The Dirac Equation Derived using 4-Vectors

It can be shown that spin comes from the Poincaré Symmetry of SR, not from a QM axiom.
Writing a 4-SpinMomentum then leads naturally to the Dirac Equation.

First, we will do a little pure mathematics:
Let $\{ (a^2 - b^2) = (a+b)(a-b) = c^2 \}$

We can multiply by an arbitrary factor (xy):
$(a^2 - b^2)(xy) = (a+b)(a-b)(xy) = c^2(xy)$

If we impose the following extra constraint:
$(a+b)x = (cy)$
$(a-b)y = (cx)$

Then the separated equations are still true when multiplied together:
$(a+b)x * (a-b)y = (cy) * (cx) \rightarrow (a+b)(a-b)(xy) = c^2(xy)$

Now add and subtract the separated equations:
$(a+b)x + (a-b)y = (cy) + (cx)$
$(a+b)x - (a-b)y = (cy) - (cx)$

Gather terms in {a,b,c}:
$a(x+y) + b(x-y) = c(x+y)$
$a(x-y) + b(x+y) = -c(x-y)$

Let $X=(x+y)$ and $Y=(y-x)=-(x-y)$, just a change in variable names
$aX - bY = cX$
$-aY + bX = cY$

Rearrange:
$aX - bY = cX$
$bX - aY = cY$

Putting into matrix form:

$$\begin{bmatrix} a & -b \\ b & -a \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

or
Putting into suggestive matrix form...

$$\left(\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} a + \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} b \right) \begin{bmatrix} X \\ Y \end{bmatrix} = c \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

And again, to confirm that this matches the original equation:
$aX - bY = cX$
$bX - aY = cY$
or
$(a-c)X = (b)Y$
$(b)X = (a+c)Y$

Multiply the terms:
(a-c)X(a+c)Y = (b)X(b)Y
(a-c)(a+c)XY = (b)(b)XY

$(a^2 - c^2)XY = (b^2)XY$
$(a^2 - c^2) = b^2$
$(a^2 - b^2) = c^2$

So, mathematically:
$(a^2 - b^2) = c^2$ is the defining equation, which holds equivalently for:

$\{ (a^2 - b^2)(xy) = (c^2)(xy) \}$
or

$$\left(\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix} a + \begin{bmatrix}0 & -1\\1 & 0\end{bmatrix} b\right)\begin{bmatrix}X\\Y\end{bmatrix} = c\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}X\\Y\end{bmatrix}$$

It is only the nature of {x,y} and {X,Y} that are different.

Pauli Matrices are a set of dimensionless 2x2 matrices that may be used in totally classical contexts.
In this regard, they are not a quantum postulate, but one of Poincaré Invariance.

$\sigma^t = \sigma^0 = \mathbf{I_2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\sigma^x = \sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\sigma^y = \sigma^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ $\sigma^z = \sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

The Pauli Spin Matrices can be written in Tensor notation, with each component itself a dimensionless 2x2 matrix
$\boldsymbol{\Sigma} = \Sigma^{\mu}{}_{\nu} = \text{Diag}[\sigma^0,\sigma^1,\sigma^2,\sigma^3] = \text{Diag}[\sigma^0,\boldsymbol{\sigma}] \rightarrow \text{Diag}[\sigma^t,\sigma^x,\sigma^y,\sigma^z]$

In classical mechanics, Pauli matrices are useful in the context of the Cayley-Klein parameters.
The 3D matrix ***P*** corresponding to the 3-position **x** of a point in space is defined in terms of the above Pauli vector matrix,

$\boldsymbol{P} = \mathbf{x} \cdot \boldsymbol{\sigma} = x\,\sigma^x + y\,\sigma^y + z\,\sigma^z$

The 4D matrix ***P*** corresponding to the 4-Position **X** of a point in spacetime is defined in terms of the above Pauli vector matrix,

$\boldsymbol{P} = \mathbf{X} \cdot \boldsymbol{\Sigma} = ct\,\sigma^t + x\,\sigma^x + y\,\sigma^y + z\,\sigma^z$

Suppose now that the vector $\mathbf{x}=(x_1,x_2,x_3)$ is rotated around an axis with unit vector $\mathbf{n}=(n_1,n_2,n_3)$ through an angle θ.

