The Fundamental Constants & Dimensional-Units:

What are the Fundamental Constants and why are they fundamental?
What are the Dimensional-Units and why are they dimensional, er.. fundamental, er.. both? :)

The answer to these questions resides within Special Relativity (SR), 4D <Time·Space>, & the 4D-Tensor Theory of Measurement.
A measurement is a way to determine the properties of an <event>, properties which are *quantities* with a particular *dimensional-unit*.
These properties have a fundamental structure which can be described using tensors, and specifically 4-Vectors {4D (1,0)-Tensors}.
There are also physical, fundamental, natural constants {4D (0,0)-Tensors} which relate these dimensional-units in various ways.

The Historical SI Units (System-International):

Start with the historical (7) SI dimensional-base-units, with all other dimensional-units derived from these:

SI Unit Symbol	Name	Unit-Dimension-Type	
[s]	second	[T]=[time]	{[T], [L], [M]} most commonly used in mechanics
[m]	meter	[L]=[length]	
[kg]	kilogram	[M]=[mass]	
[A]	ampere	[I]=[electric current]	[A = C/s] or [C = A·s]: [I] adds EM phenomena
[K]	kelvin	[Θ]=[thermodynamic temperature]	Using absolute temp scale: 0 = Absolute Minimum [Θ]
[mol]	mole	[N]=[amount of substance]	
[cd]	candela	[J]=[luminous intensity]	

SI Defining Constants (as of 2019)

Symbol	Name	Unit-Dimension	Used to Define Unit-Dim-Type
($\Delta\nu_{Cs}$)	Hyperfine transition frequency of Cs-133	[Hz = 1/s]	[T]=[time]
(c)	Speed of Light	[m/s]	[L]=[length]
(h)	Planck Constant	[Action = J·s = kg·m^2/s]/{cyc}	[M]=[mass]
(e)	Elementary EM Charge	[C]	[I]=[electric current]
(k_B)	Boltzmann Constant	[J/ºK]	[Θ]=[thermodynamic temperature]
(N_A)	Avogadro Constant	[#/mol]	[N]=[amount of substance]
(K_{CD})	Luminous efficacy of 540 THz Radiation	[683 lm/W]	[J]=[luminous intensity]

A more modern way of understanding the SI Units:

I will modify the definition of the SI dimensional-base-units just a bit, updating them with more modern ideas:
I choose the EM "charge" [C = coulomb] as being a more *fundamental* dimensional-unit to replace the "current" [A = ampere] , but it works either way. This is equivalent to replacing the dimensional-unit-type of [I]=[electric current] with [Q]=[EM charge].
One may make such changes as long as linear independence of the dimensional-unit-types is maintained.
The reason to make this choice is that charge [Q] acts more like mass [M] than does current(=moving charge), and so doing puts the dimensional-unit-types on a more similar foundation. We will see more motivation for this later, in the SR analysis.
Also, I will note at this point that [time] and [length] are fundamentally different from the other dimensional-unit-types, as they are primitives intrinsically linked by Relativity, Lorentz-Poincaré Transformations, and the LightSpeed Constant: (c) * [time] = [length]
In other words, [time] and [length] are directly intra-convertible in a way that the other dimensional-unit-types are not, being the two components of a single 4-Vector = 4D (1,0)-Tensor, the 4-Displacement $\mathbf{\Delta R} = \Delta R^\mu = (c\Delta t, \mathbf{\Delta r})$: which has a temporal part and a spatial part. There does not, however, appear to be a "particular" fundamental proper-time or proper-length. There is to-date no physical evidence of a minimal time-unit or length-unit, and hence the "natural" Planck units of time and space are of dubious value. However, the relationship between the temporal and spatial dimensions, the LightSpeed Constant (c), is absolutely measurable and does appear to be universal and physically fundamental. It can be found in quantum equations, EM equations, SR & GR equations. The other dimensional-unit-types appear as the result of Lorentz-Scalar Multiplier Constants = 4D (0,0)-Tensors between higher-index tensors.

SI Unit Symbol	Name	Unit-Dimension-Type and Explanation of Occurence
[s]	second	[T]=[time] = extent in the 1D temporal spacetime direction {t}, temporal displacement Δt
[m]	meter	[L]=[length] = extent in the 3D spatial spacetime directions {x,y,z}, spatial displacement $\lvert\mathbf{\Delta r}\rvert$
[kg]	kilogram	[M]=[mass] = count of material stuff (m) using Gravitational force, Poincaré Casimir Invariant
[C]	coulomb	[Q]=[EM charge] = count of material stuff (q) using ElectroMagnetic (EM) force, CPT Symmetry
[K] or [ºK]	kelvin	[Θ]=[thermodynamic temperature] = count of statistical information (ex. heat as molecular motion)
[mol]	mole	[N]=[amount of substance] = count of fermionic matter-particle stuff: typically "atoms/molecules"
[cd]	candela	[J]=[luminous intensity] = count of bosonic force-particle stuff : typically "photons"

The Primitive Dimensional-Unit-Types, Length and Time:

The universe can be modeled as a 4D spacetime manifold. The manifold is the collection of all possible <events>, which are points in 4D spacetime, the basic elements of the spacetime. The most basic property of an <event> is its 4-Position **R** = R^μ = (ct,**r**). Consider that the actual measurement of most physical properties actually boils down to being one or more measurements of certain positions (**r**) at certain times (t): i.e. Rods [length] and Clocks [time]. There is a deep reason for this: It enables the counting of 4D <events>.

Events have/are-subject-to: **Location, Motion, Substantiation, Alteration**:

All <events> have:

Location: described by 4-Position **R** = R^μ = (ct,**r**) and/or 4-Displacement Δ**R**, it's most basic property, providing [time] & [length].

Motion: described by 4-Velocity **U** = U^μ = γ(c,**u**) = d**R**/dτ = c**T** which gives a worldline, the evolution of <events> along a 4D-path.

An <event> may or may not have "something" at it, or **Substantiation**: {These "substances" are all 4-Scalars = 4D (0,0)-Tensors}

All of these substances can potentially "move", by multiplication with the 4-Velocity **U**. This is often called a "flux".

One or more Particles or SR "Dust" = RestNumberDensity (n_o) at an <event> gives 4-NumberFlux **N** = (n_o)**U**.

This can give [amount of substance] for fermions=matter-particles, or [luminous intensity] for bosons=force-particles.

A RestMass (m_o) at the <event> gives 4-Momentum **P** = (m_o)**U** & unit-dimension-type [mass].

A RestCharge (q=ρ_o/n_o) at the <event> gives 4-ChargeFlux **J** = (ρ_o)**U** & unit-dimension-type [EM charge].

A RestSpin (s_o) at the <event> gives 4-Spin **S** = (s_o)**Ŝ** , but no new unit-dimension-type as it uses [time] & [length] & [mass].

Note: it does give the new dimension of [action] = [mass][length]^2/[time] , but just not a new linearly-independent dimension.*

In the same way that one can choose between EM [charge] or [current] as a base, one could use [action] as base instead of [mass].

The Pauli-Lubanski PseudoVector $W^\mu=(m_o/2)\varepsilon^\mu{}_{\nu\rho\sigma}J^{\nu\rho}U^\sigma$ is how the 4-Spin moves around. ***W·U** = 0, which shows **W** to be spatial.*

