}]
[-n^{tv} - l^{xv} - l^{yv} - l^{zv} -l^{wv} 0]
While you can divide the remaining purely spatial components into two 5d-spatial vectors, there is no symmetric way to do it.

Another way to consider the tensor-vector arguments:

For any spacetime 2-index tensor of dimension N, we have (1) temporal and (N-1) spatial dimensions in an N×N matrix.

$[T^{tt},T^{tr}]$
$[T^{rt},T^{rr}]$

T^{tt} is a 1-dimensional purely-temporal scalar
T^{tr} is a (N-1) dimensional mixed-timespace vector, with each **r** component paired with t
T^{rt} is a (N-1) dimensional mixed-timespace vector, with each **r** component paired with t
T^{rr} is a (N-1) dimensional purely-spatial tensor in an (N-1)×(N-1) matrix

Based on homogeneity and isotropic arguments, we want to divide up the purely-spatial tensor T^{rr} into an integer number of (N-1) dimensional pairings.
We can pair each spatial component with the time component. ex. the top mixed row $[T^{tx},T^{ty},T^{tz},...]$
We can pair each spatial component with itself. ex. the spatial diagonal $[T^{xx},T^{yy},T^{zz},...]$
These will both always have the same number (N-1) of components.

$[T^{tt}\ T^{tx}\ T^{ty}\ T^{tz}]$
$[T^{xt}\ T^{xx}\ T^{xy}\ T^{xz}]$
$[T^{yt}\ T^{yx}\ T^{yy}\ T^{yz}]$
$[T^{zt}\ T^{zx}\ T^{zy}\ T^{zz}]$

There needs to be a symmetric way of dividing the remaining spatial components into (N-1) dimensional vectors.
The only symmetric choice is the N=4 choice of $[T^{xy},T^{xz},T^{yz}]$, which has all the remaining (N-1) components.

Any other choice will either not have all (N-1) dimensional spatial vectors, or will have to make a non-symmetric choice in the spatial components of the remaining (N-1) vectors.

4-Position	$R^{\mu} = (ct,\mathbf{r}) = \mathbf{R} \in$ <event>	[m]	$(ct,\mathbf{r}) \rightarrow (ct,x,y,z)$ only Lorentz, not Poincaré Invariant
4-Velocity	$U^{\mu} = \gamma(c,\mathbf{u}) = \mathbf{U} = d\mathbf{R}/d\tau$	[m/s]	Lorentz Gamma Factor $\gamma = 1/\sqrt{[1-(u/c)^2]} = dt/d\tau$
4-Acceleration	$A^{\mu} = \gamma(c\gamma',\gamma'\mathbf{u}+\gamma\mathbf{a}) = \mathbf{A} = d\mathbf{U}/d\tau = d^2\mathbf{R}/d\tau^2$	[m/s^2]	$A^{\mu} = (\gamma^4(\mathbf{a}\cdot\mathbf{u})/c,\gamma^4(\mathbf{a}\cdot\mathbf{u})\mathbf{u}/c^2+\gamma^2\mathbf{a}) = \gamma^2(\gamma^2(\mathbf{a}\cdot\mathbf{u})/c,\gamma^2(\mathbf{a}\cdot\mathbf{u})\mathbf{u}/c^2+\mathbf{a})$

$\gamma = 1/\sqrt{[1-\beta^2]}$
$\gamma^2 = 1/(1-\beta^2)$
$\gamma^2(1-\beta^2) = 1$
$\gamma^2 - \gamma^2\beta^2 = 1$

$\mathbf{U}\cdot\mathbf{U} = \gamma(c,\mathbf{u})\cdot\gamma(c,\mathbf{u}) = \gamma^2(c^2 - \mathbf{u}\cdot\mathbf{u}) = \gamma^2c^2(1-\mathbf{u}\cdot\mathbf{u}/c^2) = c^2$