The transformation matrix $\mathbf{R}(\theta)$ for rotations about an axis through an angle θ may be written in terms of Pauli matrices and the unit matrix. These can be gathered together in tensor notation with the 2D unit matrix = $\mathbf{I_2} = \sigma^0$.

$$\boldsymbol{R}_x(\theta) = e^{-i\theta x/2} = \sigma^0 \cos(\theta/2) - i\,\sigma^x \sin(\theta/2) = \begin{bmatrix} \cos(\theta/2) & -i\sin(\theta/2) \\ -i\sin(\theta/2) & \cos(\theta/2) \end{bmatrix}$$

$$\boldsymbol{R}_y(\theta) = e^{-i\theta y/2} = \sigma^0 \cos(\theta/2) - i\,\sigma^y \sin(\theta/2) = \begin{bmatrix} \cos(\theta/2) & -\sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{bmatrix}$$

$$\boldsymbol{R}_z(\theta) = e^{-i\theta z/2} = \sigma^0 \cos(\theta/2) - i\,\sigma^z \sin(\theta/2) = \begin{bmatrix} \cos(\theta/2) - i\sin(\theta/2) & 0 \\ 0 & \cos(\theta/2)+i\sin(\theta/2) \end{bmatrix} = \begin{bmatrix} e^{-i(\theta/2)} & 0 \\ 0 & e^{+i(\theta/2)} \end{bmatrix}$$

For an angle θ about an arbitrary axis **n**

$$\boldsymbol{R}_n(\theta) = e^{-i\theta n/2} = \sigma^0 \cos(\theta/2) - i\,(\mathbf{n} \cdot \boldsymbol{\sigma}) \sin(\theta/2) = \begin{bmatrix} \cos(\theta/2) - i\sin(\theta/2)n_3 & -\sin(\theta/2)(n_2+in_1) \\ \sin(\theta/2)(n_2-in_1) & \cos(\theta/2)+i\sin(\theta/2)n_3 \end{bmatrix}$$

All of the relativistic wave equations can be derived from a common source:
The relativistic mass-energy relation, including spin, in an Electromagnetic (EM) field.
Note that this formalism fits well with the Stern-Gerlach experiment.

4-Momentum $P = (E/c,\mathbf{p})$
4-Momentum including Spin $\mathbf{P_s} = \boldsymbol{\Sigma}\cdot\mathbf{P} = \Sigma^\mu{}_\nu P^\nu = \eta_{\alpha\beta}\Sigma^{\mu\alpha} P^\beta = Ps^\mu$
$\Sigma^\mu{}_\nu$ is the 4D (1,1)-Pauli Spin-Matrix Tensor = Diag[σ^0,$\boldsymbol{\sigma}$]
$\Sigma^{\mu\nu}$ is the 4D (2,0)-Pauli Spin-Matrix Tensor = Diag[σ^0,$-\boldsymbol{\sigma}$]

$\mathbf{P_s}$ = Diag[σ^0,$-\boldsymbol{\sigma}$]$\cdot\mathbf{P}$ = Diag[σ^0,$-\boldsymbol{\sigma}$]$\cdot(E/c,\mathbf{p}) = (\sigma^0E/c,\boldsymbol{\sigma}\cdot\mathbf{p})$

$\mathbf{P_s} = (p_s{}^0,\mathbf{p_s}) = (\sigma^0E/c,\boldsymbol{\sigma}\cdot\mathbf{p})$
with σ^0 as an identity matrix **I** of appropriate spin dimension and $\boldsymbol{\sigma}$ is the Pauli Spin Matrix Vector

4-Momentum inc. Spin in External Field $\mathbf{P_T} = (H/c,\mathbf{p_T}) = (E_T/c,\mathbf{p_T})$
with:
$H = E_T$ = Hamiltonian = Total Energy Of System
$\mathbf{p_T}$ = Total 3-momentum Of System