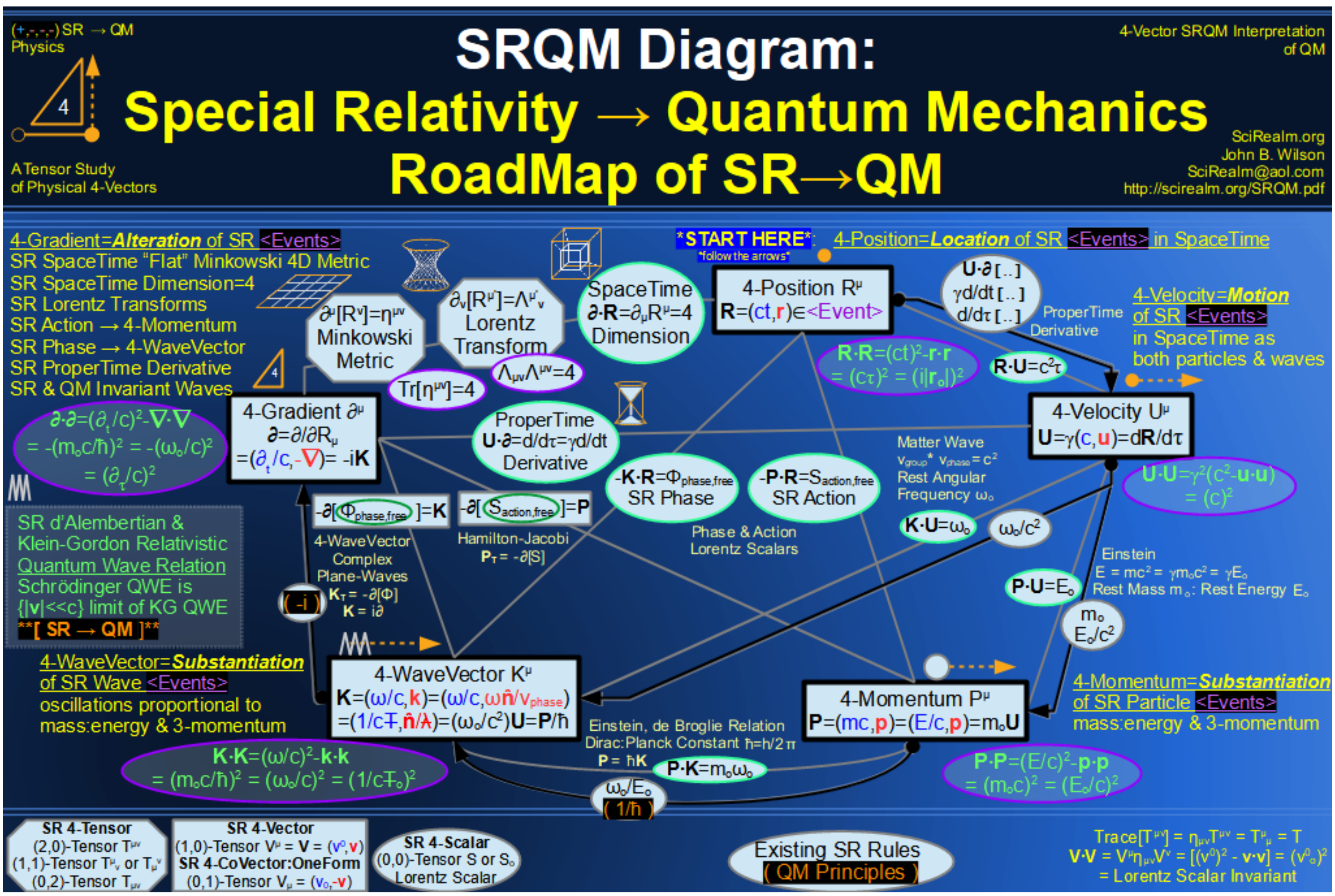
A RestTemperature (T_o) at the <event> gives 4-ThermalVector **Θ** = (1/$k_B T_o$)**U** & unit-dimension-type [thermodynamic temperature].

So, Substantiation of an <event> by a 4-Scalar can add unit-dimensionality beyond just the usual relativistic [time] & [length].

This 4-Scalar quantity (1 component) at the <event> multiplies with the 4-Velocity (3 components) to give a 4-Vector (4 components) that can have motion in SpaceTime, i.e. the 4-Scalar "substance" can move through 4D spacetime, with 4-Velocity **U**.

There is also the possibility of <event> **Alteration**: This is often related to a conservation of flux.

This can generally be described with the 4-Gradient $\partial = \partial^\mu = (\partial_t/c,-\nabla)$ by which the quantity of "substance" at the <event> changes.



Examples of Measurements:

Again, consider that the actual measurement of most physical properties actually boils down to being one or more measurements of positions (**r**) at certain times (t): i.e. Rods [length] and Clocks [time].

4-Position	$\mathbf{R} = R^\mu = (ct,\mathbf{r})$	$(\mathbf{R}\cdot\mathbf{R}) = (ct)^2 - \mathbf{r}\cdot\mathbf{r} = (ct_o)^2 = (c\tau)^2 = (i\lvert\mathbf{r_o}\rvert)^2$	Affine, only Lorentz Invar.
4-Displacement	$\Delta\mathbf{R} = \Delta R^\mu = (c\Delta t,\Delta\mathbf{r})$	$(\Delta\mathbf{R}\cdot\Delta\mathbf{R}) = (c\Delta t)^2 - \Delta\mathbf{r}\cdot\Delta\mathbf{r} = (c\Delta t_o)^2 = (c\Delta\tau)^2 = (i\lvert\Delta\mathbf{r_o}\rvert)^2$	Vector, Poincaré Invariant
4-Differential	$d\mathbf{R} = dR^\mu = (cdt,d\mathbf{r})$	$(d\mathbf{R}\cdot d\mathbf{R}) = (cdt)^2 - d\mathbf{r}\cdot d\mathbf{r} = (cdt_o)^2 = (cd\tau)^2 = (i\lvert d\mathbf{r_o}\rvert)^2$	Vector, Poincaré Invariant

{Affine|Vector} Space Rules: Affine + Affine=N/A, Affine − Affine=Vector, Affine ± Vector=Affine, Vector ± Vector=Vector

Examples: $Point_2 + Point_1$ = N/A, $Point_2 - Point_1$ = DisplacementVec, Point ± DisplacementVec = New Point, $Disp_2 \pm Disp_1 = Disp_3$

The 4-Position $\mathbf{R} = (ct,\mathbf{r})$ is a 4-Displacement $\Delta\mathbf{R} = (c\Delta t,\Delta\mathbf{r})$ for which one of the points is "pinned" to the 4-OriginZero $\mathbf{O} = (0,\mathbf{0})$.

Displacement: One makes a measurement of initial & final {position : time}, then calculate difference... $\{\Delta\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1, \Delta t = t_2 - t_1\}$

Velocity: One makes a measurement of initial & final {displacement : time}, then derive… $\{\mathbf{v}=\Delta\mathbf{r}/\Delta t\}_{\text{finite}}$ or $\{\mathbf{v}=d\mathbf{r}/dt\}_{\text{infinitesimal}}$

Acceleration: One makes a measurement of initial & final {velocity : time}, then derive… $\{\mathbf{a}=\Delta\mathbf{v}/\Delta t\}_{\text{finite}}$ or $\{\mathbf{a}=d\mathbf{v}/dt\}_{\text{infinitesimal}}$

Anything using a mechanical dial: the position of the pointer at a given time. An <event>.

Anything with light: the positions of the light points on a screen/detector at particular times. <events>.

Anything using electronics: the count of the # of electrons passing a certain point over some time interval. Counting <events>.

This leads to the SR 4-Vector approach of a single temporal-spatial entity: 4-Position $\mathbf{R} = R^\mu = (ct,\mathbf{r})$ = (c*time,**3-position**) [m]

And that leads to a more precise notion of measurement as the counting of individual SpaceTime <events>, with the 4-Position **R** being the most basic "affine" property of an <event>, and 4-Displacement Δ**R** the "vector" interval between any two <events>.

4D-Tensor Theory of Measurements (SR Poincaré Invariance = Lorentz $\Lambda^{\mu'}{}_\nu$ Invariance + SpaceTime Translation $\Delta X^{\mu'}$ Invariance):

SR 4-Vectors have a Poincaré Group 4D linear mapping {technically a linear-affine transformation due to the additive constant} $(V^{\mu'} = \Lambda^{\mu'}{}_\nu V^\nu + \Delta V[\Delta X^{\mu'}])$ which preserves interval-magnitude: $(V^{\mu'}V_{\mu'} = V^\nu V_\nu = \mathbf{V}\cdot\mathbf{V})$, which is a 4-Scalar calculated from 4-Vectors.

This idea basically says the following:

A measurement made in one reference frame has an affine relationship to the same measurement made in a different reference frame.

In other words, it is a linear relation (a) with a possible additive constant (b) mapping on (**X**): **X**'=a**X**+b {the eqn. of a line}

This means that there are certain transformations:symmetries that one can do and still get the same invariant measurement interval.