$\mathbf{U}\cdot\mathbf{A} = \gamma(c,\mathbf{u})\cdot\gamma^2(\gamma^2(\mathbf{a}\cdot\mathbf{u})/c,\gamma^2(\mathbf{a}\cdot\mathbf{u})\mathbf{u}/c^2+\mathbf{a}) = \gamma^3(\gamma^2(\mathbf{a}\cdot\mathbf{u})-\gamma^2(\mathbf{a}\cdot\mathbf{u})\mathbf{u}\cdot\mathbf{u}/c^2-\mathbf{a}\cdot\mathbf{u}) = (\mathbf{a}\cdot\mathbf{u})\gamma^3(\gamma^2-\gamma^2\mathbf{u}\cdot\mathbf{u}/c^2-1) = (\mathbf{a}\cdot\mathbf{u})\gamma^3(\gamma^2-\gamma^2\beta^2-1) = (\mathbf{a}\cdot\mathbf{u})\gamma^3(1-1) = 0$

More easily, $d(\mathbf{U}\cdot\mathbf{U}) = 2(\mathbf{U}\cdot\mathbf{A}) = d(c^2) = 0$, so $\mathbf{U}\cdot\mathbf{A} = 0$

Why only 3 orders of derivatives in the SpaceTime dimension derivation? See also **Ostrogradsky instability**.
position = $\mathbf{x} = d^0\mathbf{x}/dt^0$
velocity = $\mathbf{v} = d^1\mathbf{x}/dt^1$
acceleration $\mathbf{a} = d^2\mathbf{x}/dt^2$
Well, the process is about getting the number of independent parameters. Certainly, we couldn't do physics without positions or velocities, nor without time itself. Adding in the one more level gives accelerations, which allows:
{ interactions, speeding up, slowing down, attraction, repulsion, forces, potentials }

That empirically sounds like the universe that we observe. That this gives an answer where # of dimensions N = 1 or 4 is quite interesting to me. Remember, the argument doesn't specify the # of dimensions at the start. N = 1 or 4 is the result. In other words, not only is the universe 4D, but it has to be either 1D or 4D based on the *result*.

Even more interesting is when you then apply my good ole 4-vectors.
4-Position = $\mathbf{X} = d^0\mathbf{X}/d\tau^0$
4-Velocity = $\mathbf{U} = d^1\mathbf{X}/d\tau^1$
4-Acceleration $\mathbf{A} = d^2\mathbf{X}/d\tau^2$

Now time and space dimensions are explicitly captured by the 4-Vectors. Time becomes just another component of the 4-Position. Each 4-Vector has, on first glance, 4 independent components. So, 3 SR Vectors*4 components each =12. So, it would appear that we have 2 extra parameters not apparent in the classical vectors + time derivation.

However, SR to the rescue!

There are actually two constraint equations built-in to SR to bring the total back down to 10 independent parameters.
$\mathbf{U}\cdot\mathbf{U} = c^2$
$\mathbf{U}\cdot\mathbf{A} = 0$

I think that is a beautiful result! Not only is the time (t) incorporated into the 4-Position, but we also get a fundamental constant LightSpeed (c) out of the deal, and that 4-Velocity **U** is temporal, 4-Acceleration **A** is spatial, describing worldlines.

4-Velocity **U** is tangential to the worldline at every point and 4-Acceleration **A** is normal to the worldline at every point.
That the 4-Vector version, remembering that they are tensorial, also works with the symmetric 2-index tensor, GR's stress-energy, places the arguments on a much more rigorous level.

Now, are there situations in which the higher order derivatives become important? Perhaps... But see the **Ostrogradsky instability**.

I would bet you money that they only become important for cases in which one or more of the independent parameters is transferred from our usual 3 vectors in such a way that you still only get 10 total independent parameters.

Even in the case of the **Norton Dome**. The non-zero snap parameter is a property of the curve shape. This translates into a particular time-varying acceleration parameter in the outwards-from-axis **x**-direction. The downward **z**-acceleration is still just gravity, and the **y**-direction is just acceleration = zero. So, you still have only at most 10 independent parameters describing the motion.

Physics of Independent Parameters:
===========================
There are (10) independent Conservation Laws composed of LinearMomentum [→] P_{linear}^{μ} and AngularMomentum [↻] $M^{\mu\nu}$ parts.