4-TotalMomentum $\mathbf{P_T} = \mathbf{P} + q\mathbf{A}$
4-Momentum $\mathbf{P} = \mathbf{P_T} - q\mathbf{A}$
4-MomentumIncSpin $\mathbf{P_s} = (p_s{}^0,\mathbf{p_s}) = (\sigma^0E/c,\boldsymbol{\sigma}\cdot\mathbf{p}) = (\ \sigma^0(E_T/c - q\varphi/c),\boldsymbol{\sigma}\cdot(\mathbf{p_T} - q\mathbf{a})\ )$: Note each component is a 2x2 matrix
$\mathbf{P_s}\cdot\mathbf{P_s} = (p_s{}^0)^2 - (\mathbf{p_s})^2 = [\sigma^0(E/c)]^2 - [\boldsymbol{\sigma}\cdot(\mathbf{p})]^2 = [\sigma^0(E_T/c - q\varphi/c)]^2 - [\boldsymbol{\sigma}\cdot(\mathbf{p_T} - q\mathbf{a})]^2 = (m_oc)^2 = (E_o/c)^2$

The 4-TotalMomentum (inc. External Field Minimal-Coupling and Spin)
$\mathbf{P_s} = \boldsymbol{\Sigma}\cdot\mathbf{P} = \boldsymbol{\Sigma}\cdot(\mathbf{P_T} - q\mathbf{A}) = [\sigma^0(E_T/c - q\varphi/c),\boldsymbol{\sigma}\cdot(\mathbf{p_T} - q\mathbf{a})]$
with $\boldsymbol{\Sigma} = \Sigma^{\mu\nu}$ as the Pauli Spin Matrices, and taking the Einstein summation gives the $\sigma^0$and $\boldsymbol{\sigma}$

$\mathbf{P_s}\cdot\mathbf{P_s} = (\boldsymbol{\Sigma}\cdot\mathbf{P})^2 = [\boldsymbol{\Sigma}\cdot(\mathbf{P_T} - q\mathbf{A})]^2 = [\sigma^0(E_T/c - q\varphi/c)]^2 - [\boldsymbol{\sigma}\cdot(\mathbf{p_T} - q\mathbf{a})]^2 = (m_oc)^2$
$(\boldsymbol{\Sigma}\cdot\mathbf{P})^2 = (m_oc)^2$
$(\boldsymbol{\Sigma}\cdot\partial)^2 = -(m_o/\hbar)^2$
$(\boldsymbol{\Sigma}\cdot\partial)^2 + (m_oc/\hbar)^2 = 0$
$(\boldsymbol{\Sigma}\cdot(D-(i/h)q\mathbf{A}))^2 + (m_oc/\hbar)^2 = 0$

Now, to prove that this "Relativistic Pauli" Energy-Momentum equation can give the Dirac equation
$\mathbf{Ps}\cdot\mathbf{Ps} = [\sigma^0(E_T/c - q\varphi/c)]^2 - [\boldsymbol{\sigma}\cdot(\mathbf{p_T} - q\mathbf{a})]^2 = (p_s{}^0)^2 - (\mathbf{p_s})^2 = (m_oc)^2 = (E_o/c)^2$
$\mathbf{Ps}\cdot\mathbf{Ps} = I(E_T/c - q\varphi/c)]^2 - [\boldsymbol{\sigma}\cdot(\mathbf{p_T} - q\mathbf{a})]^2 = (p_s{}^0)^2 - (\mathbf{p_s})^2 = (m_oc)^2 = (E_o/c)^2$
$\mathbf{Ps}\cdot\mathbf{Ps} = (p_s{}^0)^2 - (\mathbf{p_s})^2 = (p_s{}^0 + \mathbf{p_s})\ (p_s{}^0 - \mathbf{p_s}) = (m_oc)^2$
$\{(p_s{}^0)^2 - (\mathbf{p_s})^2\}(xy) = (m_oc)^2(xy)$

From our math proof above, this is equivalent to:

$$\left(\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} p_s{}^0 + \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix} \mathbf{p_s}\right)\begin{bmatrix}X\\ Y\end{bmatrix} = (m_oc)I_2 \begin{bmatrix}X\\ Y\end{bmatrix}$$

or

$$\left(\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} \sigma^0p^0 + \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix} \boldsymbol{\sigma}\cdot\mathbf{p}\right)\begin{bmatrix}X\\ Y\end{bmatrix} = (m_oc)I_2 \begin{bmatrix}X\\ Y\end{bmatrix}$$