Active SR Transformations (the system being measured is changed in a certain way, the reference coordinate frame is not changed):

Rotate the system of objects. Ex. Pick an angle:axis and rotate whole experiment by that amount. $\Lambda^{\mu'}{}_\nu \to R^{\mu'}{}_\nu\,(\theta_{\text{angle}},\hat{\mathbf{n}})$

Boost the system of objects. Ex. Have the whole experiment move uniformly on a linear track. $\Lambda^{\mu'}{}_\nu \to B^{\mu'}{}_\nu\,(\boldsymbol{\beta})$ or $B^{\mu'}{}_\nu\,(\varphi_{\text{hyperangle}},\hat{\mathbf{n}})$

Translate the system of objects in space. Ex. Move everything 3 meters to the left. $\Delta X^{\mu'} \to \Delta\mathbf{x}$

Translate the system of objects in time. Ex. Take pic of clock-rod. Wait 15 mins and then measure using pic. $\Delta X^{\mu'} \to \Delta ct$

Passive SR Transformations (the system being measured is not changed, the reference coordinate frame is changed in a certain way):

Rotate the reference coordinate frame. Ex. Rotate your measuring rods about some axis, then do measurement.

Boost the reference coordinate frame. Ex. Have the measuring rods uniformly move on a linear track, then do measurement.

Translate the reference coordinate frame in space. Ex. Move the measuring rods 5 feet south, then do measurement.

Translate the reference coordinate frame in time. Ex. Take pic of object. Wait 5 seconds, then do measurement on pic.

Note: There is a symmetry to the active:passive transforms, whether the measured object or the measuring system gets "changed".

Basis Transformations (the system being measured is not changed, the reference-coordinate-frame-basis is changed):

This is another, different type of passive transformation, as the objects being measured are not changed.

Basically, these are transforms other than Poincaré transformations. They are re-scalings of basis and alternate basis-types.

Ex. Change the particular dimensional-units, ex. [in] to [ft], but not the dimensional-unit-type, still [length].

Ex. Change from rectangular Cartesian coordinates to: cylindrical coordinates, spherical coordinates, some other ortho-normal system.

Note: The time translation really applies to both the measured object and measuring system, although taking a pic sort-of fixes it.

One could try a "Twin Paradox" setup to get an actual time difference between the object and measuring system.

Or, I suppose the setup could be at a different gravitational potential (small height difference) to effect the time difference.

Atomic clocks are getting to the technological level to be able to distinguish gravity potentials at the millimeter level...

Natural vs. artificial constants used with the physically-determined dimensional-unit-types:

1) The arbitrarily-imposed human pick of reference scale or standard, which provides an artificial constant.
Example: [length] can be measured in [ft], [m], [miles], [in], [km], [lightseconds], etc.
These tie [length] to an arbitrary-chosen physical distance, with the "standard length" typically a robust object, but still composed of a conglomeration of atoms subject to change due to temperature, pressure, humidity, various other stresses, and subject to decay, theft, damage, entropy, etc. Measurements of length are then compared against this reference "object" to get a value.

A better way to pick an arbitrary scale is to use one that is atomically-based, such as the current SI system for [time]. Although time is commonly measured in dimensional-units of [s], [min], [hr], [day], etc; the [s]=[second] is at-the-time-of-this-writing based on the hyperfine transition-frequency of Cesium-133 (Caesium-133) ($\Delta\nu_{Cs}$). As all atoms:isotopes of the same type are thought to be identical (same # of protons & # of neutrons) in all other respects, they are not subject to as many measurement alterations/errors due to environmental effects or entropy. This provides a much more accurate & precise, reproducible, portable, standard-of-reference.

In most cases, one may transform from one dimensional-unit to another of the same dimensional-unit-type by using a constant scaling-multiplier, called a conversion factor.
ex. 12 [in] = 1 [ft]. Both of these dimensional-units are of dimensional-unit-type [length].
They also can be written in dimensionless form as a unit-ratio multiplier: [12 in / 1 ft] = 1.
While these scaling multipliers are "constant", they are not "fundamental". They are the result of historical, arbitrary human choices.
Also, temperature is a bit different. It is an affine transform (T' = m*T + a) with multiplier (m) and addition (a) as opposed to just a linear transform (X' = m*X) with just multiplier (m).

2) "Fundamental constants" which are determined by nature, and are all invariant 4-Scalars=4D (0,0)-Tensors.
Some of these known natural constants are:

Light Speed (Vacuum)	(c)	[m/s]	v=c/n, with n = material index-of-refraction
Rest Mass = Invariant Mass	(m_o)	[kg]	Varies depending on particle type
EM charge (= -e for electron)	(q)	[C]	Varies depending on particle type {q=n(e/3)}
Electric Constant/Permittivity (Vacuum)	(ε_o)	[F/m = $C^2 \cdot s^2/kg \cdot m^3$]	($\varepsilon_o \mu_o$) = $1/c^2$
Magnetic Constant/Permeability (Vacuum)	(μ_o)	[H/m = $kg \cdot m/C^2$]	($\varepsilon_o \mu_o$) = $1/c^2$
Planck's Reduced = Dirac's Constant	($\hbar$)	[J·s = $kg \cdot m^2/s$ = Action]/{rad}	We shall see this in the next section…
Boltzmann's Constant	(k_B)	[J/°K = $kg \cdot m^2/°K \cdot s^2$]	See the full presentation SRQM-RoadMap
Avogadro Constant	(N_A)	[#/mol]	Semi-Fundamental: fermion count
Luminous efficacy of 540 THz Radiation	(K_{CD})	[683 lm/W]	Semi-Fundamental: boson count

Constants with (Vacuum) are considered in their non-interacting state, their effective values change with matter-interaction.
<Photon speed> varies in medium (v = c/n due to atomic interaction times), ex. going through a prism, which causes refraction effects.
Note: (c) is large, but never $\to \infty$; ($\hbar$) is small, but never $\to 0$; Always use realistic limits, ex. { $|\mathbf{v}| \ll c$ } for SR→CM, RQM→ QM

A quick aside about the, IMHO, un-naturalness of "Natural Units" set to Dimensionless-Unity=1:

Many physicists have been promoting "natural units", in which the scale of the fundamental constants is set to dimensionless-unity.
A typical argument is that the SI value of LightSpeed (c) is ridiculously large at ~3×10^8 [m/s].
Setting it to 1 [light-second/second] supposedly makes "life" easier, i.e. we don't have to use large exponents.
I refute this by noting that my height of ~5.9×10^{-9} [light seconds] is ridiculous, both numerically and in chosen dimensional-units.
While setting units to the value of unity is useful in doing numerical calculation and simulations, it obscures the physics otherwise.
The SI Units have Prefixes {milli, centi, deci, kilo, mega, giga, etc.} which can be used to alleviate the large exponent issue.
Picking units that are relatively-comparable helps in understanding measurements. "Let human size-scales be the measure of all things". *Not* using Natural units allows one to continue using the power of dimensional-analysis.

Also, if one could consistently set ALL fundamental constants to the value of 1, then there might be something "natural" about it.
However, the EM fine structure constant which is made up from other fundamental constants, does *not* equal 1.
$\alpha = (1/4\pi\varepsilon_o)(e^2/\hbar c) \sim 1/137$ [dimensionless], {in any system of units, SI, Imperial, Planck, etc.} Huge argument against "natural units'.

Now then, Plank Units. Most of them give extreme values of largeness or smallness. It is unclear whether anything physical actually occurs at these values. However, the Planck Mass is about the same as a dust particle. What "natural" significance does this have? Now I grant you that dust does seem to magically appear on floors and shelves, and I could be wrong, but I am pretty sure that it is not somehow fundamental to the construction of the universe… unless we are talking about the dust gathered to form stellar systems… hmmm… Maybe there is something to it… :) But seriously, I have yet to see anything significant about any Planck [Unit].

Origin of Dimensional-Unit-Types:

a) Two dimensional-unit-types are 4D <Time·Space> dual to one another: [time] & [length], the components of 4-Position $\mathbf{R} = (ct,\mathbf{r})$.

t = [time] is the 1D positive part of the 4D Minkowski Metric $\eta_{\mu\nu} = \eta^{\mu\nu} \rightarrow \mathrm{Diagonal}[+1,-1,-1,-1]_{(\mathrm{Cartesian})}$ in Metric Signature (+,-,-,-)
The fundamental time invariant is Rest:Proper Time ($t_o = \tau$), which depends on a particular measurement arrangement.

r = [length] is the 3D negative part of the 4D Minkowski Metric $\eta_{\mu\nu} = \eta^{\mu\nu} \rightarrow \mathrm{Diagonal}[+1,-1,-1,-1]_{(\mathrm{Cartesian})}$ in Metric Signature (+,-,-,-)
The fundamental space invariant is Rest:Proper Length ($r_o = |\mathbf{r}_o|$), which depends on a particular measurement arrangement.