4-LinearMomentum $P_{linear}^{\mu} = (E_{linear}/c, \mathbf{p}_{linear})$ has (4) independent linear parameters.

4-Position $R^{\mu} = (ct, \mathbf{r})$ has (4) independent parameters.
4-RotationalMomentum $P_{rotational}^{\mu} = (E_{rotational}/c, \mathbf{p}_{rotational})$ has (4) independent parameters. (not the 4-AngularMomentum 4-Tensor)
This would normally give a total of (8) independent angular parameters…

However, there are (2) tensor invariants for the combination:
4-AngularMomentum $M^{\mu\nu} = R^{\mu} \wedge P_{rotational}^{\nu} = R^{\mu} P_{rotational}^{\nu} - R^{\nu} P_{rotational}^{\mu}$
=
$[M^{tt}\ M^{tx}\ M^{ty}\ M^{tz}]$
$[M^{xt}\ M^{xx}\ M^{xy}\ M^{xz}]$
$[M^{yt}\ M^{yx}\ M^{yy}\ M^{yz}]$
$[M^{zt}\ M^{zx}\ M^{zy}\ M^{zz}]$
=
$[\ 0\ \ x^0p^1-x^1p^0\ \ x^0p^2-x^2p^0\ \ x^0p^3-x^3p^0]$
$[x^1p^0-x^0p^1\ \ 0\ \ x^1p^2-x^2p^1\ \ x^1p^3-x^3p^1\ ]$
$[x^2p^0-x^0p^2\ \ x^2p^1-x^1p^2\ \ 0\ \ x^2p^3-x^3p^2\ ]$
$[x^3p^0-x^0p^3\ \ x^3p^1-x^1p^3\ \ x^3p^2-x^2p^3\ \ 0\ ]$
=
This leaves (6) total independent angular parameters.

Thus, [→] [↻]
Conservation of 4-LinearMomentum P_{linear}^{μ} (4) + Conservation of 4-AngularMomentum $M^{\mu\nu} = R^{\mu} \wedge P_{rotational}^{\nu}$ (6) = (10) independent.

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Particle Dynamics using 4-Vectors = 4D (1,0)-Tensors:
4-Position $R^{\mu} = (ct, \mathbf{r})$ has (4) independent parameters.
4-Velocity $U^{\mu} = \gamma(c, \mathbf{u})$ has (3) independent parameters, due to invariant $U^{\mu} \cdot U^{\nu} = c^2$
4-Acceleration $A^{\mu} = \gamma(c\gamma', \gamma'\mathbf{u}+\gamma\mathbf{a})$ has (3) independent parameters, due to invariant $U^{\mu} \cdot A^{\nu} = 0$
Thus,
(4) + (3) + (3) = (10) independent parameters.

Can we link these to the conservation laws for a particle?
We can once again decompose into linear [→] and angular [↻] parts.
4-Position R^{μ} (4) → directly to 4-Position R^{μ} (4)
4-Velocity U^{μ} (3) + RestMass m_o (1) → to 4-LinearMomentum $P_{linear}^{\mu} = m_o U_{linear}^{\mu}$ (4)
4-Acceleration A^{μ} has (3) independent parameters, due to invariant $U_{rotational}^{\mu} \cdot A_{rotational}^{\nu} = 0$
So, we have (4)+(4)+(3) = (11)
However, apply the constraint $\{-R^{\mu} \cdot A^{\nu} = U_{rotational}^{\mu} \cdot U_{rotational}^{\nu} = \text{constant}\}$ for hyperbolic motion, or $-r^j \cdot a^k = u^j \cdot u^k$ for circular motion
Note the familiar rotational acceleration $|\mathbf{a}|=|\mathbf{u}^2|/|\mathbf{r}|$
1) Take the RestMass m_o used in the linear part to give hyperbolic $-(m_o)^2 R^{\mu} \cdot A^{\nu} = (m_o)U_{rotational}^{\mu} \cdot (m_o)U_{rotational}^{\nu} = P_{rotational}^{\mu} \cdot P_{rotational}^{\nu}$
or
2) Take the RestMass m_o used in the linear part to give circular $-(m_o)^2 r^j \cdot a^k = (m_o)u_{rotational}^{j} \cdot (m_o)u_{rotational}^{k} = p_{rotational}^{j} \cdot p_{rotational}^{k}$
The constraint either way lowers the system back down to (10) independent parameters.