Putting into highly suggestive matrix form...

$$\left(\begin{bmatrix}\sigma^0 & 0\\ 0 & -\sigma^0\end{bmatrix} p^0 + \begin{bmatrix}0 & -\boldsymbol{\sigma}\\ \boldsymbol{\sigma} & 0\end{bmatrix} \cdot\mathbf{p}\right)\begin{bmatrix}X\\ Y\end{bmatrix} = (m_oc)I_2 \begin{bmatrix}X\\ Y\end{bmatrix}$$

let Spinor $\Psi = \begin{bmatrix} X \\ Y \end{bmatrix}$ and note that $\sigma^0 = I_2$

this is equivalent to Dirac Gamma Matrices (in Dirac Basis)...

$$\left(\begin{bmatrix} I_2 & 0 \\ 0 & -I_2 \end{bmatrix} p^0 + \begin{bmatrix} 0 & -\boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{bmatrix} \cdot \mathbf{p} \right) \Psi = (m_o c) I_2 \Psi$$

$(\gamma^0 p^0 - \boldsymbol{\gamma} \cdot \mathbf{p})\Psi = (m_o c) I \Psi$

$(\boldsymbol{\Gamma} \cdot \mathbf{P})\Psi = (m_o c) I \Psi$

$(\boldsymbol{\Gamma} \cdot \mathbf{P}) = (m_o c)$

$(\Gamma^\mu P_\mu)\Psi = (m_o c)\Psi$

$i\hbar(\Gamma^\mu \partial_\mu)\Psi = (m_o c)\Psi$

The Dirac Relativistic Quantum Equation for spin 1/2 particles

To recap, the SR 4-Vectors have some very simple, fundamental, and Invariant Lorentz Scalar relations between one another:

4-Position	$\mathbf{R} = R^\mu = (ct,\mathbf{r}) \in$ <Event> $\in$ <Time·Space>	$\rightarrow (E_o/c^2) \twoheadrightarrow \mathbf{P}= (E/c,\mathbf{p})$
4-Velocity	$\mathbf{U} = U^\mu = \gamma(c,\mathbf{u}) = (\mathbf{U}\cdot\partial)\mathbf{R}=(d/d\tau)\mathbf{R}$	/
4-Momentum	$\mathbf{P} = P^\mu = (E/c,\mathbf{p}) = (m_o)\mathbf{U}$	$\mathbf{U}= \gamma(c,\mathbf{u})$ (ħ)↑ ↓(1/ħ)
4-WaveVector	$\mathbf{K} = K^\mu = (\omega/c,\mathbf{k}) = (1/\hbar)\mathbf{P}$	\
4-Gradient	$\partial = \partial^\mu = (\partial_t/c,-\nabla) = (-i)\mathbf{K}$	$\rightarrow (\omega_o/c^2) \rightsquigarrow \mathbf{K}= (\omega/c,\mathbf{k})$

Analysis of Dirac's Constant ($\hbar = h/2\pi$) : Planck's Constant ($h = 2\pi\hbar$), and (E_o/ω_o) in the context of SRQM:

It is an empirical (observational) fact that the Lorentz Scalar Invariant $(E_o/\omega_o) = (\gamma E_o/\gamma\omega_o) = (E/\omega) \Longrightarrow (\hbar)$ [J·s]/{rad}
for all known experimental measurements. The SR 4D-Tensor rules show that one doesn't need a quantum axiom for this.
(ħ) is actually an empirically-measurable quantity, just like (c), (e), (G), (k_B), (μ_o), (ε_o) or the other fundamental constants, which are also 4D Lorentz Scalar Invariants. (ħ) can be measured classically {without need of quantum axioms} from the photoelectric effect, from the inverse photoelectric effect, from LED's (injection electroluminescence), from Atomic Line Spectra (Bohr Atom), from the Duane-Hunt Law in Bremsstrahlung, from Electron Diffraction in crystals, from the Watt/Kibble-Balance, from the Sagnac Effect, from Incandescent Blackbody Intensity-Temperature Relations, from Compton Scattering, from the Stern-Gerlach experiment, etc.