SR:Minkowski SpaceTime show that these are components of some fundamental 4-Vectors=4D (1,0)-Tensors:

4-Position	$\mathbf{R} = R^\mu = (ct,\mathbf{r})$	[m] ∈ [length]	Alt. $\mathbf{X} = X^\mu = (ct,\mathbf{x})$	:	only Lorentz, not Poincaré Invariant
4-Displacement	$\Delta\mathbf{R} = \Delta R^\mu = (c\Delta t,\Delta\mathbf{r})$	[m] ∈ [length]	Finite $\Delta\mathbf{R} = \mathbf{R}_2 - \mathbf{R}_1$	:	fully Poincaré Invariant
4-Differential	$d\mathbf{R} = dR^\mu = (cdt,d\mathbf{r})$	[m] ∈ [length]	Infinitesimal $d\mathbf{R}$	:	fully Poincaré Invariant

$(\mathbf{R}\cdot\mathbf{R}) = (ct)^2 - \mathbf{r}\cdot\mathbf{r} = (ct_o)^2 = (c\tau)^2 = (i|\mathbf{r}_o|)^2$: Proper Time ($t_o = \tau$) [s] : Proper Length ($r_o = |\mathbf{r}_o|$) [m] : { conversion factor c [m/s] : $r_o = ct_o$ }

The fundamental constant (c) = LightSpeed = [length]/[time] relate these two dimensional-unit-types and allows one to Lorentz-Boost transform from one to the other. Relativistic effects show that these dimensional-units are tied directly to the 4D physical dimensions.

$(\mathbf{R}\cdot\mathbf{R}) = (ct)^2 - \mathbf{r}\cdot\mathbf{r} = (ct_o)^2 = (c\tau)^2 = (i|\mathbf{r}_o|)^2$ with dimensional-units $[\text{length}]^2$
$(\mathbf{R}\cdot\mathbf{R})/c^2 = (t)^2 - \mathbf{r}\cdot\mathbf{r}/c^2 = (t_o)^2 = (\tau)^2 = (i|\mathbf{r}_o|/c)^2$ with dimensional-units $[\text{time}]^2$

A sort of "balanced" time-space equality version would be:
$(\mathbf{R}\cdot\mathbf{R})/c = c(t)^2 - \mathbf{r}\cdot\mathbf{r}/c = c(t_o)^2 = c(\tau)^2 = (i|\mathbf{r}_o|)^2/c$ with dimensional-units [time] · [length]

"Unit"Temporal 4-Vector $\bar{\mathbf{T}} = \gamma(1,\boldsymbol{\beta})$, with Lorentz Scalar Invariant $\bar{\mathbf{T}}\cdot\bar{\mathbf{T}} = T^\mu T_\mu = \gamma^2[1^2 - \boldsymbol{\beta}\cdot\boldsymbol{\beta}] = +1$ $\quad\bar{\mathbf{T}} = \mathbf{U}/c$: $\boldsymbol{\beta} = \mathbf{v}/c$
Null 4-Vector $\bar{\mathbf{N}} \sim (\pm|\mathbf{a}|,\mathbf{a}) = a(\pm 1,\hat{\mathbf{n}})$, with Lorentz Scalar Invariant $\bar{\mathbf{N}}\cdot\bar{\mathbf{N}} = N^\mu N_\mu = a^2[1^2 - \hat{\mathbf{n}}\cdot\hat{\mathbf{n}}] = 0$
"Unit"Spatial 4-Vector $\bar{\mathbf{S}} = \gamma_{\beta\hat{n}}(\boldsymbol{\beta}\cdot\hat{\mathbf{n}},\hat{\mathbf{n}})$, with Lorentz Scalar Invariant $\bar{\mathbf{S}}\cdot\bar{\mathbf{S}} = S^\mu S_\mu = \gamma_{\beta\hat{n}}^2[(\boldsymbol{\beta}\cdot\hat{\mathbf{n}})^2 - \hat{\mathbf{n}}\cdot\hat{\mathbf{n}}] = -1$ $\quad\bar{\mathbf{T}}\cdot\bar{\mathbf{S}} = (\gamma^*\gamma_{\beta\hat{n}})[\boldsymbol{\beta}\cdot\hat{\mathbf{n}} - \boldsymbol{\beta}\cdot\hat{\mathbf{n}}] = 0$
$\bar{\mathbf{T}}\cdot\bar{\mathbf{S}} = 0 \leftrightarrow (\bar{\mathbf{T}} \perp \bar{\mathbf{S}})$

Time-like separated <Events>	Null-like separated <Events>	Space-like separated <Events>
Invariant Temporal Causality=Time-ordering	Invariant Null LightCone	Invariant Spatial Topology=Space-ordering
Moving Clock = ←|Time Dilation|→	||Invariant LightSpeed (c)||	Moving Ruler = →|Length Contraction|←
Relativity of Stationarity = non-Topological	Causal & Topological	Relativity of Simultaneity = non-Causal

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b) Two of the SI dimensional-unit-types are formed by "origin or substance of fundamental force" types: [EM charge] & [mass].
These both are related by aspects of Poincaré Symmetry: CPT Invariance and Casimir Invariance [↗],[∪] .

q = [EM charge] is the "charge" used by the ElectroMagnetic force of SR, described by U(1) Symmetry, typically measured in SI [C].
It can be generated from SR's CPT Invariance, as it is one of the Discrete Lorentz-Transforms $\Lambda^{\mu'}{}_{\nu} \rightarrow C^{\mu'}{}_{\nu}$ the combination of {C=PT} (P)arity=Space Reversal and (T)ime-Reversal Discrete Lorentz-Transforms, and is the source of NormalMatter ←☯→ AntiMatter.
The fundamental constant is Rest Charge (q_o)=(q) which varies according particle type, in discrete units of EM electron charge (e)
ex. electron (-1 e), positron (+1 e), up-quark (+2/3 e), down-quark (-1/3 e), anti-up-quark (-2/3 e), etc. {q = (n/3)*e : n = integer}

m = [mass] is the "charge" used by the Gravitational force of Einstein's GR, typically measured in SI [kg].
It can be generated from SR's Poincaré Invariance. It is from the "linear" [↗] Casimir Invariant $\mathbf{P}\cdot\mathbf{P}$ (1 of 2) of Poincaré Symmetry.
The fundamental constant is Rest Mass (m_o) which varies according particle type.
ex. Photon (RestMass=0), electron, muon, tauon, up-quark, down-quark, the various neutrinos, etc.
{This Casimir invariant may actually be inertial mass, as opposed to GR gravitational mass...}

s = [spin] is not an independent dimensional-unit-type, SI [Action = J·s = kg·m²/s], but a combo: $[\text{mass}]*[\text{length}]^2/[\text{time}]$
It can be generated from SR's Poincaré Invariance, from the "rotational" [∪] Casimir Invariant $\mathbf{W}\cdot\mathbf{W}$ (2 of 2) of Poincaré Symmetry.
Spin is the discrete quantity giving the internal "rotation" or "angular momentum" of particles.
The fundamental constant is Rest Spin (s_o) which varies according particle type, in discrete units of Dirac's/Planck's reduced (ħ)
ex. Fermions {matter-like} have spin of (n/2)*ħ, Bosons {force-like} have spin (n)*ħ : n = integer
ex. Higgs is spin-0, Electron is spin-1/2, Photon is spin-1, hypothetical Graviton is spin-2, etc.

Neglecting the weak and strong forces, all fundamental-particles and interestingly also black-holes (BH's) are totally described by:
4-Position $\mathbf{R} = R^{\mu} = (ct,\mathbf{r})$
4-Momentum $\mathbf{P} = P^{\mu} = (E/c,\mathbf{p}) = m_o\mathbf{U} = m_oc\overline{\mathbf{T}}$, which incorporates momentum direction with [mass], from linear [↗] Casimir Invariant
4-Spin $\mathbf{S} = S^{\mu} = (s^0,\mathbf{s}) = s_o\overline{\mathbf{S}}$, which incorporates an spatial axis with the [spin], from rotational [↻] Casimir Invariant
Charge q, which is directionless [EM charge], from CPT
with 4-Velocity $\mathbf{U} = U^{\mu} = \gamma(c,\mathbf{u})$: 4-"Unit"Temporal $\overline{\mathbf{T}} = \overline{T}^{\mu} = \gamma(1,\boldsymbol{\beta})$: 4-"Unit"Spatial $\overline{\mathbf{S}} = \overline{S}^{\mu} = \gamma_{\beta\hat{n}}(\boldsymbol{\beta}\cdot\hat{\mathbf{n}},\hat{\mathbf{n}})$
========================

c) Two of the dimensional-unit-types are formed by counting particle types: [amount of substance] & [luminous intensity]
It sorts them into two different categories based on their [spin], which again comes from a Casimir Invariant of the Poincaré Group.