So,
$R^{\mu} = R^{\mu}$: (4)
$P_{linear}^{\mu} = m_o U_{linear}^{\mu}$: (4) because (1) RestMass + (3) 4-Velocity
$P_{rotational}^{\mu} \cdot P_{rotational}^{\nu} = -(m_o)^2 R^{\mu} \cdot A^{\nu}$: (2) =because (3) 4-Acceleration – (1) Hyperbolic/Circular Acceleration Constraint
(4) + (4) + (2) = (10) independent parameters.

Note the $R^{\mu} \wedge P_{rotational}^{\nu}$ has (4) + (2) = (6) for angular [↻]
and P_{linear}^{μ} has (4) for linear [→]
(6) + (4) = (10) total independent parameters.

==

$T^{\mu\nu}$ =
$[T^{00}\ T^{01}\ T^{02}\ T^{03}]$
$[T^{10}\ T^{11}\ T^{12}\ T^{13}]$
$[T^{20}\ T^{21}\ T^{22}\ T^{23}]$
$[T^{30}\ T^{31}\ T^{32}\ T^{33}]$

$A^{\mu\nu}$ =
$[\ 0\ \ A^{01}\ \ A^{02}\ \ A^{03}]$
$[-A^{01}\ \ 0\ \ A^{12}\ A^{13}]$
$[-A^{02}\ -A^{12}\ \ 0\ \ A^{23}]$
$[-A^{03}\ -A^{13}\ -A^{23}\ \ 0\ ]$

Ostrogradsky_instability:
https://en.wikipedia.org/wiki/Ostrogradsky_instability
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Basically, it is a prescription for showing that physics typically uses at most two time derivatives to describe dynamic systems.
I got to thinking about the form of equations of motion using different techniques.
The usual SR 4-Vectors:

4-'Position'Gradient	$\partial = \partial^\mu = (\partial_t/c, -\nabla) = \partial_\mathbf{r} = \partial/\partial R_\mu$
4-'Velocity'Gradient	$(\partial_{u\_t}/c, -\nabla_\mathbf{u}) = \partial_\mathbf{u} = \partial/\partial U_\mu$
4-Position	$\mathbf{R} = R^\mu = (ct, \mathbf{r})$
4-Velocity	$\mathbf{U} = U^\mu = \gamma(c, \mathbf{u})$
4-Acceleration	$\mathbf{A} = A^\mu = \gamma(c\gamma', \gamma'\mathbf{u} + \gamma\mathbf{a})$

$(d/d\tau) = (\mathbf{U}\cdot\partial)$

1) Acceleration-based:
$\mathbf{A} = (d/d\tau)\mathbf{U} = (d^2/d\tau^2)\mathbf{R} = (d/d\tau)(d/d\tau)\mathbf{R} = (\mathbf{U}\cdot\partial)(\mathbf{U}\cdot\partial)\mathbf{R}$

2) Euler-Lagrange-based:
$\partial_\mathbf{r} = (d/d\tau)\partial_u = (\mathbf{U}\cdot\partial_\mathbf{r})\partial_u$

3) d'Alembertian wave-based
$\partial\cdot\partial$ = Constant Invariant

Note that all of these have two 4-Gradients ∂ involved.
These 4-Gradient open forms assume that there exist solutions:

$\{\partial_\mathbf{r}[L] = (d/d\tau)\partial_u[L]$ = Euler-Lagrange$\}$
$\{\partial\cdot\partial[\psi] = \text{const}[\psi]$ = d'Alembertian wave Eqn.$\}$