For physics simulations of measurements of Dirac's : Planck's Constant (ħ : h), see:

http://scirealm.org/Physics-PlanckConstantViaAtomicLineSpectra.html	(transitions of electron energy-levels)
http://scirealm.org/Physics-PlanckConstantViaComptonScattering.html	(photon:electron collisions)
http://scirealm.org/Physics-PlanckConstantViaElectronDiffraction.html	(electron:atomic crystal scattering)
http://scirealm.org/Physics-PlanckConstantViaGravityInducedInterferometry.html	(gravity interaction)
http://scirealm.org/Physics-PlanckConstantViaIncandescence.html	(temperature interaction)
http://scirealm.org/Physics-PlanckConstantViaLEDs.html	(electron:photon interaction)
http://scirealm.org/Physics-PlanckConstantViaSagnacEffect.html	(rotation interaction)
http://scirealm.org/Physics-PlanckConstantViaSternGerlach.html	(magnetic-field interaction)

$\{E_o/\omega_o = E/\omega = \hbar\}$ implies the following SR (wave:particle-like □ ·) relation:
$\mathbf{P} = \hbar\mathbf{K} = (E/c,\mathbf{p}) = \hbar(\omega/c,\mathbf{k}) =(\hbar\omega/c,\hbar\mathbf{k})$
The temporal part $\{E = \hbar\omega\}$ gives Einstein's photoelectric quantum "postulate". Again, emphasis: <u>derived</u> from SR.
The spatial part $\{\mathbf{p} = \hbar\mathbf{k}\}$ gives de Broglie's matter-wave quantum "postulate". Again, emphasis: <u>derived</u> from SR.

$\partial = -i\mathbf{K} = (\partial_t/c,-\nabla) = -i(\omega/c,\mathbf{k})$
The temporal part $\{\partial_t = -i\omega\}$ or $\{\omega = i\partial_t\}$ gives temporal:frequency complex planewave change:operator
The spatial part $\{\nabla = i\mathbf{k}\}$ or $\{\mathbf{k} = -i\nabla\}$ gives spatial:wavenumber complex planewave change:operator

The standard Schrödinger QM Relations derived from SR:

Again, we examine these SR 4-Vector relations derived above… and by simply combining them...
$\mathbf{P} = \hbar\mathbf{K}$ (a relation which is entirely empirical, based on just SR arguments, shown above)
$\mathbf{K} = i\partial$ (which is a relation for complex plane-waves, used in classical EM and elsewhere)

$\mathbf{P} = i\hbar\partial = (E/c,\mathbf{p}) = i\hbar(\partial_t/c,-\nabla)$
The temporal part $\{E = i\hbar\partial_t = i\hbar\ \partial/\partial t\}$ gives unitary QM time evolution operator. Again, emphasis: <u>derived</u> from SR.
The spatial part $\{\mathbf{p} = -i\hbar\nabla\}$ gives the QM momentum operator. Again, emphasis: <u>derived</u> from SR.

Relativistic version (Relativistic Pauli Equation)

$\mathbf{Ps}\cdot\mathbf{Ps} = [\underline{\sigma}^0 (E_T/c - q\varphi/c)]^2 - [\underline{\boldsymbol{\sigma}}\cdot(\mathbf{p_T} - q\mathbf{a})]^2 = (m_o c)^2 = (E_o/c)^2$

$[\underline{\sigma}^0 (E_T/c - q\varphi/c)]^2 - [\underline{\boldsymbol{\sigma}}\cdot(\mathbf{p_T} - q\mathbf{a})]^2 = (m_o c)^2$

$[\underline{\mathbf{I}}(E_T/c - q\varphi/c)]^2 - [\underline{\boldsymbol{\sigma}}\cdot(\mathbf{p_T} - q\mathbf{a})]^2 = (m_o c)^2$