The mole [mol] is a SI unit of [amount of substance], giving a count of fermionic matter-particles with spin (n/2)*ħ.
Avogadro's Constatnt (N_A) is a semi-fundamental constant related to [amount of substance].

The candela [cd] is a SI unit of [luminous intensity], giving a count of bosonic force-particles with spin (n)*ħ. {particularly photons}
The luminous efficacy of 540 THz radiation (K_{CD}) is a semi-fundamental constant related to [luminous intensity].

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d) One of the dimensional-unit-types is formed by the count of statistical information: [thermodynamic temperature].

The absolute temperature T is measured in [ºK], the SI unit of [thermodynamic temperature].
Boltzmann's Constant (k_B) is a fundamental constant related to [thermodynamic temperature].
Entropy $S_{Ent} = k_B \ln \Omega$: Energy ~ k_BT : Ideal Gas Law $PV = n\, k_BT$: Boltzmann factor $p_i \sim e^{(-E/k_BT)} / Z$, with Z = partition function

<u>Fundamental constants are 4-Scalar multipliers between fundamental physical 4-Vectors:</u>

Now, bringing the SR 4-Vectors & 4-Scalars back into the picture.
All of the fundamental constants are 4-Scalars=4D (0,0)-Tensors, giving natural scaling-multipliers between functions of 4-Vectors=4D (1,0)-Tensors and 4-Dyadics=4D (2,0)-Tensors.

$\mathbf{U} = (c)\overline{\mathbf{T}}$	LightSpeed (c) is between 4-Velocity $\mathbf{U} = \gamma(c,\mathbf{u})$ and 4-UnitTemporal $\overline{\mathbf{T}} = \gamma(1,\boldsymbol{\beta})$
$\mathbf{P} = (m_o)\mathbf{U} = (m_oc)\overline{\mathbf{T}}$	RestMass (m_o) is between 4-Momentum $\mathbf{P} = (E/c,\mathbf{p})$ and 4-Velocity $\mathbf{U} = \gamma(c,\mathbf{u})$
$\mathbf{S} = (s_o)\overline{\mathbf{S}}$	RestSpin (s_o) is between 4-Spin $\mathbf{S} = (s^0,\mathbf{s})$ and 4-UnitSpatial $\overline{\mathbf{S}} = \gamma_{\beta\hat{n}}(\boldsymbol{\beta}\cdot\hat{\mathbf{n}},\hat{\mathbf{n}})$: discrete unit of $(\hbar)$
$\mathbf{J} = (q)\mathbf{N}$	RestCharge (q) is between 4-ChargeFlux $\mathbf{J} = (\rho c,\mathbf{j})$ and 4-(Dust)NumberFlux $\mathbf{N} = (nc,\mathbf{n})$: discrete unit of (e)
$\mathbf{P} = (\hbar)\mathbf{K}$	Planck's Constant $(\hbar)$ is between 4-Momentum $\mathbf{P} = (E/c,\mathbf{p})$ and 4-WaveVector $\mathbf{K} = (\omega/c,\mathbf{k})$
$\partial\cdot F^{\alpha\beta} = (\mu_o)\mathbf{J}$	The Magnetic Constant (μ_o) is between Divergence of Faraday 4-Tensor $\partial\cdot F^{\alpha\beta}$ and 4-ChargeFlux $\mathbf{J} = (\rho c,\mathbf{j})$
$\mathbf{U}\cdot F^{\alpha\beta} = (1/q)\mathbf{F}$	RestCharge (q) is between Temporal Component of Faraday 4-Tensor $\mathbf{U}\cdot F^{\alpha\beta}$ and 4-Force $\mathbf{F} = \gamma(\dot{E}/c,\mathbf{f})$
$\boldsymbol{\Theta} = (1/k_BT_o)\mathbf{U}$	Boltzmann's Constant (k_B) is between 4-ThermalVector $\boldsymbol{\Theta} = (c/k_BT,\mathbf{u}/k_BT)$ and 4-Velocity $\mathbf{U} = \gamma(c,\mathbf{u})$
$\mathbf{S_{Ent}} = (k_B \ln \Omega)\mathbf{N}$	Boltzmann's Constant (k_B) is between 4-EntropyFlux $\mathbf{S_{Ent}} = (s_{Ent}c,\mathbf{s_{Ent}})$ and 4-(Dust)NumberFlux $\mathbf{N} = (nc,\mathbf{n})$
$G^{\mu\nu} = (8\pi G/c^4)T^{\mu\nu}$	Gravitational Constant (G), between 4D (2,0)-Tensors: The Einstein Tensor $G^{\mu\nu}$ & Stress-Energy Tensor $T^{\mu\nu}$

It turns out that the Electric (ε_o) and Magnetic (μ_o) Constants are related and not independent: $(\varepsilon_o\mu_o) = 1/c^2$
Note: Faraday EM 4-Tensor $F^{\alpha\beta} = \partial^{\alpha}A^{\beta} - \partial^{\beta}A^{\alpha} = (\partial\wedge\mathbf{A})$ $F^{\alpha\beta}$ = AntiSymmetric 4D-(2,0)-Tensor

That these fundamental constants are all 4-Scalars mean that all inertial observers must measure the same value for these constants.
Unlike the human-made arbitrary units and their constant multipliers, which are only between the same dimensional-unit-types, these natural-constant-multipliers add and allow transformations between different combinations of dimensional-unit-types.

<u>Arbitrary/Artificial/Human-Imposed:</u>
ex. [12 in / 1 ft] = 1 : transforms only between [length] & [length]
ex. [60 s / 1 min] = 1 : transforms only between [time] & [time]

<u>Physical/Natural/Fundamental:</u>
ex. (c) transforms between [time] & [length]
ex. (m_o) allows:generates the dimensional-unit-type of [mass]
ex. (q) allows:generates the dimensional-unit-type of [EM charge]
ex. $(\hbar)$ transforms between [mass]·[length] / [time] & {angle} / [length]
ex. (k_B) transforms between [mass]·[length]^2 / [time]^2 & [thermodynamic temperature]

Argument against VLS {Variable LightSpeed $c = f(t,\mathbf{x})$} theories:

4-Position $\mathbf{X} = X^\mu = (ct,\mathbf{x})$

4-Differential $d\mathbf{X} = dX^\mu = (cdt,d\mathbf{x})$

4-Velocity $\mathbf{U} = U^\mu = \gamma(c,\mathbf{u})$

$cd\tau = \sqrt{[g_{\mu\nu}dX^\mu dX^\nu]} = \sqrt{[g_{\mu\nu}(dX^\mu/dt)(dX^\nu/dt)]}dt = \sqrt{[g_{\mu\nu}(dX^\mu/d\tau)(dX^\nu/d\tau)]}d\tau = \sqrt{[g_{\mu\nu}U^\mu U^\nu]}d\tau = \sqrt{[\mathbf{U}\cdot\mathbf{U}]}d\tau$

==

For the Minkowski "Flat" SpaceTime Metric $\eta_{\mu\nu} = \mathrm{Diag}[1,-1,-1,-1]$

$\mathbf{U}\cdot\mathbf{U} = U^\mu\eta_{\mu\nu}U^\nu = \gamma^2[c^2 - \mathbf{u}\cdot\mathbf{u}] = (c^2)$ since $\gamma = 1/\sqrt{[c^2 - \mathbf{u}\cdot\mathbf{u}]}$, which is the normal definition of the Lorentz Gamma Factor

==

For the GR "Weak-Field" Metric:

GR "Weak-Field" limiting-case… or alternately viewed as a small perturbation field ($h_{\mu\nu}$) on the SR Minkowski Metric ($\eta_{\mu\nu}$):