And experiment shows that these solutions can and do exist for various circumstances
$\mathbf{A} = (d/d\tau)\mathbf{U} = (d^2/d\tau^2)\mathbf{R} = (d/d\tau)(d/d\tau)\mathbf{R} = (\mathbf{U}\cdot\partial)(\mathbf{U}\cdot\partial)\mathbf{R}$

$\{\ \mathbf{U} = d\mathbf{R}/d\tau = (d/d\tau)\mathbf{R}\}$: gives a 4D tensorial dynamics of particles
$\{\partial_\mathbf{r}[L] = (d/d\tau)\partial_u[L]$ = Euler-Lagrange$\}$: gives a 4D tensorial dynamics of almost anything

$\partial\cdot\partial$ = Invariant. This is the d'Alembertian Wave Equation.
$\{\partial\cdot\partial$ = d'Alembertian wave Eqn.$\}$ gives a 4D tensorial dynamics of waves

N-dimensional fluidic 2-index symmetric tensor $S^{\mu\nu}$ has N(N+1)/2 independent components. i.e. Stress-Energy Tensor $T^{\mu\nu}$
There is a way to use this as a "single-particle" fluid.

This can be decomposed into a particulate form:
{angular} 2-index anti-symmetic tensor $A^{\mu\nu}$ with N(N-1)/2 independent components, i.e. N-AngularMomentum [⥁]
+
{linear} 1-index tensor V^μ with N independent components, i.e. N-LinearMomentum [→]

$[N(N-1)/2]+[N] = N^2/2-N/2+N = N^2/2+N/2 = N(N+1)/2$ total independent components (angular+linear=symmetric)

The {angular} 2-index anti-symmetic tensor N(N-1)/2 independent components can be constructed from two 1-index tensors $\{X^\mu,P^\nu\}$of N components each using $A^{\mu\nu} = X^\mu \wedge P^\nu = X^\mu P^\nu - X^\nu P^\mu$

So, we have three 1-index tensors required to equate with the original 2-index symmetric tensor.
Normally there would be 3N total independent components. But, there is the possibility of some tensor constraints C.

We want:
3N - # of constraints C = N(N+1)/2 independent components of symmetric 2-index tensor.

$3N - C = N(N+1)/2 = N^2/2 + N/2$
$6N - 2C = N^2 + N$
$N^2 - 5N + 2C = 0$
$N = (5\pm\sqrt{[25-4*2C]})/2 = (5\pm\sqrt{[25-8C]})/2$

if C=0, $N = (5\pm\sqrt{[25]})/2 = (5\pm5)/2 = \{0,5\}$
if C=1, $N = (5\pm\sqrt{[17]})/2$ = non-integer
if C=2, $N = (5\pm\sqrt{[9]})/2 = (5\pm3)/2 = \{1,4\}$
if C=3, $N = (5\pm\sqrt{[1]})/2 = (5\pm1)/2 = \{2,3\}$
if C>=4, N = complex

One of the 1-index tensors must be the N-Position $\mathbf{R} = (ct,r^0,\ldots,r^{N-1}) = (ct,\mathbf{r})$

Take a scalar invariant derivative (d/dτ)
N-Velocity $\mathbf{U} = (d/d\tau)\mathbf{R} = (d/d\tau)[(ct,r^0,...r^{N-1})] = (dt/d\tau)(d/dt)[(ct,r^0,...r^{N-1})] = (dt/d\tau)(c,u^0,\ldots,u^{N-1}) = \gamma(c,u^0,\ldots,u^{N-1}) = \gamma(c,\mathbf{u})$
$\mathbf{U} = d/d\tau[(ct,r^0,...r^{N-1})] = (dt/d\tau)(d/dt)[(ct,r^0,...r^{N-1})] = (dt/d\tau)(c,u^0,\ldots,u^{N-1}) = \gamma(c,u^0,\ldots,u^{N-1}) = \gamma(c,\mathbf{u})$
$\mathbf{U}\cdot\mathbf{U} = \text{invariant} = \gamma(c,\mathbf{u})\cdot\gamma(c,\mathbf{u}) = \gamma^2(c^2 - \mathbf{u}\cdot\mathbf{u})$
In a rest frame, $(\mathbf{u_{rest}} = \mathbf{0})$ and $(t=\tau)$ and $(dt/d\tau) = 1 = \gamma_{rest}$
$\mathbf{U}\cdot\mathbf{U} = \text{invariant} = c^2$
There is at least 1 constraint regardless of # of dimensions N