$[\underline{\mathbf{I}}(E_T/c - q\varphi/c)]^2 - [\underline{\mathbf{I}}\cdot(\mathbf{p_T} - q\mathbf{a})]^2 - i[\underline{\boldsymbol{\sigma}}\cdot((\mathbf{p_T} \text{ x } - q\mathbf{a}) + (- q\mathbf{a} \text{ x } \mathbf{p_T}))] = (m_o c)^2$

$[\underline{\mathbf{I}}(E_T/c - q\varphi/c)]^2 - [\underline{\mathbf{I}}\cdot(\mathbf{p_T} - q\mathbf{a})]^2 - i[\underline{\boldsymbol{\sigma}}\cdot i\hbar q((-\nabla_T \text{ x } \mathbf{-a}) + (\mathbf{-a} \text{ x } \mathbf{-}\nabla_T))] = (m_o c)^2$

$[\underline{\mathbf{I}}(E_T/c - q\varphi/c)]^2 - [\underline{\mathbf{I}}\cdot(\mathbf{p_T} - q\mathbf{a})]^2 - i[\underline{\boldsymbol{\sigma}}\cdot i\hbar q(\mathbf{B})] = (m_o c)^2$

$[\underline{\mathbf{I}}(E_T/c - q\varphi/c)]^2 - [\underline{\mathbf{I}}\cdot(\mathbf{p_T} - q\mathbf{a})]^2 + \hbar q[\underline{\boldsymbol{\sigma}}\cdot\mathbf{B}] = (m_o c)^2$

$[\underline{\mathbf{I}}(E_T/c - q\varphi/c)]^2 = [\underline{\mathbf{I}}\cdot(\mathbf{p_T} - q\mathbf{a})]^2 - \hbar q[\underline{\boldsymbol{\sigma}}\cdot\mathbf{B}] + (m_o c)^2$

Non-relativistic version (Standard Pauli Equation)

$[\underline{\mathbf{I}}(E_T/c - q\varphi/c)]^2 - [\underline{\boldsymbol{\sigma}}\cdot(\mathbf{p_T} - q\mathbf{a})]^2 = (m_o c)^2$

$[\underline{\mathbf{I}}(E_T/c - q\varphi/c)]^2 = [\underline{\boldsymbol{\sigma}}\cdot(\mathbf{p_T} - q\mathbf{a})]^2 + (m_o c)^2$

$[\underline{\mathbf{I}}(E_T/c - q\varphi/c)] = (\pm) \text{ Sqrt } [[\underline{\boldsymbol{\sigma}}\cdot(\mathbf{p_T} - q\mathbf{a})]^2 + (m_o c)^2]$

$[\underline{\mathbf{I}}(E_T/c - q\varphi/c)] = (\pm) \text{ Sqrt } [[\underline{\boldsymbol{\sigma}}\cdot(\mathbf{p_T} - q\mathbf{a})]^2 + (m_o c)^2]$

$[\underline{\mathbf{I}}(E_T/c - q\varphi/c)] \sim (\pm) [[\underline{\boldsymbol{\sigma}}\cdot(\mathbf{p_T} - q\mathbf{a})]^2/(2m_o c) + (m_o c)]$

$[\underline{\mathbf{I}}(E_T/c - q\varphi/c)] \sim (\pm) [([(\mathbf{p_T} - q\mathbf{a})]^2 - \hbar q[\underline{\boldsymbol{\sigma}}\cdot\mathbf{B}])/(2m_o c) + (m_o c)]$

$[\underline{\mathbf{I}}(E_T - q\varphi)] \sim (\pm) [([(\mathbf{p_T} - q\mathbf{a})]^2 - \hbar q[\underline{\boldsymbol{\sigma}}\cdot\mathbf{B}])/(2m_o) + (m_o c^2)]$

$E_T \sim q\varphi (\pm) [([(\mathbf{p_T} - q\mathbf{a})]^2 - \hbar q[\underline{\boldsymbol{\sigma}}\cdot\mathbf{B}])/(2m_o) + (m_o c^2)]$

where the $- (\hbar q[\underline{\boldsymbol{\sigma}}\cdot\mathbf{B}])/(2m_o)$ is the Stern-Gerlach term