SpaceTime Metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$: $\eta_{\mu\nu} = \mathrm{Diag}[1,-1,-1,-1]$: $h_{\mu\nu} = (2\varphi/c^2)\delta_{\mu\nu}$: $g_{00} = (1+2\varphi/c^2)$ & $g_{ii} = (-1+2\varphi/c^2)$: Gravity potential (φ)

$cd\tau = \sqrt{[g_{\mu\nu}dX^\mu dX^\nu]} = \sqrt{[g_{\mu\nu}(dX^\mu/dt)(dX^\nu/dt)]}dt = \sqrt{[(1+2\varphi/c^2)c^2+(-1+2\varphi/c^2)\mathbf{u}\cdot\mathbf{u}]}dt = \sqrt{[c^2+2\varphi-\mathbf{u}\cdot\mathbf{u}+2\varphi\mathbf{u}\cdot\mathbf{u}/c^2]}dt$

$d\tau = \sqrt{[1+2\varphi/c^2-\mathbf{u}\cdot\mathbf{u}/c^2+2\varphi\mathbf{u}\cdot\mathbf{u}/c^4]}dt \sim \sqrt{[1+2\varphi/c^2-\mathbf{u}\cdot\mathbf{u}/c^2+\{0...\}]}dt = dt/\gamma_{WeakGrav}$ Assume $O[1/c^4]$ factor ~ {0...}

So, "Weak-Gravity" Lorentz Factor $\gamma_{WeakGrav} = 1/\sqrt{[1+2\varphi/c^2-\mathbf{u}\cdot\mathbf{u}/c^2]}$ and the Metric effectively is: $g_{00} = (1+2\varphi/c^2)$ & $g_{ii} \to (-1)$

$\Delta\tau = 1/\gamma_{WeakGrav}\,\Delta t = \sqrt{[1+2\varphi/c^2-\mathbf{u}\cdot\mathbf{u}/c^2]}\Delta t$: Aging rate slows down lower in gravity potential (φ) and at higher velocity ($\mathbf{u}$)

$\mathbf{U}\cdot\mathbf{U} = g_{\mu\nu}U^\mu U^\nu = (1+2\varphi/c^2)(u^0)^2 + (-1)(u^i\cdot u^j) = \gamma^2[(1+2\varphi/c^2)c^2 + (-1)\mathbf{u}\cdot\mathbf{u}] = \gamma^2[(c^2+2\varphi c^2/c^2)-(\mathbf{u}\cdot\mathbf{u})] = \gamma^2[c^2+2\varphi-\mathbf{u}\cdot\mathbf{u}]$

$= \gamma^2c^2[1+2\varphi/c^2-\mathbf{u}\cdot\mathbf{u}/c^2] = (c^2)$ since $\gamma \to \gamma_{WeakGrav}$ for this metric, a "Weak-Field" Lorentz Gamma Factor

==

In both cases, one can view {$\mathbf{U}\cdot\mathbf{U} = c^2$} as a constraint equation for the variable $\mathbf{U}$.

If (c) is a constant, then it takes the normally 4 independent components of a 4-Vector down to only 3 for the 4-Velocity $\mathbf{U}$.

Other 4-Vectors (with 4 independent components) can then be formed by multiplying a 4-Scalar times the 4-Velocity $\mathbf{U}$.

4-Momentum $\mathbf{P} = (mc,\mathbf{p}) = m_o\mathbf{U} = m_o\gamma(c,\mathbf{u}) = m_o$ * 4-Velocity $\mathbf{U}$

This 4-Vector has 4 independent components, {1 for m and 3 for $\mathbf{p}$} or {1 for m_o and 3 for $\mathbf{U}$}, with the (c) being a constant.

$\mathbf{P}\cdot\mathbf{P} = (m_o)^2\mathbf{U}\cdot\mathbf{U} = (m_oc)^2$

The constraint given by the Lorentz Scalar Product of $\mathbf{P}$ has a variable m_o, so $\mathbf{P}$ still has 4 independent components.

If, however, c is not constant, then 4-Momentum $\mathbf{P}$ would have 5 independent components, {1 for c, 1 for m and 3 for $\mathbf{p}$}

This would essentially break tensor rules, as one is trying to put 2 variables (c & m) in a slot only built for one (the temporal).

Also, this phenomenon is not experimentally observed. All the 4-Vectors have maximum limit of 4 independent components.

There are many physical 4-Vectors made by multiplying the 4-Velocity with different 4-Scalars

4-Momentum	$\mathbf{P} = P^\mu = (E/c,\mathbf{p}) = (E_o/c^2)\mathbf{U} = m_o\mathbf{U}$
4-WaveVector	$\mathbf{K} = K^\mu = (\omega/c,\mathbf{k}) = (\omega_o/c^2)\mathbf{U}$
4-(Dust)NumberFlux	$\mathbf{N} = N^\mu = (nc,\mathbf{n}) = (n_o)\mathbf{U}$
4-Current(Density)=4-ChargeFlux	$\mathbf{J} = J^\mu = (\rho c,\mathbf{j}) = (\rho_o)\mathbf{U}$
4-(EM)VectorPotential	$\mathbf{A} = A^\mu = (\varphi/c,\mathbf{a}) = (\varphi_o/c^2)\mathbf{U}$
4-ThermalVector	$\boldsymbol{\Theta} = \Theta^\mu = (c/k_BT,\boldsymbol{\theta}) = (1/k_BT_o)\mathbf{U}$

etc. These are all "Temporal" 4-Vectors, as 4-Velocity $\mathbf{U} = c\mathbf{T} = c$ * 4-"Unit"Temporal $= c\gamma(1,\boldsymbol{\beta}) = \gamma(c,\mathbf{u})$

One also has the same trick with a "Spatial" 4-Vector $\mathbf{S} = \gamma_{\beta\hat{n}}(\boldsymbol{\beta}\cdot\hat{\mathbf{n}},\hat{\mathbf{n}})$.

The "Unit"Spatial 4-Vector $\mathbf{S}$ has 3 independent components.

There is a 4-Scalar RestSpin (s_o), 1 independent component.

4-Spin $\mathbf{S} = S^\mu = (s^0,\mathbf{s}) = (s_o)\mathbf{S}$, has 4 independent components.

Aside: $\mathbf{P}\cdot\boldsymbol{\Theta} = m_o\mathbf{U}\cdot(1/k_BT_o)\mathbf{U} = (m_o/k_BT_o)\mathbf{U}\cdot\mathbf{U} = (m_o/k_BT_o)(c^2) = (E_o/k_BT_o) = (E/c*c/k_BT - \mathbf{p}\cdot\boldsymbol{\theta}) = (E / k_BT - \mathbf{p}\cdot\boldsymbol{\theta})$

which gives the (E / k_BT) factor seen in particle distribution functions, which are invariant counting operations.

Ideal Gas Law in Manifestly-Invariant-Tensor-Form

$(p_o)\boldsymbol{\Theta} = \mathbf{N}$ $[(N/m^2)\cdot(1/N\cdot s) = (\#/m^2\cdot s)]$ The Lorentz Scalar Pressure * 4-ThermalVector = 4-"Dust"NumberFlux

$(p_o)(1/[k_BT_o])\mathbf{U} = (n_o)\mathbf{U}$

$(p_o/[k_BT_o]) = (n_o)$

$(p_o/[k_BT_o]) = (N_o/V_o)$

$p_oV_o = N_ok_BT_o$

Tensor Types:
This idea is mostly based on the tensor argument that there are only so many independent components that fit in a given tensor.

4-Scalar S: a 4D (0,0)-Tensor has at most ($4^0 = 1$) independent component. S can be a function of spacetime, such as $S = S(t,\mathbf{x})$, meaning that it's value at different spacetime <events> can vary, but all inertial observers must measure the same value at each of those 4D points.

4-Vector $\mathbf{V} = V^\mu$: a 4D (1,0)-Tensor has at most ($4^1 = 4$) independent components. V^μ could be $V^\mu(t,\mathbf{x})$. Each component could be a function of time and space. But, the values are constrained; the Lorentz Scalar Product $V^\mu \cdot V^\nu$ must be an invariant scalar. When building the various physical 4-Vectors, one notices that they all have 4 independent components except for the 4-"Unit"Temporal $\bar{\mathbf{T}}$, the 4-Velocity $\mathbf{U}$, and the 4-"Unit"Spatial $\bar{\mathbf{S}}$, which only have 3 independent components. Other physical 4-Vectors are then created by multiplying by a Lorentz 4-Scalar (1 component). (3+1=4). So 4-Momentum, 4-WaveVector, 4-VectorPotential, 4-Acceleration, 4-Force, 4-Position, 4-Gradient etc. all have 4 independent components.