Take another scalar invariant derivative (d/dτ)
N-Acceleration $\mathbf{A} = (d/d\tau)\mathbf{U}$
$d[\mathbf{U}\cdot\mathbf{U}] = d[c^2] = 0 = \mathbf{A}\cdot\mathbf{U}+\mathbf{U}\cdot\mathbf{A} = 2(\mathbf{A}\cdot\mathbf{U})$, hence $(\mathbf{A}\cdot\mathbf{U}) = 0$ is another constraint
So, there are at least 2 constraints.
This also matches the idea of **U** tangent to the worldline and **A** normal to the worldline. $(\mathbf{A}\cdot\mathbf{U}) = 0$ implies $(\mathbf{A}\perp\mathbf{U})$

In the classical limit, we know that we have one time {t} scalar of Dim=(1) and three spatial vectors $\{\mathbf{r},\mathbf{u},\mathbf{a}\}$ of Dim=(N-1)

$1+3(N-1) = N^2/2 + N/2$
$2+6(N-1) = N^2 + N$
$2+6N-6 = N^2 + N$
$N^2 + N - 6N + 4 = 0$
$N^2 - 5N + 4 = 0$
$(N-4)(N-1) = 0$
$N = \{1,4\}$

So, this derivation implies that, based on tensor mathematics, a physical system can be described with:
a) one symmetric 2-index tensor (including possibility of “one-particle” fluid)
b) one (angular [⥁]) anti-symmetric 2-index tensor + one (linear [→]) 1-index tensor – Conservation of Angular/Linear Momentum
c) three 1-index tensors, using just two ProperTime derivatives (Ostrogradsky’s Instability) – $\mathbf{R}$, $\mathbf{U}=d\mathbf{R}/d\tau$, $\mathbf{A}=d^2\mathbf{R}/d\tau^2$

Mass Density $\rho[x^\alpha] = \int(m_o/\sqrt{[-g[x^\alpha]]})\delta^4[x^\alpha - \gamma^\alpha[\tau]]d\tau$

$T^{\mu\nu}[x^\alpha] = \rho[x^\alpha](u^\mu)[\tau](u^\nu)[\tau]$ with $(u^\mu)=(d\gamma^\mu/d\tau)[\tau]$ with $\gamma^\mu[\tau]$ as the particle worldline

$T^{\mu\nu}[x^\alpha] = \rho[x^\alpha](d\gamma^\mu/d\tau)[\tau](d\gamma^\nu/d\tau)[\tau]$

$T^{\mu\nu}[x^\alpha] = \int(m_o/\sqrt{[-g[x^\alpha]]})(d\gamma^\mu/d\tau)[\tau](d\gamma^\nu/d\tau)[\tau]\delta^4[x^\alpha - \gamma^\alpha[\tau]]d\tau$

$T^{\mu\nu}[t,x^j] = \int(m_o/\sqrt{[-g[t,x^j]]})(d\gamma^\mu/d\tau)[t'](d\gamma^\nu/d\tau)[t']\delta(t-t')\delta^3[x^j - \gamma^j[t]](d\tau/dt)[t']dt'$

$T^{\mu\nu}[t,x^j] = (m_o/\sqrt{[-g[t,x^j]]})(d\gamma^\mu/d\tau)[t](d\gamma^\nu/d\tau)[t]\delta^3[x^j - \gamma^j[t]](d\tau/dt)[t]$

mass conservation implies the alignment of the four-velocity with the worldline for a "single-particle" fluid

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N-Differential $d\mathbf{R} = (cdt,dr^0,\ldots,dr^{N-1}) = (cdt,dr)$