4-Tensor $\boldsymbol{T} = T^{\mu\nu}$: a 4D (2,0)-Tensor has at most ($4^2 = 16$) independent components. These can be decomposed into symmetric and anti-symmetric parts in an relativistically invariant way. Symmetric 4-Tensors have at most 10 independent components (ex. Stress-Energy Tensor), Anti-Symmetric 4-Tensors have at most 6 independent components (ex. Faraday EM Tensor, 4-Angular-Momentum Tensor). For a long time I was confused by the Faraday tensor $F^{\mu\nu} = \partial \wedge \mathbf{A}$ because it is made from the 4-Gradient ∂ and the 4-VectorPotential $\mathbf{A}$. (4+4=8) components.
However, there are two EM invariants: ($F^{\mu\nu}F_{\mu\nu} = 2\{(\mathbf{b}\cdot\mathbf{b})-(\varepsilon_o\mu_o\mathbf{e}\cdot\mathbf{e})\}$) & ($\mathrm{Det}[F^{\mu\nu}] = \{(\mathbf{e}\cdot\mathbf{b})/c\}^2$),
which act as constraint equations, removing 2 of the 8 components, giving just 6 as the Anti-Symmetric tensor $F^{\mu\nu}$ must have.
Likewise, the 4-Angular-Momentum Tensor $M^{\mu\nu} = \mathbf{R} \wedge \mathbf{P}$ (4+4=8) has 2 invariants, again dropping the number of components to 6.
Thinking along the same line, the 10 components of the Symmetric Stress energy tensor are matched by the 10 Conservation Laws known from Poincare symmetry. This is also matched by the 6 components of the Anti-Symmetric 4-AngularMomentum $M^{\mu\nu}$ + the 4 components of the 4-LinearMomentum P^μ.

Building SpaceTime:

"Unit"Temporal 4-Vector $\bar{\mathbf{T}} = \gamma(1,\boldsymbol{\beta})$, with Lorentz Scalar Invariant $\bar{\mathbf{T}}\cdot\bar{\mathbf{T}} = T^\mu T_\mu = \gamma^2[1^2 - \boldsymbol{\beta}\cdot\boldsymbol{\beta}] = +1$ $\quad \bar{\mathbf{T}} = \mathbf{U}/c$
Null 4-Vector $\bar{\mathbf{N}} \sim (\pm|\mathbf{a}|,\mathbf{a}) = a(\pm 1,\hat{\mathbf{n}})$, with Lorentz Scalar Invariant $\bar{\mathbf{N}}\cdot\bar{\mathbf{N}} = N^\mu N_\mu = a^2[1^2 - \hat{\mathbf{n}}\cdot\hat{\mathbf{n}}] = 0$
"Unit"Spatial 4-Vector $\bar{\mathbf{S}} = \gamma_{\beta\hat{n}}(\boldsymbol{\beta}\cdot\hat{\mathbf{n}},\hat{\mathbf{n}})$, with Lorentz Scalar Invariant $\bar{\mathbf{S}}\cdot\bar{\mathbf{S}} = S^\mu S_\mu = \gamma_{\beta\hat{n}}^2[(\boldsymbol{\beta}\cdot\hat{\mathbf{n}})^2 - \hat{\mathbf{n}}\cdot\hat{\mathbf{n}}] = -1$ $\quad \bar{\mathbf{T}}\cdot\bar{\mathbf{S}} = (\gamma^*\gamma_{\beta\hat{n}})[\boldsymbol{\beta}\cdot\hat{\mathbf{n}} - \boldsymbol{\beta}\cdot\hat{\mathbf{n}}] = 0$
$\bar{\mathbf{T}}\cdot\bar{\mathbf{S}} = 0 \leftrightarrow (\bar{\mathbf{T}} \perp \bar{\mathbf{S}})$

So, one can think of SpaceTime as being built from the primitive 4-Vectors:
4-"Unit"Temporal $\bar{\mathbf{T}} = \gamma(1,\boldsymbol{\beta})$
4-"Unit"Spatial $\bar{\mathbf{S}} = \gamma_{\beta\hat{n}}(\boldsymbol{\beta}\cdot\hat{\mathbf{n}},\hat{\mathbf{n}})$
and the various 4-Scalar Invariants which Substantiate these <events>.

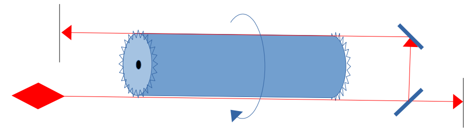
Further Study:

Dimensional Analysis
Directed Dimensions
Orientational Analysis
Buckingham π-Theorem
Dimensionless Parameters
Dimensional Equivalences
Affine vs. Vector Quantities, Position vs. Displacement
Fundamental Coldness (& possible negative temperature) Thermodynamic Beta β vs Temperature T

Invariant Energy equivalents:

Energy E_o	Fundamental Constant	Particle/Field Dependent Invariant
$= c^2 m_o$	LightSpeed (c)	Rest Mass (m_o)
$= \hbar\omega_o$	Dirac Constant ($\hbar$)	Rest Angular Frequency of Wave (ω_o)
$= k_B T_o$	Boltzmann Constant (k_B)	Rest Temperature (T_o)
$= q\varphi_o$	Elementary Charge (e)	Charge ($q=n^*e/3$) & Rest Scalar Potential (φ_o)
$= -GMm/r$	Gravitational Constant (G)	Mass & Length
$= (1/2)n_o[\varepsilon_o e^2 + \mu_o b^2]$	Electric , Magnetic Constants (ε_o,μ_o)	EM field ($\mathbf{e},\mathbf{b}$) from Faraday Tensor, #Density (n_o)

Measuring Both One-way LightSpeeds (c) with single Rod-Clock = (Spinning Rotor):
Laser Beam from left-to-right, through rotor, split, mirror, back thru rotor right-to-left, velocities = v_{beam} to be determined



Rotor cylinder:
length: L = End - Start
radius: R
rotation rate: ω

Start with the simple case of a single set of paired notches at Start and End of Rotor:
Rotor Start notch angle = θ_s
Rotor End notch angle = θ_e

Shine a laser parallel to rotor length such that it is blocked by a disc unless it goes through a notch.
If the laser passes both notches (start & end), it will shine (creating a dot) on a screen/detector beyond rotor.
If there is a light on screen during rotation, then:
Rotor length (L) is the Rod: standard of [length]
Rotor angular velocity (ω) is the Clock: standard of [time]

Rotor cylinder rotates at constant angular velocity ω
Angular Position of Rotor at time t: $\theta = \omega t$

Beam packet position along Rotor: $x = v_{beam} * t$
Beam packet begins at Start Disk: T = 0
Beam packet reaches End Disc: $T = L / v_{beam}$
Time that End Rotor Notch Position = Start Rotor Notch Position: $T = (\theta_e - \theta_s) / \omega$

Solve: $(\theta_e - \theta_s) / \omega = L / v_{beam}$
$v_{beam} = L\omega / (\theta_e - \theta_s)$

Ideally, we want the notches to be at the same angular position $(\theta_e - \theta_s)=2\pi$, so that a full rotation is required.
This way, a stationary Rotor passes the light beam, and there is no complication of measuring notch positions.
In the ideal case, in which the Rotor makes one complete 2π rotation during the light beam down its length:
$v_{beam} = L\omega / 2\pi$
$v_{rotation} = \omega R$, with R the radius of the Rotor and $v_{rotation}$ the max outer edge speed

We want $v_{beam} \gg v_{rotation}$, so that relativistic effects of rotation are minimized.
$L\omega / 2\pi \gg \omega R$
$L / 2\pi \gg R$
$L \gg 2\pi R = C$
We want the Length of the Rotator ≫ Circumference of the Rotator

Now, the technological complications:
To get best balance of Rotor, and the least acceleration issues, use N equally-spaced notches instead of a single notch on both ends.
This also reduces the necessary angular velocity to something more technologically feasible.
Let all notch-pairs line up during during zero rotation, so again, no "measurement" required.
Then, $v_{beam} = L\omega / (\theta_{n+1} - \theta_n) = NL\omega / 2\pi$

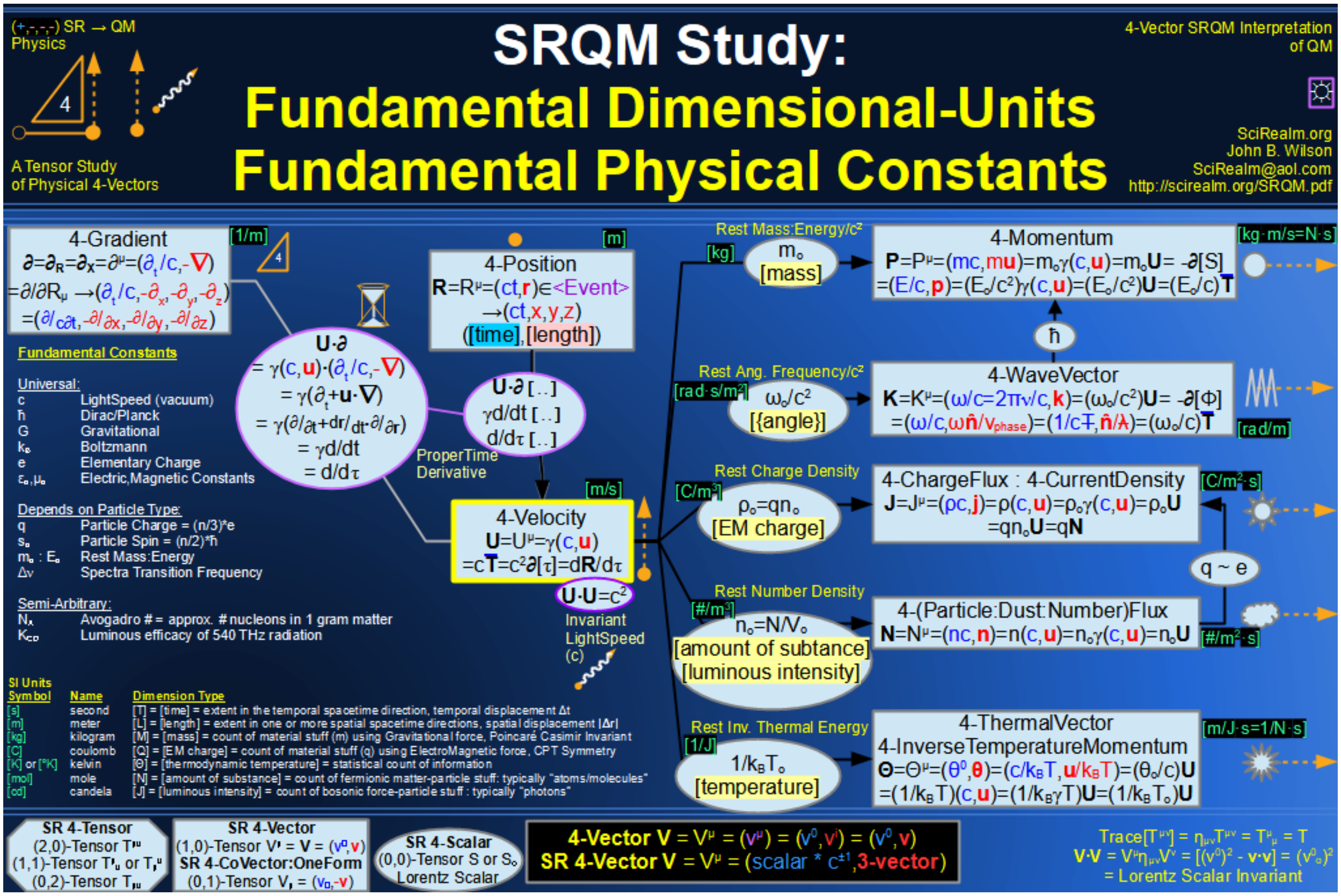
Now, let's add in another aspect, a return beam.
By adding in a beam-splitter and mirror, we should be able to arrange the setup so that a right-to-left beam also goes along the Rotor. This beam would be the from the original that made it left-to-right thru Start notch (n) and End notch (n+1). After bouncing off a splitter and mirror, one can arrange for the return beam to go through End notch (m) and Start notch (m+1). If one is able to simply get a dot on the Start side detector, then the velocity of the return beam should match the velocity of the initial beam. The exact timing of when a dot appears on the screen is not important and doesn't have to be measured. It is simply the case that if one gets dots on both screens that the one-way speeds are equivalent and isotropy is the case. Then, of course, the whole experimental setup can be rotated, translated, etc. to prove full 3D or 4D isotropy.

Alternately, one might just use a set of several lasers, some LtR and some RtL. If dots appear on all detectors, then isotropy is the case.

Realistic values of experiment, based on technological aspects:
Smallest Drilled Holes ~ 6 [microns], or $6*10^{-6}$ [m]
https://phys.org/news/2005-11-precision-breakthrough-world-smallest-hole.html
Fastest Rotor ~ $1*10^{6}$ [rpm]
https://www.livescience.com/3075-spin-record-set-1-million-rpm.html
https://www.celeroton.com/en/technology/magnetic-bearings.html
https://www.celeroton.com/fileadmin/user_upload/technologie/tech-blog/Design_of_ultra-high_speed_optical_beam_chopper_rotors.pdf

L ~ 1 [dm] = 0.1 [m]
$\omega \sim 10^{6}$ [rpm] = 104719.755 [rad/sec] = (10^{6} rpm)(2π rad/1 rev)(1 min/60 sec) ~ 10^{5} [rad/sec]
So, $L\omega \sim 10^{4}$ [rad·m/sec]
But, $v_{beam} = c \sim 3 * 10^{8}$ [m/sec] is very fast
θ_e has to be very close to θ_s : $(\theta_e - \theta_s) \sim 10^{-4}$ [rad] ~ 10^{-3} [deg]
However, measuring angular displacement to this accuracy is technologically feasible.

Chart:


SRQM Study:
Fundamental Dimensional-Units
Fundamental Physical Constants
(+,-,-,-) SR → QM Physics
A Tensor Study of Physical 4-Vectors
4-Vector SRQM Interpretation of QM
SciRealm.org
John B. Wilson
SciRealm@aol.com
http://scirealm.org/SRQM.pdf
4-Gradient
4-Position
4-Velocity
4-Momentum
4-WaveVector
4-ChargeFlux : 4-CurrentDensity
4-(Particle:Dust:Number)Flux
4-ThermalVector
4-InverseTemperatureMomentum
Rest Mass:Energy/c²
Rest Ang. Frequency/c²
Rest Charge Density
Rest Number Density
Rest Inv. Thermal Energy
ProperTime Derivative
Invariant LightSpeed (c)
Fundamental Constants
Universal:
c LightSpeed (vacuum)
ħ Dirac/Planck
G Gravitational
k_B Boltzmann
e Elementary Charge
ε₀,μ₀ Electric,Magnetic Constants
Depends on Particle Type:
q Particle Charge = (n/3)*e
s₀ Particle Spin = (n/2)*ħ
m₀ : E₀ Rest Mass:Energy
Δν Spectra Transition Frequency
Semi-Arbitrary:
N_A Avogadro # = approx. # nucleons in 1 gram matter
K_cd Luminous efficacy of 540 THz radiation
SI Units
Symbol Name Dimension Type
[s] second [T] = [time] = extent in the temporal spacetime direction, temporal displacement Δt
[m] meter [L] = [length] = extent in one or more spatial spacetime directions, spatial displacement |Δr|
[kg] kilogram [M] = [mass] = count of material stuff (m) using Gravitational force, Poincaré Casimir Invariant
[C] coulomb [Q] = [EM charge] = count of material stuff (q) using ElectroMagnetic force, CPT Symmetry
[K] or [°K] kelvin [Θ] = [thermodynamic temperature] = statistical count of information
[mol] mole [N] = [amount of substance] = count of fermionic matter-particle stuff: typically "atoms/molecules"
[cd] candela [J] = [luminous intensity] = count of bosonic force-particle stuff: typically "photons"
SR 4-Tensor
SR 4-Vector
SR 4-CoVector:OneForm
SR 4-Scalar
Lorentz Scalar
4-Vector V = V^μ = (v^μ) = (v⁰,v^i) = (v⁰,v)
SR 4-Vector V = V^μ = (scalar * c^±1, 3-vector)
= Lorentz Scalar Invariant