$d\mathbf{R}\cdot d\mathbf{R} = (cdt,dr)\cdot(cdt,dr) = c^2dt^2 - d\mathbf{r}\cdot d\mathbf{r} = c^2d\tau^2$
$d\mathbf{R}\cdot d\mathbf{R}/dt^2 = c^2dt^2/dt^2 - d\mathbf{r}\cdot d\mathbf{r}/dt^2 = c^2d\tau^2/dt^2$
$c^2 - \mathbf{u}\cdot\mathbf{u} = c^2d\tau^2/dt^2$
$1 - \mathbf{u}\cdot\mathbf{u}/c^2 = d\tau^2/dt^2$
$\sqrt{[1 - \mathbf{u}\cdot\mathbf{u}/c^2]} = d\tau/dt$
$1/\sqrt{[1 - \mathbf{u}\cdot\mathbf{u}/c^2]} = dt/d\tau = \gamma$

//======================================

N-Differential $d\mathbf{R} = (cdt^0,\ldots,cdt^M , dr^0,\ldots,dr^{N-M}) = (c\mathbf{dt},\mathbf{dr})$ with possibility of multiple temporal dimensions

$d\mathbf{R}\cdot d\mathbf{R} = (c\mathbf{dt},\mathbf{dr})\cdot(c\mathbf{dt},\mathbf{dr}) = c^2d\mathbf{t}\cdot d\mathbf{t} - d\mathbf{r}\cdot d\mathbf{r} = c^2d\tau^2$
$d\mathbf{R}\cdot d\mathbf{R}/(d\mathbf{t}\cdot d\mathbf{t}) = c^2(d\mathbf{t}\cdot d\mathbf{t})/(d\mathbf{t}\cdot d\mathbf{t}) - d\mathbf{r}\cdot d\mathbf{r}/(d\mathbf{t}\cdot d\mathbf{t}) = c^2d\tau^2/(d\mathbf{t}\cdot d\mathbf{t})$
$c^2 - \mathbf{u}\cdot\mathbf{u} = c^2d\tau^2/(d\mathbf{t}\cdot d\mathbf{t})$ with $\mathbf{u}\cdot\mathbf{u} = (d\mathbf{r}\cdot d\mathbf{r})/(d\mathbf{t}\cdot d\mathbf{t})$
$1 - \mathbf{u}\cdot\mathbf{u}/c^2 = d\tau^2/(d\mathbf{t}\cdot d\mathbf{t})$
$\sqrt{[1 - \mathbf{u}\cdot\mathbf{u}/c^2]} = d\tau/(d\mathbf{t}\cdot d\mathbf{t})$
$1/\sqrt{[1 - \mathbf{u}\cdot\mathbf{u}/c^2]} = (d\mathbf{t}\cdot d\mathbf{t})/d\tau = \gamma$

==

$\mathbf{R}\cdot\mathbf{U} = c^2\tau = \gamma(c^2t - \mathbf{r}\cdot\mathbf{u})$

$d/d\tau[\mathbf{R}\cdot\mathbf{U}]$
$= d/d\tau[\mathbf{R}]\cdot\mathbf{U} + \mathbf{R}\cdot d/d\tau[\mathbf{U}]$
$= \mathbf{U}\cdot\mathbf{U} + \mathbf{R}\cdot\mathbf{A}$
$= c^2 + \mathbf{R}\cdot\mathbf{A}$
$= d/d\tau[\gamma(c^2t - \mathbf{r}\cdot\mathbf{u})]$
$= \gamma d/dt[\gamma(c^2t - \mathbf{r}\cdot\mathbf{u})]$
$= \gamma[\ d/dt[\gamma](c^2t - \mathbf{r}\cdot\mathbf{u}) + \gamma d/dt[(c^2t - \mathbf{r}\cdot\mathbf{u})]\ ]$
$= \gamma[\ d/dt[\gamma](c^2t - \mathbf{r}\cdot\mathbf{u}) + \gamma[(c^2 - \mathbf{u}\cdot\mathbf{u} - \mathbf{r}\cdot\mathbf{a})]\ ]$